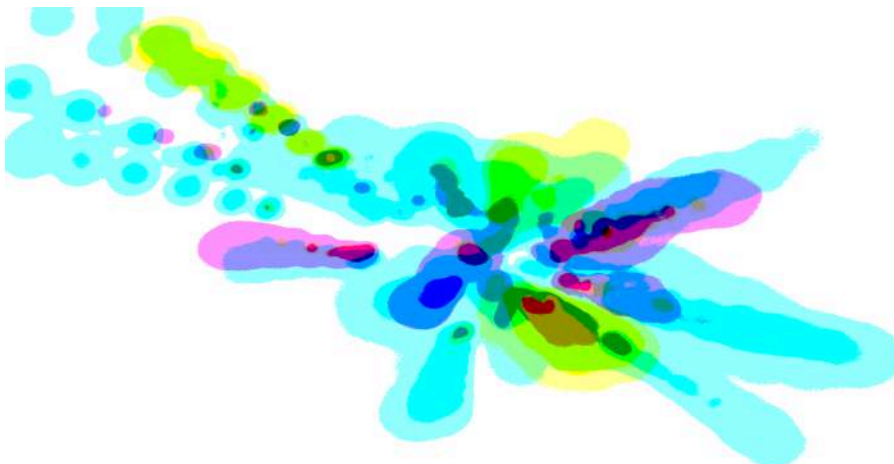
Separating fact from fantasy*: The chiral anomaly and the proton spin puzzle



Raju Venugopalan
(BNL)

Arizona State Online Theory Colloquium, August 19, 2020

Separating fact from fantasy*: The chiral anomaly and the proton spin puzzle

R. L. Jaffe

A. Manohar

30th anniversary of their seminal paper which strongly inspired the work reported here:

The G(1) Problem: Fact and Fantasy on the Spin of the Proton.
Nucl. Phys., B337:509–546, 1990

Work* in collaboration with Andrey Tarasov (The OSU and CFNS)

* Our first paper will be on arXiv tonight (6 pm ASU time) and two others are in preparation

Talk outline

- Polarized DIS – preliminaries

- Iso-singlet axial vector current and topology of QCD vacuum: the role of the triangle graph

- Fun with world-lines

- World-line effective action: i) Finding a triangle in a box in Bjorken and Regge asymptotics
  ii) Hidden triangles dress infrared poles; the WZW term
  iii) Small x_{Bj} effective action for polarized DIS:
  Extended PCAC and the $\eta^{'}$ as an emergent axion-like field
  iv) QCD evolution of the small x effective action

- Conjecture: How spin diffuses from polarized large x partons to small x via sphalerons transitions

- General considerations/outlook

On the DOE web page: January 9, 2020

U.S. Department of Energy Selects Brookhaven National Laboratory to Host Major New Nuclear Physics Facility

WASHINGTON, D.C. – Today, the **U.S. Department of Energy (DOE)** announced the selection of Brookhaven National Laboratory in Upton, NY, as the site for a planned major new nuclear physics research facility.

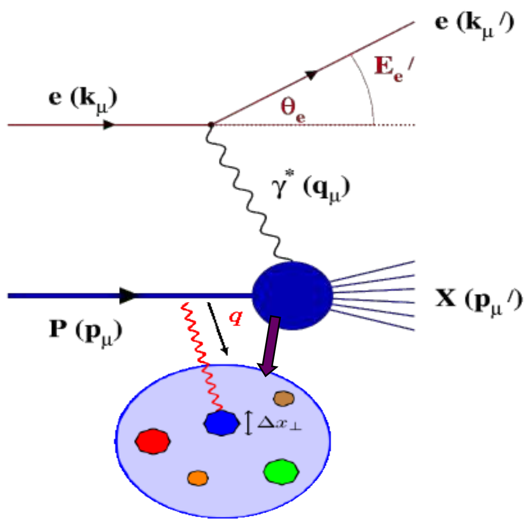
The Electron Ion Collider (EIC), to be designed and constructed over ten years at an estimated cost between $1.6 and $2.6 billion, will smash electrons into protons and heavier atomic nuclei in an effort to penetrate the mysteries of the “strong force” that binds the atomic nucleus together.

“The EIC promises to keep America in the forefront of nuclear physics research and particle accelerator technology, critical components of overall U.S. leadership in science,” said **U.S. Secretary of Energy Dan Brouillette**. “This facility will deepen our understanding of nature and is expected to be the source of insights ultimately leading to new technology and innovation.”
“

Secretary Brouillette also approved Critical Decision-0, “Approve Mission Need,” for the EIC on December 19, 2019

What’s the Electron-Ion Collider ?

The deeply inelastic scattering (DIS) femtoscope



$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

Measure of resolution power

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

Bjorken variable: Measure of momentum fraction of struck quark

Inclusive DIS measurements: e+↑p/A → e'+X
Measure only the scattered lepton in the detector: structure functions (pdfs: F2,FL,g1,g2,…)

Semi-inclusive DIS (SIDIS) measurements: **e+↑p/A → e'+h(p,K,p,jet)+X**
Measure electrons in coincidence with identified hadrons/jets (Transverse momentum dependent dists. -TMDs)

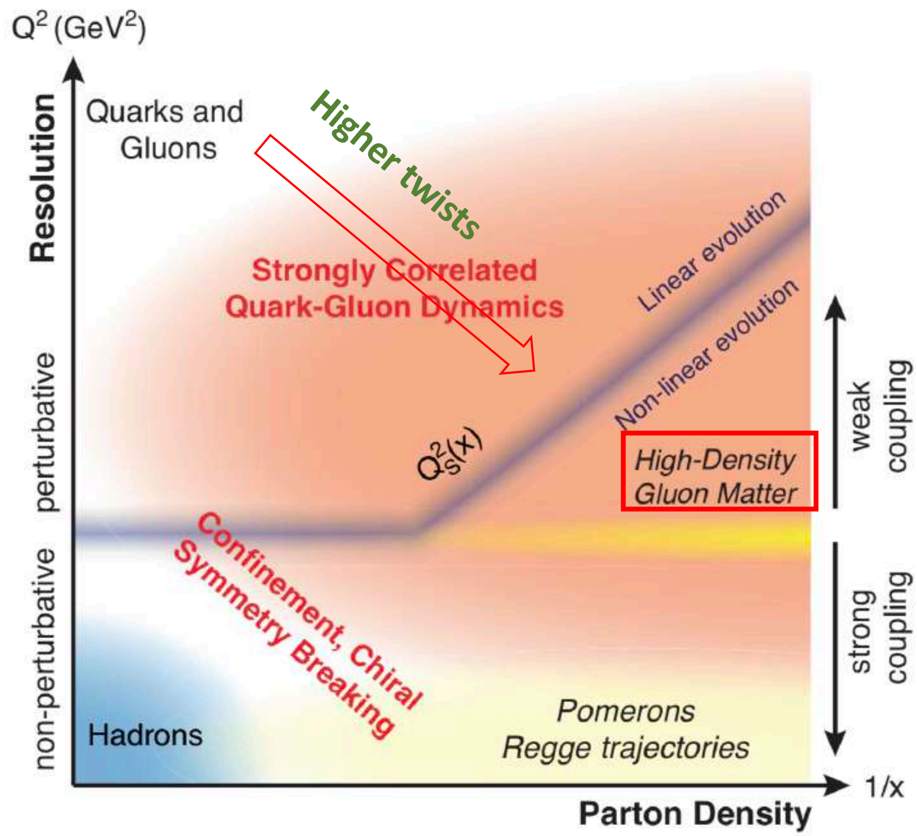
Exclusive measurements (DVCS,…): e+↑p/A → e'+ photon/hadron + p/A
Measure electron, photon (or e.g., vector meson), intact proton/nucleus (Generalized parton dists. -GPDs)

Diffractive measurements: **e+↑p/A → e'+ hadrons/jets + rapidity gap (coherent diff.→ intact p/A)**
As for exclusive, but with rapidity veto on particle production (Diff. structure functions, F_2^D, F_L^D)

Also, measure nuclear fragments, multiparticle correlations, …

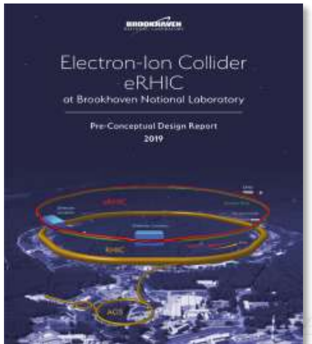
Landscape of scattering in the strong interaction

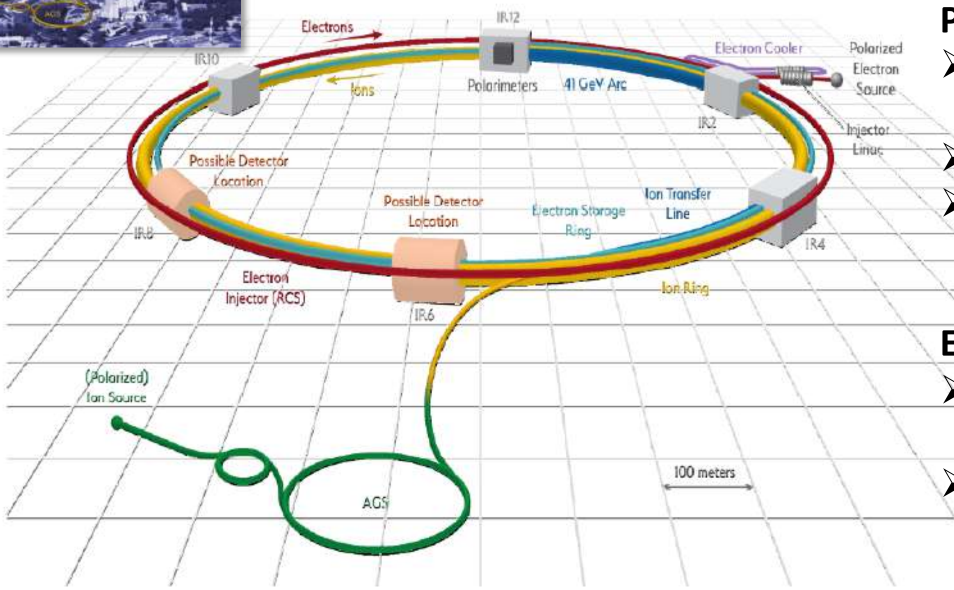
Aschenauer et al., arXiv:1708.01527
Rep.Prog. Phys. 82, 024301 (2019)



Many open questions: three-D structure, spin and orbital dynamics, many-body correlations, small x

The Electron-Ion Collider





- ❖ Electron storage ring with frequent injection of fresh polarized electron bunches
- ❖ Hadron storage ring with strong cooling or frequent injection of hadron bunches

Polarized protons up to 275 GeV; Nuclei up to ~ Z/A*275 GeV/n

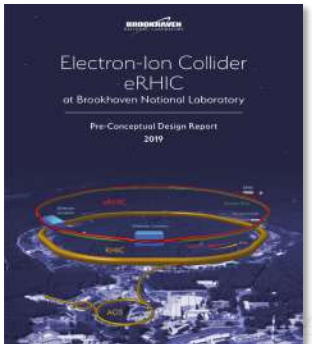
- ➢ Existing RHIC complex: Storage (Yellow), injectors (source, booster, AGS)
- ➢ Need few modifications
- ➢ RHIC beam parameters fairly close to those required for EIC@BNL

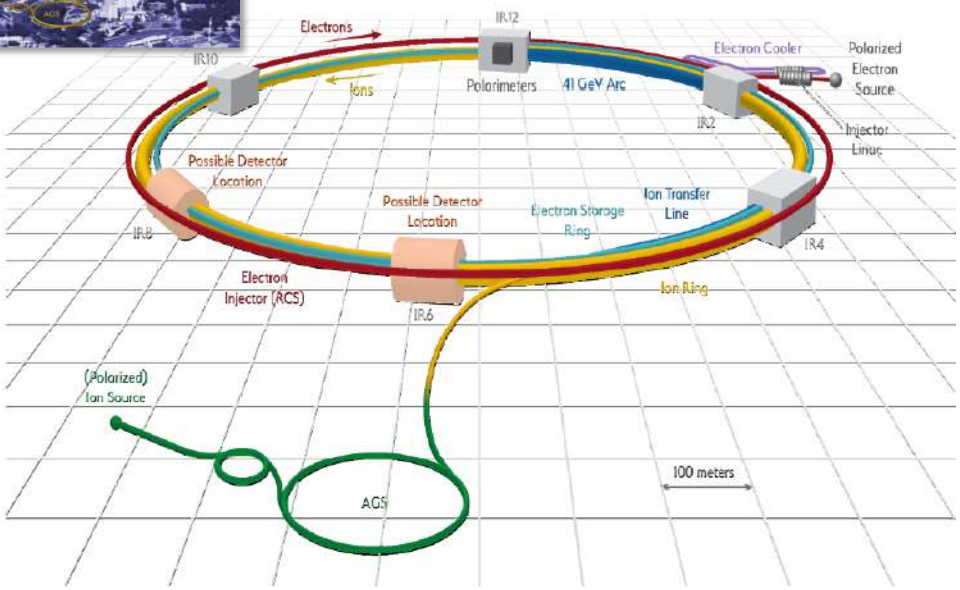
Electrons up to 18 GeV

- ➢ Storage ring, provides the range sqrt(s) = 20-140 GeV. Beam current limited by RF power of 10 MW
- ➢ Electron beam with variable spin pattern (s) accelerated in on-energy, spin transparent injector (Rapid-Cycling-Synchrotron) with 1-2 Hz cycle frequency
- ➢ Polarized e-source and a 400 MeV s-band injector LINAC in the existing tunnel

Design optimized to reach 10^{34} $cm^{-2}sec^{-1}$

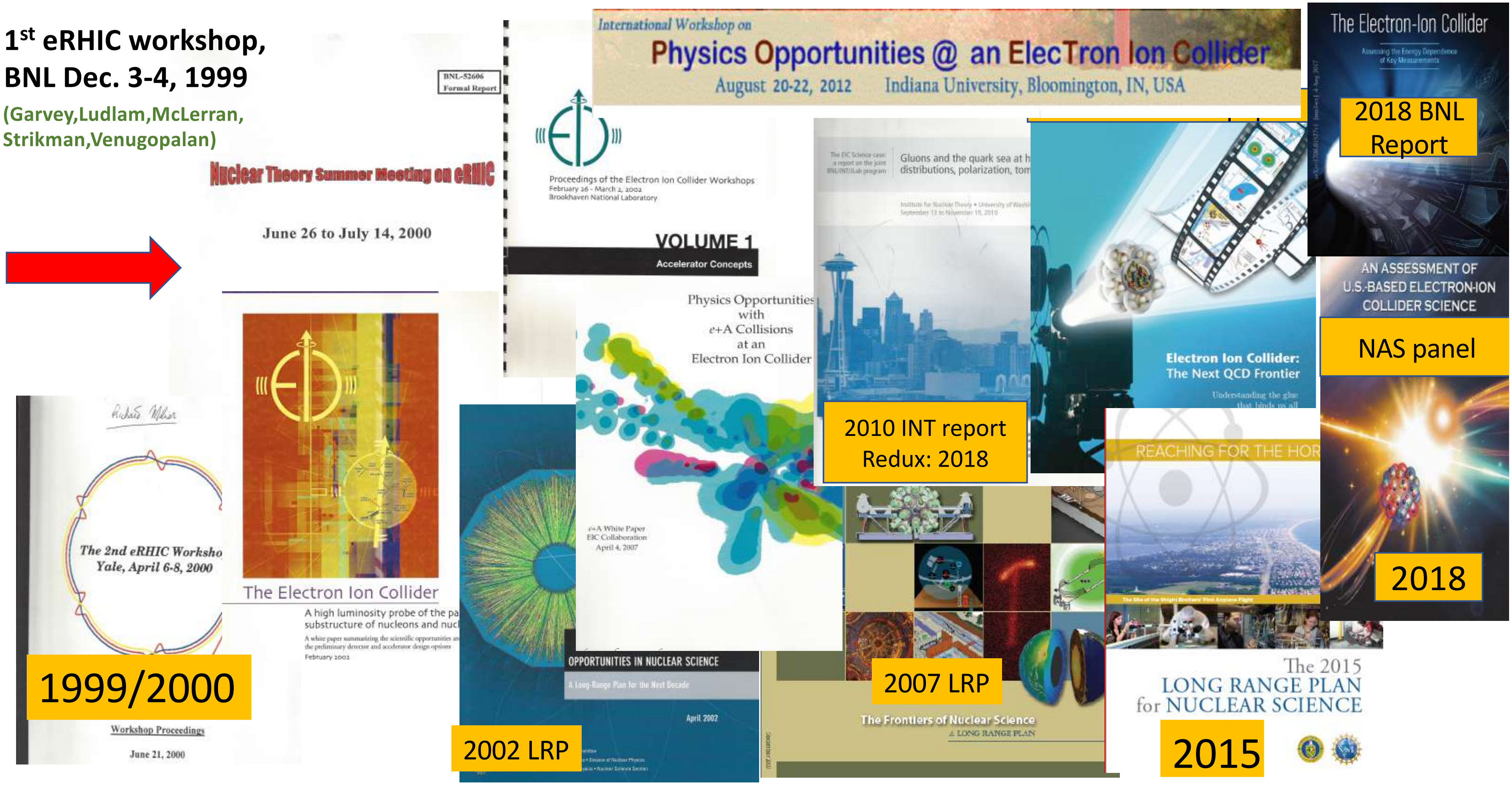
The Electron-Ion Collider





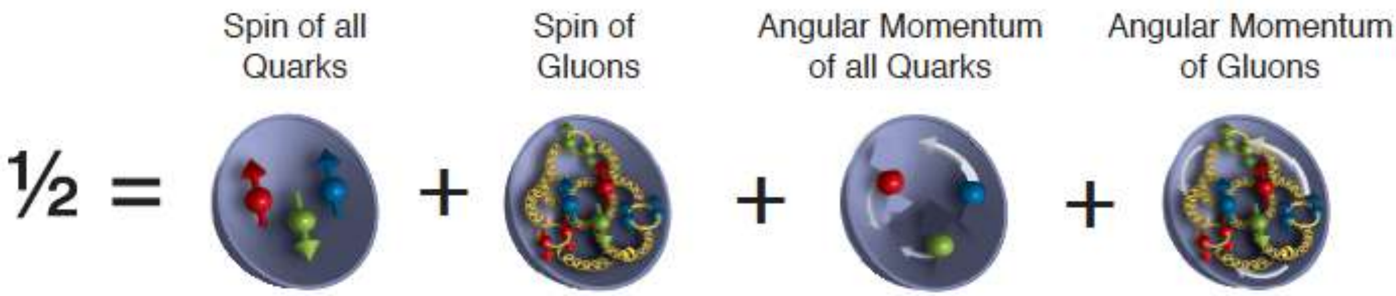
- First polarized electron-polarized proton collider
- First electron-nucleus collider
- Luminosities up to ~ 1000 times that of HERA
- Fine resolution inside the proton down to ~10^{-18} m

A long and winding road…



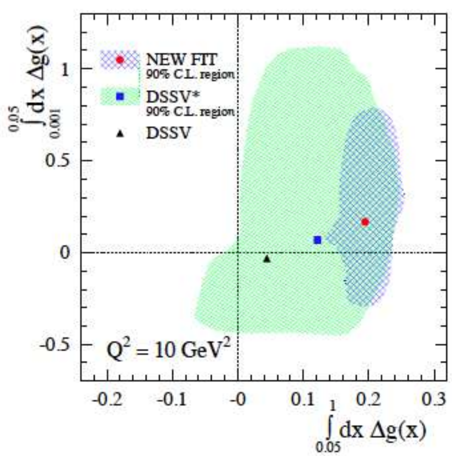
Polarized DIS at small x – reaching into terra incognita with EIC

The proton's spin puzzle: a many-body picture

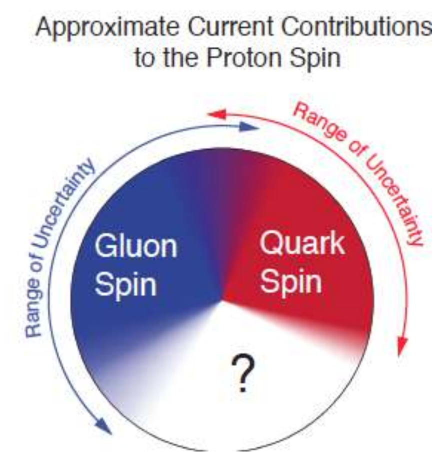


Fixed target DIS experiments showed that quarks ($\Delta\Sigma$) carry only about 30% of the proton's spin

"Spin crisis": failure of the quark model ("Ellis-Jaffe sum rule") picture of relativistic "constituent" quarks

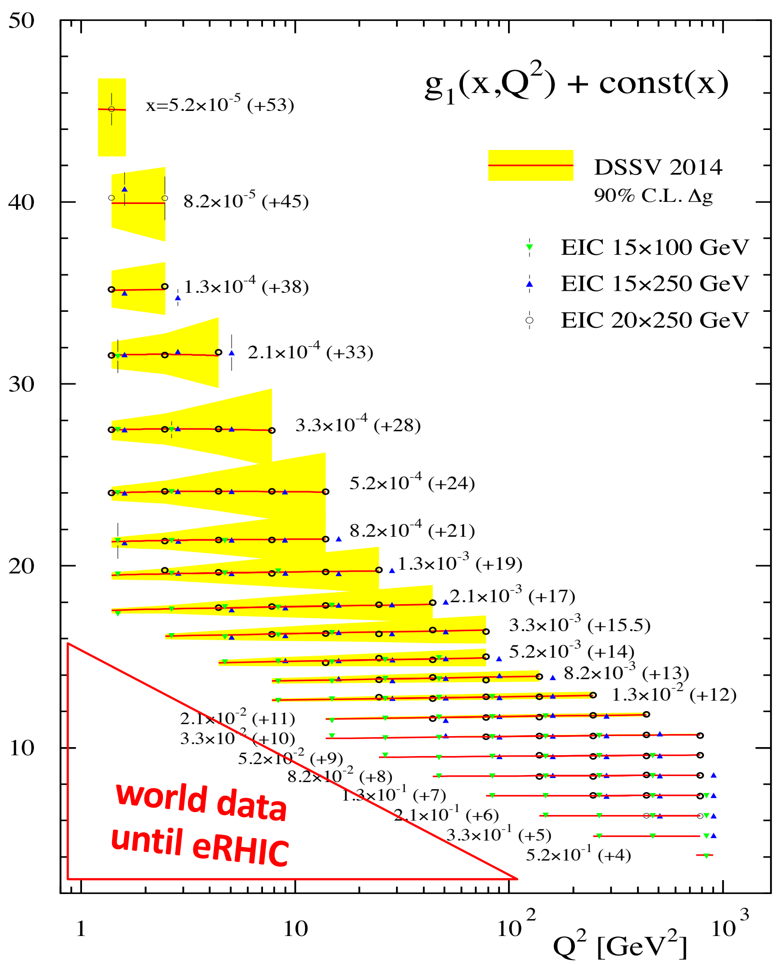


Evidence for gluon spin (ΔG) from RHIC but large uncertainties from small x

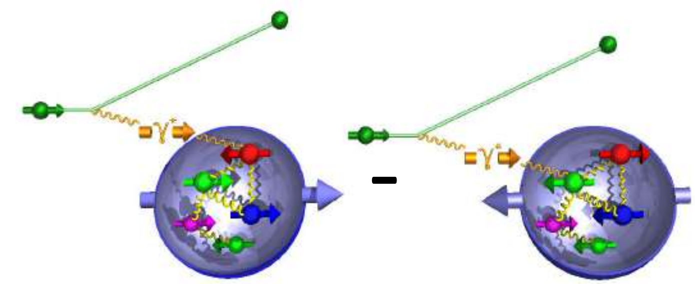


D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)

Resolving the proton's spin puzzle: the g_1 structure function

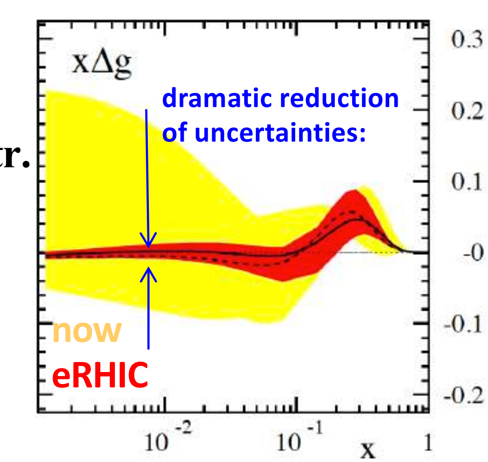


g_1 extracted from longitudinal spin asymmetry

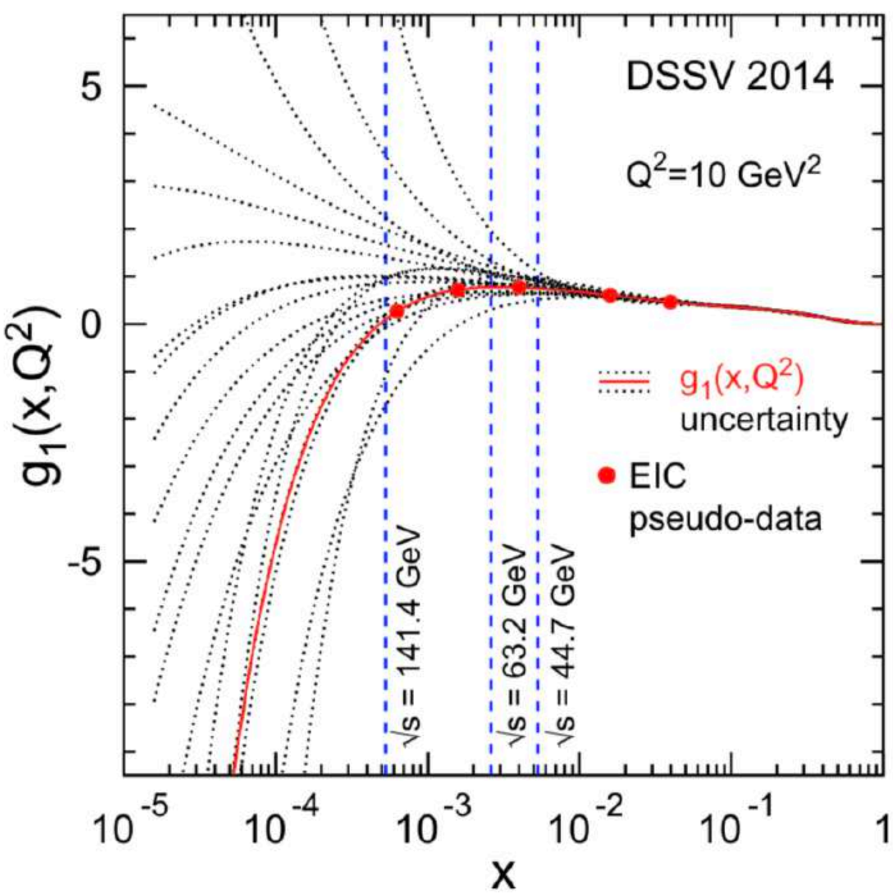


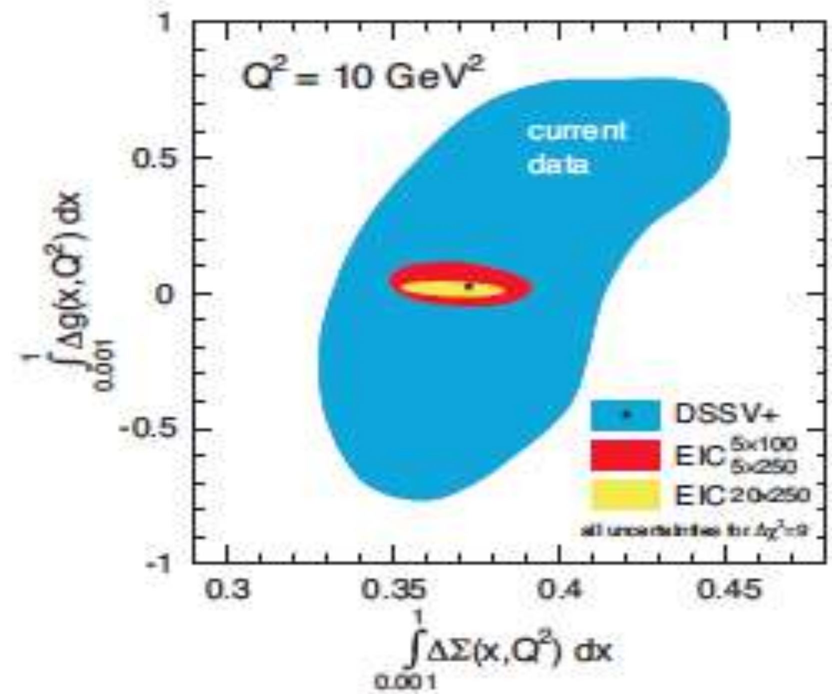
$$\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x,Q^2) \to \text{quark contribution}$$

$$\frac{dg_1}{d\log(Q^2)} \overset{?}{\sim} -\Delta g(x,Q^2) \to \textbf{gluon contr.}$$



Resolving the proton's spin puzzle: the g_1 structure function



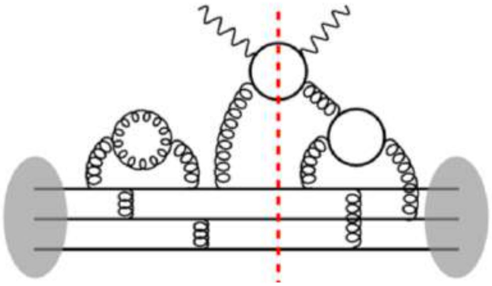


Aschenauer et al., arXiv:1708.01527
Rep. Prog. Phys. 82, 024301 (2019)

Iso-singlet axial vector current and topology of the QCD vacuum

Reviews:
S. D. Bass, RMP, hep-ph/0411005
G. M. Shore, hep-ph/0701171

g_1 structure function: formal definitions



Hadron tensor $W^{\mu\nu}$ = $\mathrm{Im}\,\frac{i}{\pi}\int d^4\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}}\langle P,S|\mathbb{T}\,\hat{j}^{\mu}(\mathbf{x})\hat{j}^{\nu}(0)|P,S\rangle$

with $j^{\mu} = \sum_f e_f \bar{\psi}_f \gamma^{\mu}\psi_f$

Most generally, $g_1(x,Q^2) = \frac{1}{8\lambda}\epsilon_T^{\mu\nu}\tilde{W}_{\mu\nu}(q,P,S)$

where $\widetilde{W}^{\mu\nu}$ is the antisymmetric part of $W^{\mu\nu}$
$S^{\mu} = \frac{2\lambda}{m_P}P^{\mu}$ and $\lambda = \pm 1/2$

Generalized parton model ("leading twist"): $g_1(x_B,Q^2) = \frac{1}{2}\sum_f e_f^2\left(\Delta q_f(x_B,Q^2) + \Delta\bar{q}_f(x_B,Q^2)\right)$

Where the quark helicity pdf is defined to be

$$\Delta q(x) = \frac{1}{4\pi}\int dy^-\, \mathrm{e}^{-iy^- xP^+}\langle P,S|\bar{\psi}(0,y^-,\mathbf{0}_\perp)\,\gamma^+\gamma_5\,\psi(0)|P,S\rangle$$

$\Delta q(x) =$ 

Iso-singlet axial vector current

In general, the first moment of g_1:

$$\int_0^1 g_1(x,Q^2) = \frac{1}{18}\left(\underbrace{3F + D} + 2\,\Sigma(Q^2)\right)$$

Combination of triplet axial vector current (gives g_A) measured in β decay and octet axial vector current measured in hyperon decays

with the iso-singlet quark helicity given by

$$\Delta\Sigma(Q^2) \;=\; \sum_q^{N_f} \int_0^1 dx \left(\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)\right)$$

In the parton model picture, this mixes, under evolution, with other isospin blind moment $\Delta G = \int_0^1 dx\, \Delta g(x, Q^2)$

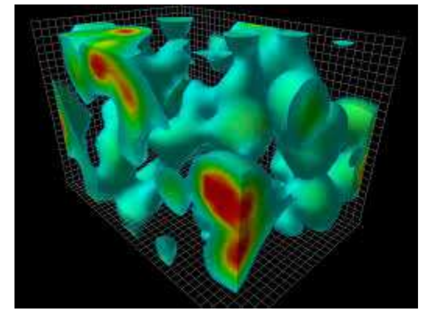
$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma} & 2N_f \Delta P_{qG} \\ \Delta P_{Gq} & \Delta P_{GG}\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

Splitting functions known to high loop order

Moch, Rogal, Vermaseren, Vogt
Talk by Vogelsang at POETIC IX, LBNL; DeFlorian, Vogelsang (2019)

Iso-singlet axial vector current and the chiral anomaly

$$S^\mu \, \Delta\Sigma = \langle P, S \,|\, \bar{\psi}\, \gamma^\mu \gamma_5 \, \psi \,|\, P, S\rangle \equiv \langle P, S \,|\, j_5^\mu \,|\, P, S\rangle$$



$U_A(1)$ violation from the chiral anomaly:

$$\partial_\mu J_5^\mu = 2 n_f \partial_\mu K^\mu + \sum_{i=1}^{n_f} 2 i m_i \bar{q}_i \gamma_5 q_i$$

where the Chern-Simons current

$$K_\mu = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right]$$

For massless quarks, conserve $J_5^\mu - 2\, n_f K^\mu$ → $\propto$ $\Delta\Gamma(Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} N_f \Delta g(Q^2)$

So then the “real” ΔΣ is

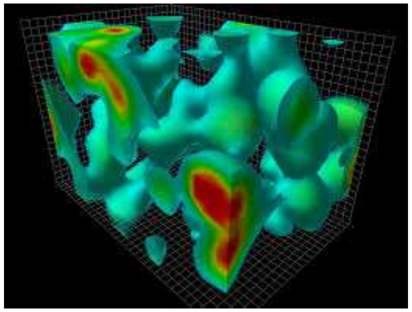
$$\Sigma(Q^2) = \tilde{\Sigma}(Q^2) - \frac{\alpha_s(Q^2)}{2\pi} N_f \Delta g(Q^2)$$

offering a possible explanation of empirical small ΔΣ (in addition to flavor SU(3) violation)…

ca., 1988
Efremov, Teryaev
Altarelli, Ross
Carlitz, Collins, Mueller

Iso-singlet axial vector current and the chiral anomaly

$$S^\mu \, \Delta\Sigma = \langle P, S \,|\, \bar{\psi}\, \gamma^\mu \gamma_5 \, \psi \,|\, P, S \rangle \equiv \langle P, S \,|\, j_5^\mu \,|\, P, S \rangle$$



$U_A(1)$ violation from the anomaly:

$$\partial_\mu J_5^\mu = 2n_f \partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i \bar{q}_i \gamma_5 q_i$$

where the Chern-Simons current

$$K_\mu = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right]$$

But, identification of CS charge with ΔG is intrinsically ambiguous
… *the latter is gauge invariant, the former is not*

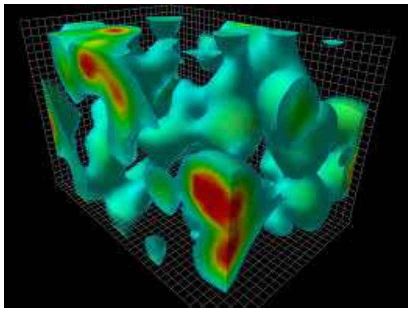
Jaffe, Manohar (1990)

$$K_\mu \to K_\mu + i \frac{g}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\nu \left(U^\dagger \partial^\alpha U A^\beta \right) + \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \left[(U^\dagger \partial^\nu U)(U^\dagger \partial^\alpha U)(U^\dagger \partial^\beta U) \right]$$

"Large gauge transformation"
- deep consequence of topology

R. Jaffe: identification of K^μ with ΔG
a source of much confusion
in the literature (Varenna lectures, 2007)

Iso-singlet axial vector current and the chiral anomaly



However, identifying CS − charge with ΔG "works" remarkably well ...
(Vogelsang, POETIC IX)

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma\\ \Delta\Gamma\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma}(a_s) & 0\\ -\frac{1}{2N_f}\Delta P_{\Sigma\Sigma}(a_s) & 0\end{pmatrix}\begin{pmatrix}\Delta\Sigma\\ \Delta\Gamma\end{pmatrix}$$

$\propto\ \Delta G$

NNLO splitting function computations confirm this

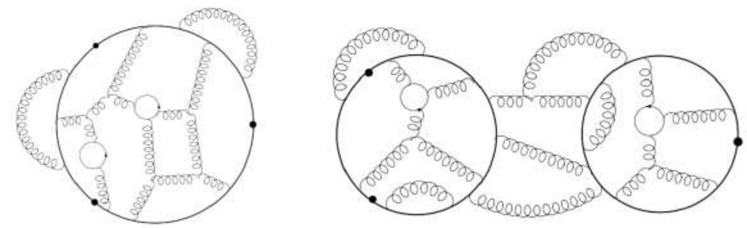
At LO, Altarelli, Lampe (1990)
NNLO: Vogt, Moch, Rogal, Vermaseren, arXiv:0807.1238

This suggests that $\Delta\Sigma$ only mixes with itself; likewise ΔG as defined, only depends on $\Delta\Sigma$ with the same splitting function

We will discuss later the deeper physics behind these expressions and their relation to the topology of the QCD vacuum

Alternative picture: topological charge screening of spin

Shore, Veneziano, PLB (1990); NPB (1992)
Narison, Shore, Veneziano, hep-ph/9812333



Anomalous chiral Ward identities
and extended PCAC from $U_A(1)$ breaking

Example: Witten-Veneziano formula $m^2_{\eta'} = \frac{2\, n_f}{f_\pi^2} \chi_{\rm YM}(0) + O((\frac{n_f}{N_c})^2)$

where the topological susceptibility $\chi_{\rm YM}(p^2) = i \int dx\, e^{iP\cdot x} \langle 0|T(Q(x)Q(0)|0\rangle$

with $Q(x) = \frac{\alpha_S}{8\pi} {\rm Tr} \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$

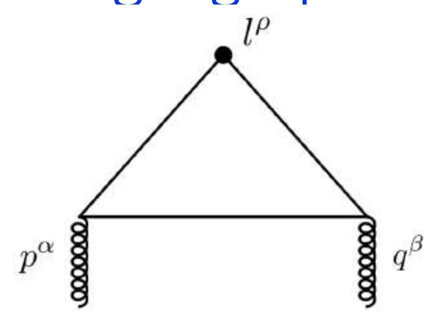
In this picture,
$$\Sigma(Q^2) = \frac{1}{3m_N} \Delta C_1^S(\alpha_S) \left(g_{QNN} \chi(0) + g_{\eta' NN} \sqrt{\chi'(0)} \right)$$

In chiral limit $\chi(0) \to 0$, $\Delta\Sigma$ "controlled" by the slope χ' at p^2=0 – estimated to be small by Veneziano et al.

What about dynamics – can we reconcile this picture with the perturbative one ?

Perturbative & nonperturbative interplay: The triangle graph

The key role of the $U_A(1)$ anomaly is seen from the structure of the triangle graph
In the off-forward ($l^\mu \to 0$, $t = l^2 \to 0$) matrix element of $< P'$,S$| J^5_\mu |$P$, S >$



$$\langle P', S | J_5^\mu | P, S \rangle = G_A(t) S_\mu + l \cdot S \, l_\mu G_P(t)$$

Jaffe. Manohar (1990)

The computation of this has an infrared pole proportional to $\frac{l^\mu}{l^2} F\tilde{F}$

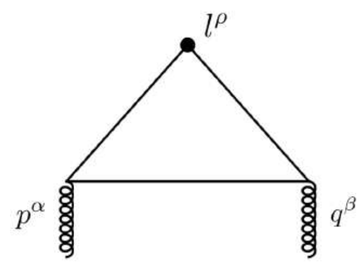
The perturbative/nonperturbative interplay gives a highly non-trivial result:

$$\langle P', S | J_5^\mu | P, S \rangle \to \frac{l \cdot S \, l^\mu}{l^2} \kappa(t) + \left(S^\mu - \frac{l \cdot S \, l^\mu}{l^2} \right) \lambda(t)$$

The infrared pole in G_A(t) must be canceled by a pole in G_P(t) – the corresponding Wess-Zumino-Witten term for the η'

For that to hold, one must have $\kappa(0) = \lambda(0) \propto F\tilde{F}$ the topological charge density

Perturbative & nonperturbative interplay: The triangle graph



with the (manifestly) gauge invariant result for the forward matrix element

$$\Sigma(Q^2) = \frac{n_f\,\alpha_s}{2\pi\,M_N} \lim_{l_\mu \to 0} \langle P', S| \frac{1}{il\cdot s} \mathrm{Tr}\left(F\tilde{F}\right)(0)|P,S\rangle$$

Suprisingly (since known to Jaffe+Manohar & Veneziano et al) not addressed in a lot of the "pQCD" literature
It is however deeply profound since it is intimately tied to the mechanism whereby the η' gets its mass

This result, generalization to $g_1(x,Q^2)$, and the interplay with non-perturbative physics
can be explored efficiently in a worldline framework

Thinking properly about anomalies with worldlines

Review: Schubert, Phys. Repts. (2001)
N. Mueller, RV: 1701.03331.1702.01233,1901.10492
Tarasov, RV: 1903.11624 and in preparation

The worldline formulation of QFT is equivalent to the string amplitude formalism of Bern and Kosower, as shown by Strassler

Bern,Kosower, NPB 379 (1992) 145;
Bern, TASI lectures, hep-ph/9304249
Strassler, NPB 385 (1992) 145

World-line formalism: vector and axial vector fields

$$S[A, A_5] = \int d^4x\, \bar{\psi}\, (i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5)\, \psi$$

One loop effective action $\Gamma[A, A_5] = \log\det(\chi)$ with $\chi = i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5$

Heat kernel representation of χ: *sandwich* between $0+1-\mathrm{D}$ boson (x, p) and fermion (Grassmann) coherent states

Berezin, Marinov (1976);
Brink et al; Barducci et al;
Balachandran et al (1976-77);
D'Hoker, Gagne (1996)

Wigner-Weyl formalism: $\hat{a}_i^{\pm} \equiv \frac{1}{2}(\gamma_i \pm i\gamma_{i+2})$ i=0,1 $\hat{a}_i^{-}|\theta\rangle = \theta_i|\theta\rangle$ $\hat{a}_i^{+}|\theta^*\rangle = \theta_i^*|\theta^*\rangle$

Grassmann Majorana fermions: $\psi_i \equiv \frac{1}{\sqrt{2}}(\theta_i + \theta_i^*)$ $\psi_{i+2} \equiv \frac{i}{\sqrt{2}}(\theta_i - \theta_i^*)$

World-line formalism: vector and axial vector fields

$$S[A, A_5] = \int d^4x\, \bar{\psi}\, (i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5)\, \psi$$

One loop effective action $\Gamma[A, A_5] = \log\det(\chi)$ with $\chi = i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 A\!\!\!/_5$

$$\Gamma[A, A_5] = \Gamma_R + i\Gamma_I$$

→ *Phase of the determinant..*
An elegant way to represent the chiral anomaly which Fundamentally arises from non-invariance of path integral measure under a chiral rotation

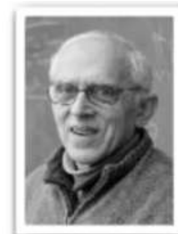
Remarkably, this phase can be re-expressed in a form that is nearly identical to Γ_R !

Useful mnemonic: *Odd* powers of γ_5 contribute to Γ_I and *Even* powers to Γ_R

D'Hoker, Gagne, hep-ph/9512080
See also Mondragon, Nellen, Schmidt, Schubert, hep-th/9502125

The triangle anomaly in the worldline formalism

Tarasov, RV



Steven Adler

John S. Bell

Roman Jackiw

Point particle Bose and Grassmann path integrals

$$\Gamma[A, A_5] = -\frac{1}{2}\mathrm{Tr_c}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x \int_{AP} \mathcal{D}\psi$$

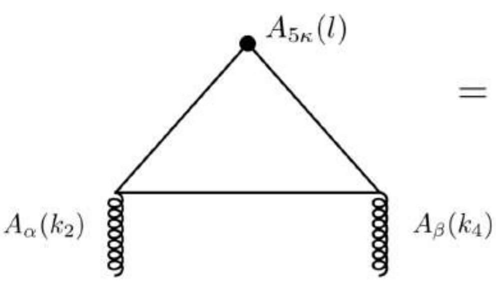
$$\times \exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi_\mu\dot{\psi}^\mu + ig\dot{x}^\mu A_\mu - ig\psi^\mu\psi^\nu F_{\mu\nu} - 2i\psi_5\dot{x}^\mu\psi_\mu\psi_\nu A_5^\nu + i\psi_5\partial_\mu A_5^\mu + (D-2)A_5^2\Big)\Big\}$$

Wilson line

Spin precession

Axial vector couplings

$$\langle P', S|J_5^\kappa|P, S\rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)}\Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

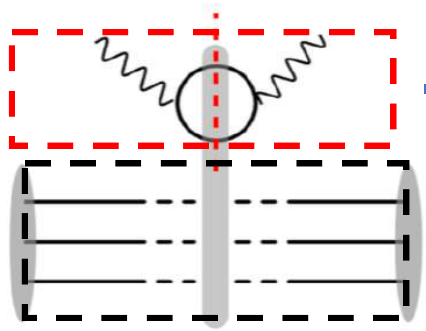


$$= \frac{1}{4\pi^2}\frac{l^\kappa}{l^2}\int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\,\mathrm{Tr_c}F_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\;(2\pi)^4\delta^4(l+k_2+k_4)$$

Famous infrared pole of anomaly

William A. Bardeen

Polarized DIS in the worldline formalism

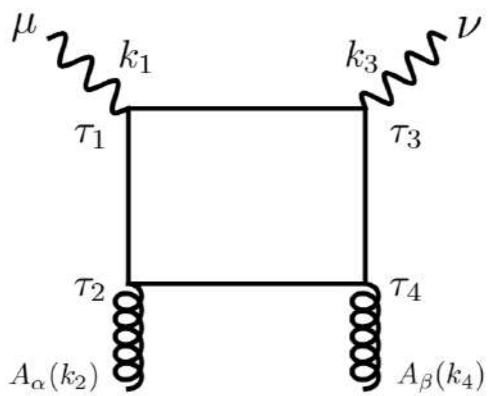


Polarization tensor from world-line Hamiltonian

Gauge (gluon) background

The box diagram for polarized DIS ($g_1(x,Q^2)$)

DIS with worldlines, Tarasov, RV (2019)



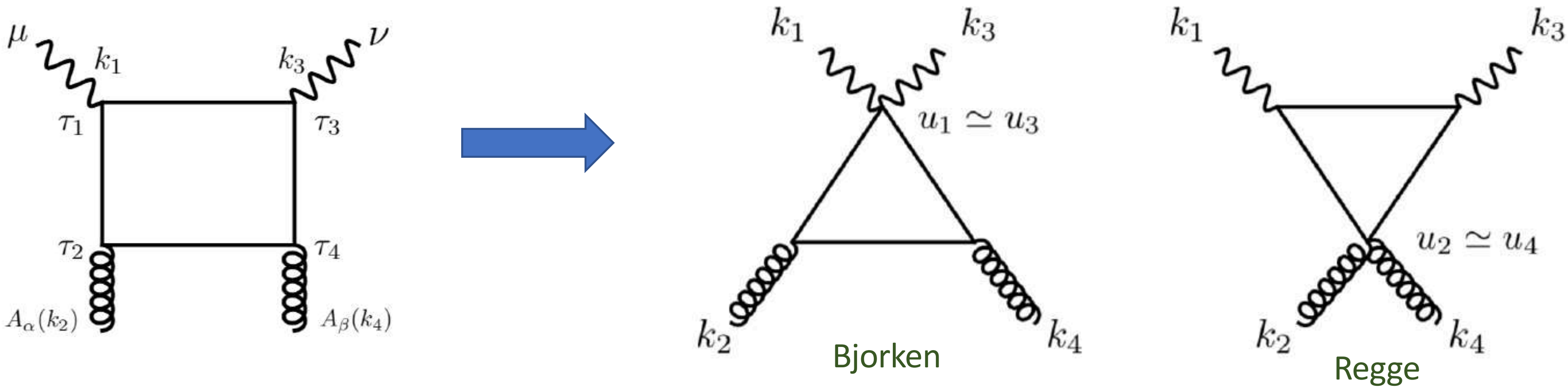
$$\Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4}\, \Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\ \mathrm{Tr}_c(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$$

Polarization tensor (antisymmetric piece)

Box diagram

We can compute the box explicitly in both the Bjorken limit of QCD ($Q^2 \to \infty, s \to \infty, x = \text{fixed}$) and the Regge limit ($x \to 0, s \to \infty, Q^2 = \text{fixed}$). The latter result is new

Finding triangles in boxes in Bjorken and Regge asymptotics



Remarkably, $g_1(x_B,Q^2)$ has the same structure in both limits, dominated by the triangle anomaly !

$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle \quad + \text{non-pole } \frac{\Lambda^2{}_{QCD}}{Q^2}$$

$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle \quad + \text{non-pole } \frac{{Q_S}^2}{M^2} \ll 1$$

Hence g_1 is topological in both asymptotic limits of QCD…its relation to $\Delta g(x, Q^2)$ is unclear

The r.h.s in the previous expressions is infrared divergent – how is this cured to give a finite result?

The role of pseudoscalar fields in resolving the $U_A(1)$ problem

We **did not** previously write down the most general form of the **imaginary part** of the worldline effective action :

D'Hoker, Gagne,hep-th/9508131

$$W_{\Im}[\Phi,\Pi,A] = \frac{1}{8}\int_{-1}^{1} d\alpha \int_0^\infty dT\,\mathcal{N}\int \mathcal{D}x\mathcal{D}\psi\,\mathrm{Tr}_c\,\mathcal{J}(0)\,\mathcal{P}e^{-\int_0^T d\tau\,\mathcal{L}_\alpha}$$

with $\mathcal{L}_\alpha = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi_A\dot{\psi}_A - i\dot{x}\cdot A + \frac{i}{2}\mathcal{E}\psi_\mu F_{\mu\nu}\psi_\nu + \frac{1}{2}\mathcal{E}\alpha^2\Phi^2 + \frac{1}{2}\mathcal{E}\Pi^2$

$$+ i\,\varepsilon\,\psi_\mu\psi_5 D_\mu\Pi + \alpha\,\varepsilon\,\psi_5\psi_6[\Pi,\Phi]$$

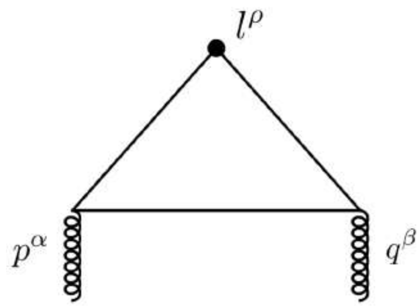
where Φ is the chiral condensate

and $\mathcal{J}(0) \propto \psi_5\psi_6\{\Pi,\Phi\}$

Expanding out the worldline Lagrangian, the first nontrivial contribution to W_I is the Wess-Zumino-Witten term !

$$W_{\Im}[\Pi^5] = -\frac{i}{5}\int_{p^1,\ldots,p^5}(2\pi)^4\delta^{(4)}(p^1+\cdots+p^5)\,(-4im)\,\mathrm{Tr}_c(\tilde{\Pi}_1\cdots\tilde{\Pi}_5)\varepsilon_{\mu_1\cdots\mu_4}p^1_{\mu_1}\cdots p^4_{\mu_4}I'(p^i)$$

The role of pseudoscalar fields in resolving the $U_A(1)$ problem



The WZW term has a contribution $\propto$ $\eta' \, F\tilde{F}$ (Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000))

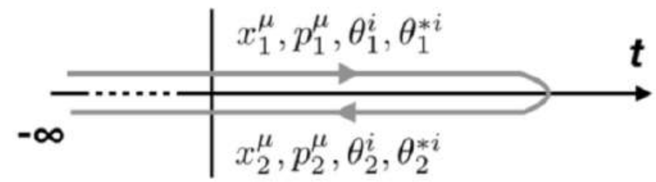
The zero mode part of it exactly cancels the $\frac{l^\mu}{l^2}$ *"perturbative" contribution to the anomaly*

$g_1(x, Q^2) \rightarrow \left\langle P' \right| \frac{1}{il * S} \omega \left| P \right\rangle$ where the topological charge density $\omega(X) = \frac{\alpha_s}{8\pi} \mathrm{Tr}\left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$

Tarasov, RV, in preparation

Computing the worldline matrix element

$$\langle \mathcal{O} \rangle = \mathrm{Tr}\{\mathcal{O}\mathcal{R}\} \quad \Rightarrow \quad \langle \mathcal{O} \rangle = \int \mathcal{D}A\; \mathcal{O}[A]\; \mathcal{R}[A]\; e^{iS_{YM}[A]}$$



$$\mathcal{R}_q[A] = \mathrm{tr}_c \int d^4 z_{\max} \int d^4 z_{\min}\; e^{iP^+(z^-_{\max} - z^-_{\min})}\; e^{-iP_\perp(z_{\perp\max} - z_{\perp\min})}$$

$$\times \int_0^\infty dT \int_{z_{\min}}^{z_{\max}} \mathcal{D}z \int \mathcal{D}\psi\; \mathcal{R}^c_{\mathrm{q\ init.}} \otimes \mathcal{R}^f_{\mathrm{q\ init.}}\; \exp\left[- \int_0^T d\tau \left(\frac{1}{4}\dot{z}^2 + \frac{1}{2}\psi\dot{\psi} + igA\dot{z} - ig\psi F \psi \right) \right]$$

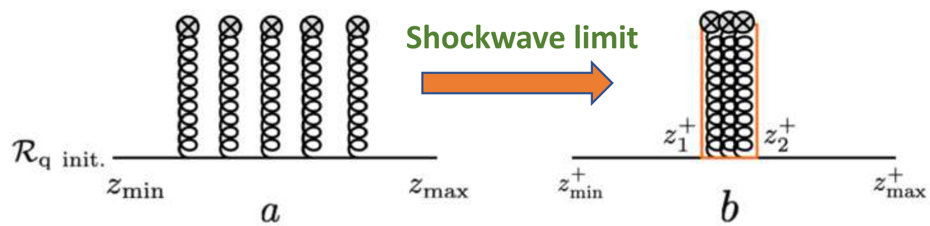
$$\frac{1}{4}\left(1 + \lambda \psi_5\right)\left(1 + 2\psi^- \psi^+\right)$$

Wilson line

Spin precession

Berezin,Marinov (1976)
Tarasov,RV

See also
Bakker,Leader,Trueman,
Hep0ph/0406139



Spin precession and smearing of width of shock wave become important to obtain helicity flip contributions

But the key effect is γ_5 which induces a zero mode in W_l that couples to the topological charge

Worldline quark trajectory: polarized case

$$\mathcal{R}_q[A] = \mathrm{tr_c} \int d^4 z_{\max} \int d^4 z_{\min}\, e^{iP^+(z^-_{\max} - z^-_{\min})}\, e^{-iP_\perp(z_{\perp \max} - z_{\perp \min})}$$
$$\times \int_0^\infty dT \int_{z_{\min}}^{z_{\max}} \mathcal{D}z \int \mathcal{D}\psi\, \mathcal{R}^c_{\mathrm{q\ init.}} \otimes \mathcal{R}^f_{\mathrm{q\ init.}}\ \exp\left[-\int_0^T d\tau \left(\frac{1}{4}\dot{z}^2 + \frac{1}{2}\psi\dot{\psi} + ig\dot{z}^+ A^-\right)\right]$$
$$\times\left(1 + ig\int_0^T d\tau \psi^\mu \psi^\nu F_{\mu\nu}(x(\tau_1)) - g^2 \int_0^T d\tau_1 \psi^\mu\psi^\nu F_{\mu\nu}(x(\tau_1)) \int_0^{\tau_1} d\tau_2 \psi^\xi \psi^\eta F_{\xi\eta}(x(\tau_2)) + \dots\right)$$

Only this term survives in the density matrix – giving $F\tilde{F}$

ψ_5 changes anti-periodic b.c.'s to periodic b.c's for Majorana modes
Anomaly term arises from the surviving zero mode

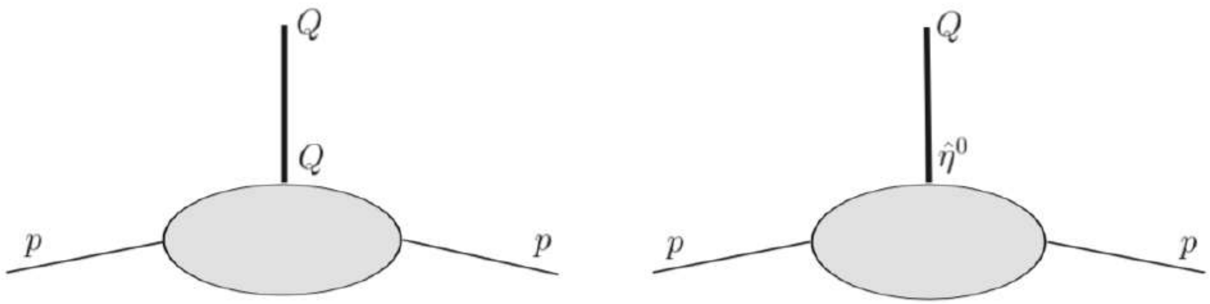
$$\mathcal{R}_q \equiv ig^2 \int d^4x\, \theta(x) \mathrm{Tr}_c F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ig^2 \int d^4x\, \phi_5(x) \mathrm{Tr}_c F_{\mu\nu}\tilde{F}^{\mu\nu}$$

from the WZW term in the worldline effective action…

Emergent ”axion” effective action for polarized DIS at small x

Tarasov, RV: in preparation

Conjecture: “Axion-like” effective action for polarized DIS

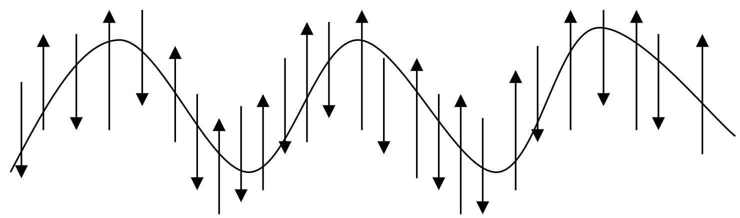


$$\Delta\Sigma(Q^2) = \frac{2}{3}\frac{1}{2m_N}\Delta C_1^S(\alpha_s)\left(\langle 0|T\,Q\,Q|0\rangle\, g_{QNN} + \langle 0|T\,Q\,\hat{\eta}^0|0\rangle\, g_{\hat{\eta}^0 NN}\right)$$

≈ 0 $\chi'(p^2 = 0)$

Narison, Shore and Veneziano (see hep-ph/0701171) argue that the result is dominated by the derivative of the topological susceptibility (χ'), which they compute (using QCD sum rules) to be in agreement with the HERMES and COMPASS data

Conjecture: "Axion-like" effective action for Regge limit



If we now assume (at small x), the background field couples to a large # of quark and gluon world-line trajectories, one can construct the *effective action for an ensemble of spinning, colored partons at a given x<< 1*:

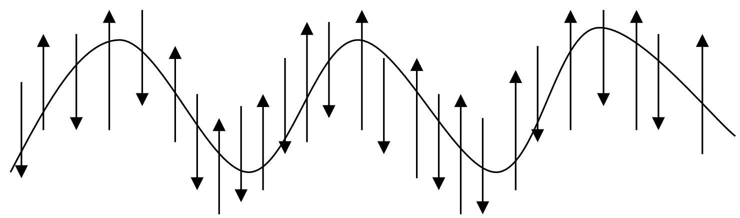
$$g_1(x,Q^2) \propto \int [D\rho] W_Y^P[\rho] \int [D\psi] \tilde{W}_P^{P,S}[\psi]$$
$$\times \int [dA] \int d^4X\, \omega(X) \exp\left(iS_{\rm YM}[A] + \frac{i}{N_c} \mathrm{Tr}_c \left(\rho\, U_{-\infty,\infty}\right)\right)$$
$$\times \exp\left(\int d^4X \left(-\frac{\omega^2}{2\,\chi_{\rm YM}} - \sqrt{\frac{N_f}{2}}\omega\,\psi + \frac{1}{2} F^2 \psi \partial^2 \psi \right)\right)$$

Veneziano, Mod. Phys. Lett. (1989)
Hatsuda, PLB (1990)

Topological susceptibility: $\chi_{\rm YM} = \int d^4X \langle \omega(X)\omega(0)\rangle_A$

$\omega(X)$ *is the topological charge density*, $F\ \psi$ *is the* η' field and F is the η' decay constant

Conjecture: "Axion-like" effective action for Regge limit



The term $\exp\left(\int d^4X \left(-\frac{\omega^2}{2\,\chi_{\rm YM}} - \sqrt{\frac{N_f}{2}}\omega\,\psi + \frac{1}{2}F^2\psi\partial^2\psi\right)\right)$

can be rewritten as the "Veneziano effective action" of a glueball and η' field

$$\exp\left(-\int d^4X \left(-\frac{G^2}{2\chi_{\rm YM}} + \frac{1}{2}\eta'\left(\partial^2 + m_{\eta'}^2\right)\eta'\right)\right)$$

ChPT formulation for the nonet, Herrera-Siklody et al, hep-ph/9610549

with $m_{\eta'}$ given by the Witten-Veneziano formula $m_{\eta'}^2 = \frac{2N_f\,\chi_{\rm YM}}{F^2}$

or equivalently as $\exp\left(\int d^4X \left(F^2\psi\partial^2\psi + (\psi+\theta)\,\omega + \chi_{\rm YM}\,(\psi+\theta)^2\right)\right)$

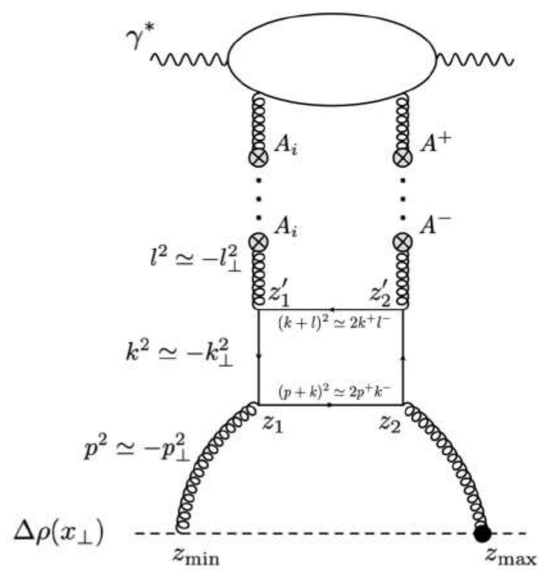
Spin diffusion to small x

A lot of impressive work on perturbative computations of g_1 at small x…

Kirschner Lipatov (1980’s); Bartels, Ermolaev, Ryskin (1990’s); Kovchegov, Pitonyak, Sievert (2015-), Kovchegov,Cougoulic (2019); Boussarie, Hatta, Yuan (2019)

Can we reconcile this work with the dynamics of the chiral anomaly at small x ?

QCD evolution to small x



After smearing of the target in x^-, the effective action reproduces itself (in a double log approximation)

– renormalizing the "axion effective action" at small x, one should recover the triangle structure in the "box diagram"

Fascinating interplay between gluon saturation (controlled by the saturation scale Q_S) and spin diffusion (controlled by the topological charge density ω)

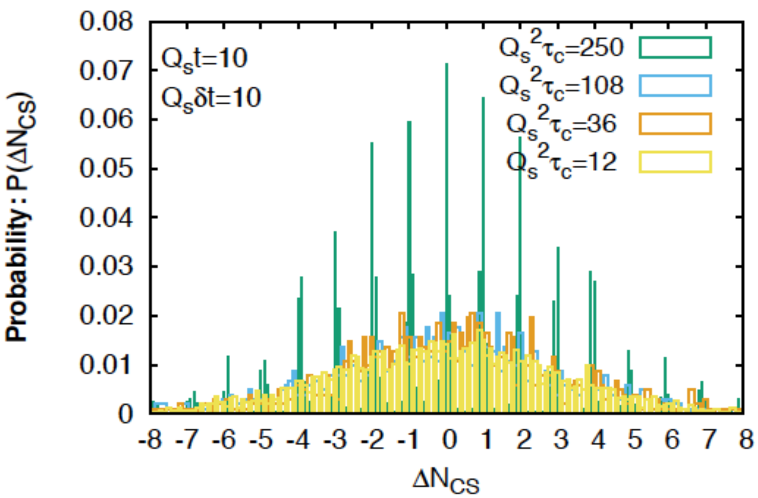
Spin diffusion via sphaleron transitions

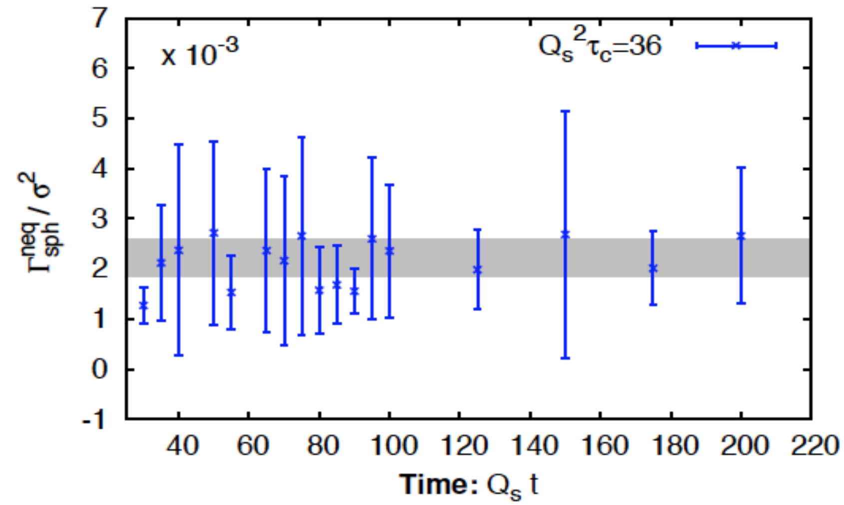
In a hot plasma (McLerran,Mottola,Shaposhnikov, 1990), diffusion of topological charge

$$\langle \dot{Q}_{\rm ch} \rangle = -12 n_f \frac{\Gamma_{\rm sph}}{T^3} \langle Q_{\rm ch} \rangle$$

Out of equilibrium, for overoccupied glue,
the scale is set by the spatial string tension and gluon saturation scale **Q_S**

Mace, Schlichting, RV: 1601.07342

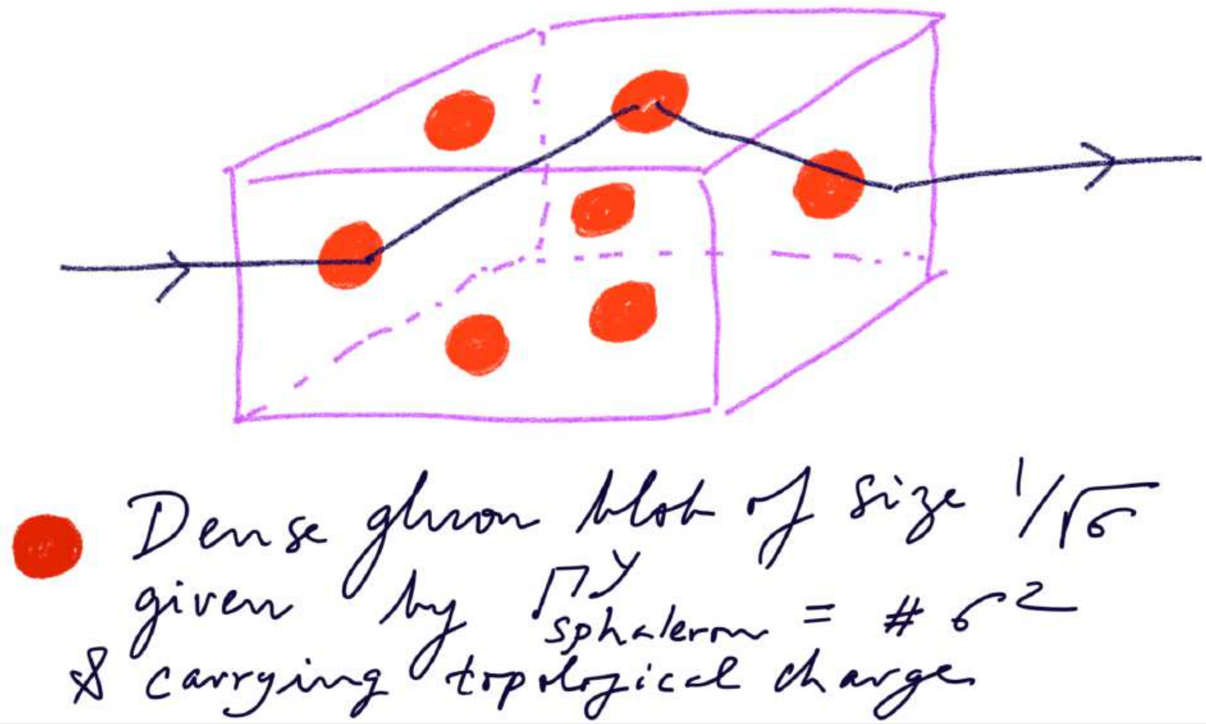




The analogy of time here is the rapidity Y = Ln(1/x)

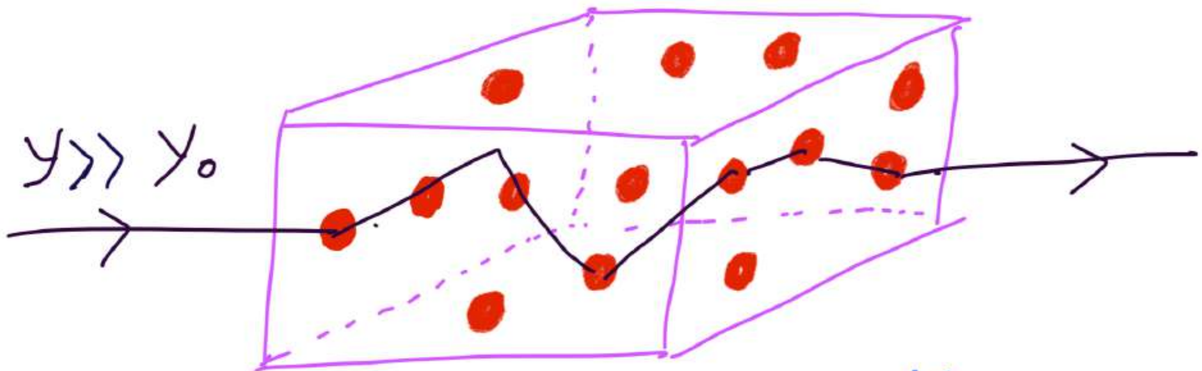
So the diffusion to small x is controlled by the rate of sphaleron transitions

Spin diffusion via sphaleron transitions in topologically disordered media



Helicity flip for massless quarks given by $n_L - n_R = 2\, N_f\, \nu$
where $\Gamma^{Y}_{\text{sphaleron}} \propto \langle \nu^2 \rangle$

Spin diffusion via sphaleron transitions in topologically disordered media

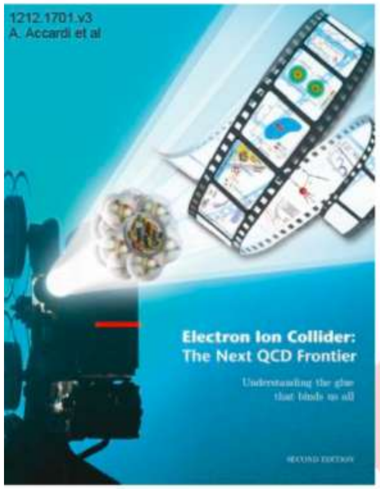


As x decreases ($y >> y_0$), the blobs become smaller ($Q_s(y) > Q_s(y_0)$) and denser with more topological charge

Helicity flip for massless quarks given by $n_L - n_R = 2\, N_f\, \nu$

where $\Gamma^{Y}_{sphaleron} \propto \langle \nu^2 \rangle$

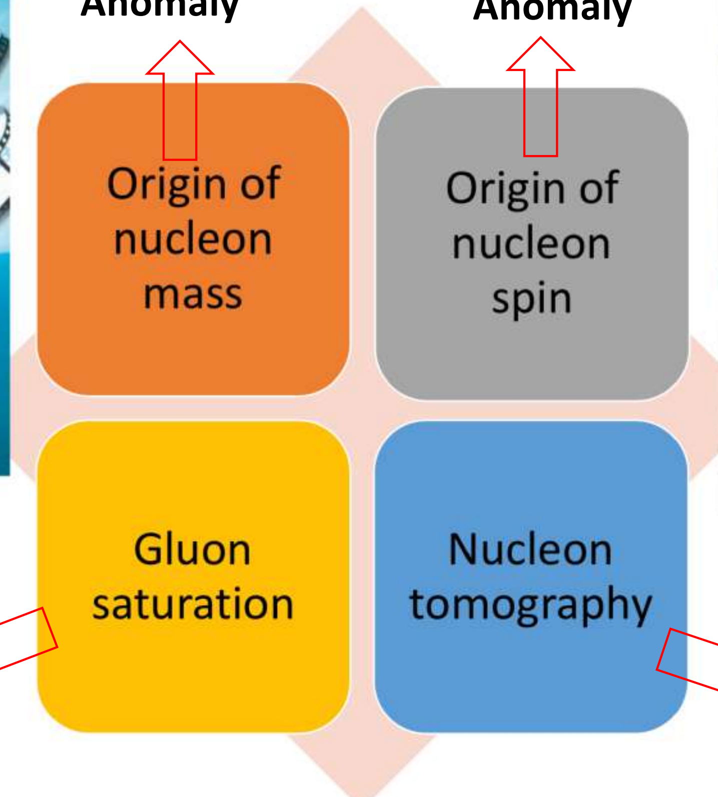
Outlook: these ideas can be tested at the EIC !



White paper
arXiv:1212.1701

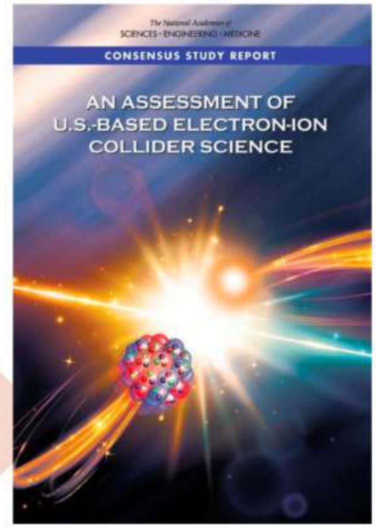
Conformal Anomaly

Chiral Anomaly



Color Glass Condensate

Confined many-body parton dynamics



NAS report
July 2018

Precision probes of the strong interplay between perturbative many-body parton dynamics and non-perturbative structure ("the ether") of the QCD vacuum

Outlook

Semi-classical approximations

Worldline QCD+QED Hamiltonian: $$H[A;a] = P^2 + ig\psi^\mu F_{\mu\nu}[A]\psi^\nu + ie\psi^\mu F_{\mu\nu}[a]\psi^\nu \,.$$

$$P_\mu \;=\; p_\mu \;-\; gA_\mu(x) \;-\; ea_\mu(x)$$

Powerful semi-classical intuition from corresponding Hamilton equations:

- *Bargmann-Michel-Telegdi equations for spinning particles in gauge backgrounds,*
- *Wong equations for colored particles precessing in gauge field backgrounds*
- *Papapetrou-Dixon equations for spinning massive particles in gravity*

Useful in semi-classical ("Wigner function") approaches in theories with internal symmetries

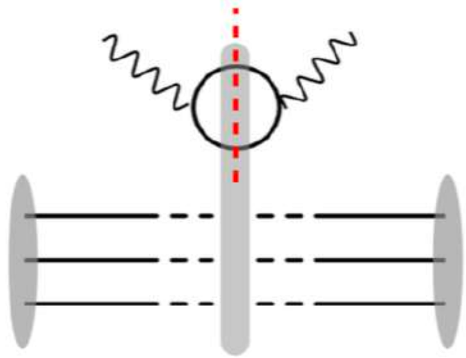
Mueller, Venugopalan, arXiv:1901.10492, and references therein
Applications to chiral kinetic theory:
Mueller, Venugopalan, arXiv:1701.03331, arXiv: 1702.01233

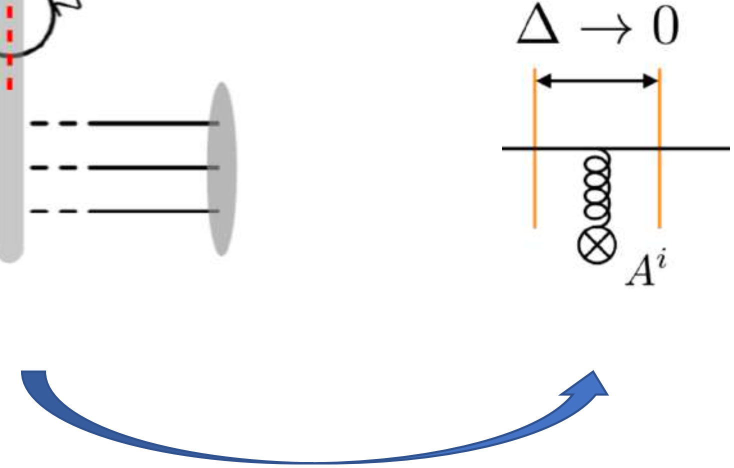
Also, "world-line instanton" techniques can be used to compute pair-production (Schwinger mechanism)

Dunne,Schubert, hep-th/0507174

Worldline inspired "axion" effective action for polarized DIS

Tarasov, RV: in preparation





Spin-dependent contributions at high energies are "sub-eikonal"

- in addition to shockwave Coulomb/Glauber field A^+, there is a $\Delta = 1/P^+ \to 0$ suppressed contribution A^i

World-line formalism: preliminaries

Based on Schwinger's proper time trick:

Review: Corradini, Schubert, arXiv:1512.08694
Also, Strassler, NPB385 (1992) 145

$$\log(\sigma) = \int_1^\sigma \frac{dy}{y} \equiv \int_1^\sigma dy \int_0^\infty dt\, e^{-yt} = -\int_0^\infty \frac{dt}{t}\left(e^{-\sigma t} - e^{-t}\right)$$

One loop effective action of massless scalar field coupled to background Abelian field

$$\mathcal{L} = \Phi^\dagger D^2 \Phi \qquad D_\mu = \partial_\mu - igA_\mu$$

$$\Gamma[A] = -\log\left[\det(-D^2)\right] \equiv -\mathrm{Tr}\left(\log(-D^2)\right) = \int_0^\infty \frac{dT}{T} \mathrm{Tr}\, \exp(-TD^2)$$

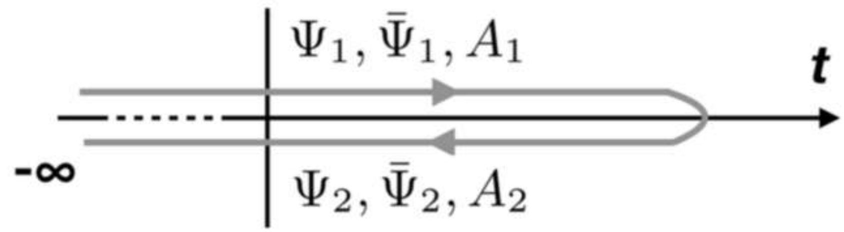
$$= \int_0^\infty \frac{dT}{T} \mathcal{N} \int \mathcal{D}x\, \mathcal{P} \exp\left[-\int_0^T d\tau \left(\frac{1}{2\varepsilon}\dot{x}^2 + igA[x(\tau)]\cdot\dot{x}\right)\right]$$

with $\mathcal{N} = \int \mathcal{D}p\, \exp(-\frac{1}{2}\int_0^T d\tau \epsilon\, p^2)$

ε is the Einbein:
square root of 1D metric

Computing structure functions: world-line formalism

"Schwinger-Keldysh" approach to computing real-time correlations of quantum fields



Break up problem into two parts:

$$Z = \int dA_1 dA_2 \langle A_1 | \hat{\rho}_{\rm YM} | A_2 \rangle \int_{A_1}^{A_2} \mathcal{D}A\, Z_f[A] \exp\left\{ i S_{\mathcal{C}}^{\rm YM} \right\}$$

Initial conditions for gluon fields

Yang-Mills action

with $$Z_f[A] \equiv \int d\Psi_1 d\Psi_2 \langle \Psi_1 | \hat{\rho}_V | \Psi_2 \rangle \int_{\Psi_1}^{\Psi_2} \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \exp\left\{ i S_{\mathcal{C}}^{q} \right\}$$

Initial conditions for "valence" quarks

Dirac action

Z_f [A] can be rexpressed as the effective action corresponding to a quantum mechanical "world-line" Hamiltonian describing *spinning and colored Grassmann point particles (qubits) in gluon fields*