QCD phase structure from fluctuations of conserved charges

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Theoretical Physics Colloquium
hosted by Prof. Igor Shovkovy at the Arizona State University

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QCD phase structure

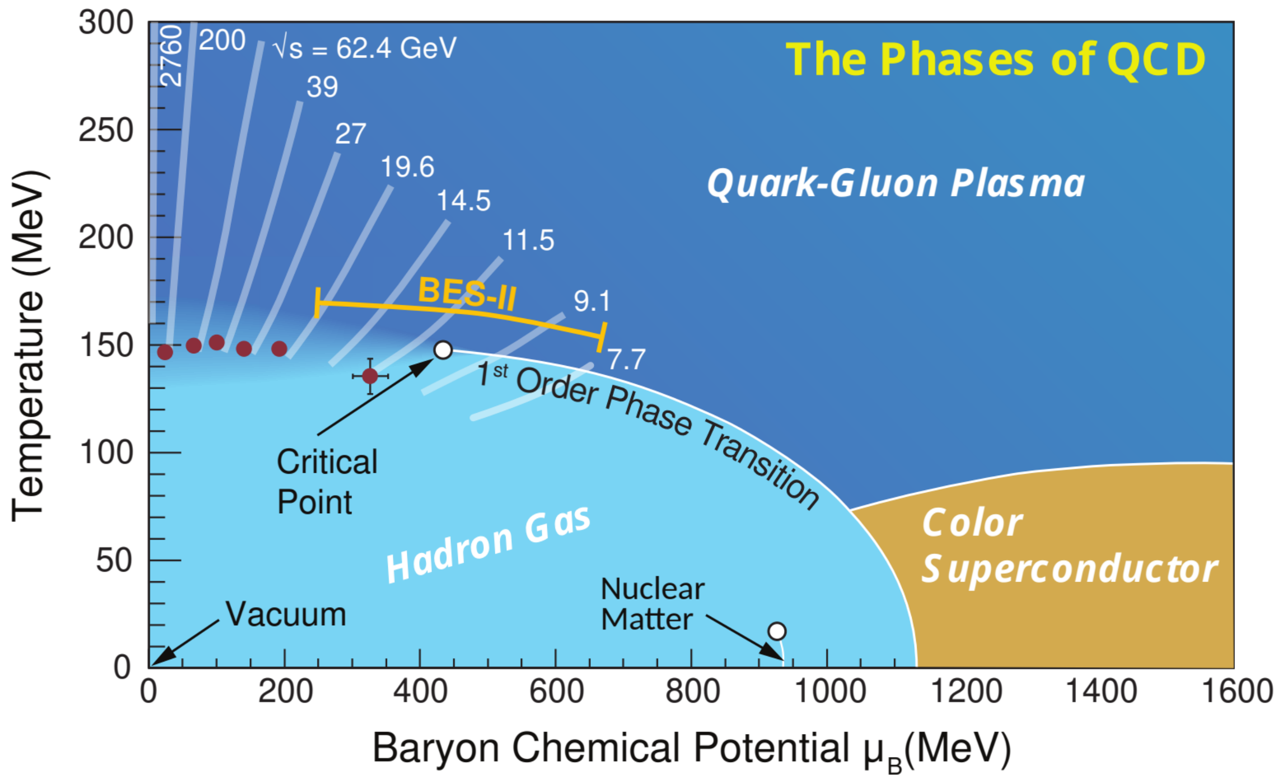


Figure from Bzdak et al., Phys. Rept. '20

- Dilute hadron gas at low T & ρ_B due to confinement, quark-gluon plasma high T & ρ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

What is the nature of the quark-hadron transition?

QCD transition from lattice QCD

$$Z = \mathrm{Tr}(e^{-(\hat{H}-\mu\hat{N})/T}) = \int DU \det M[U,\mu]\, e^{-S_{YM}}$$

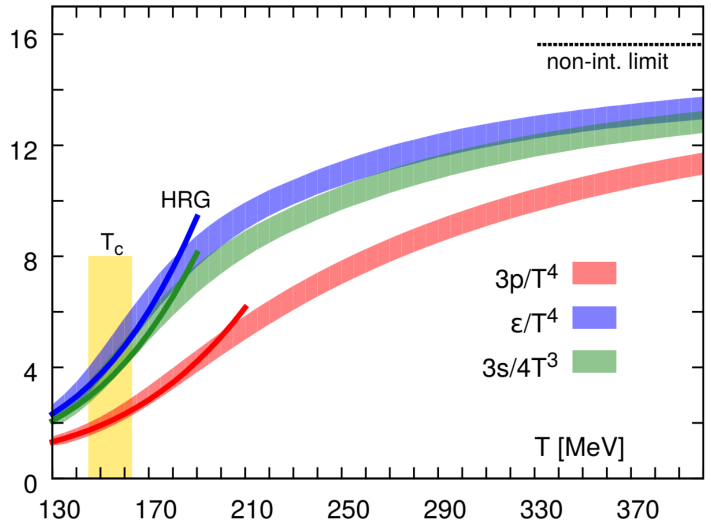


Figure from HotQCD coll., PRD '14

lattice QCD

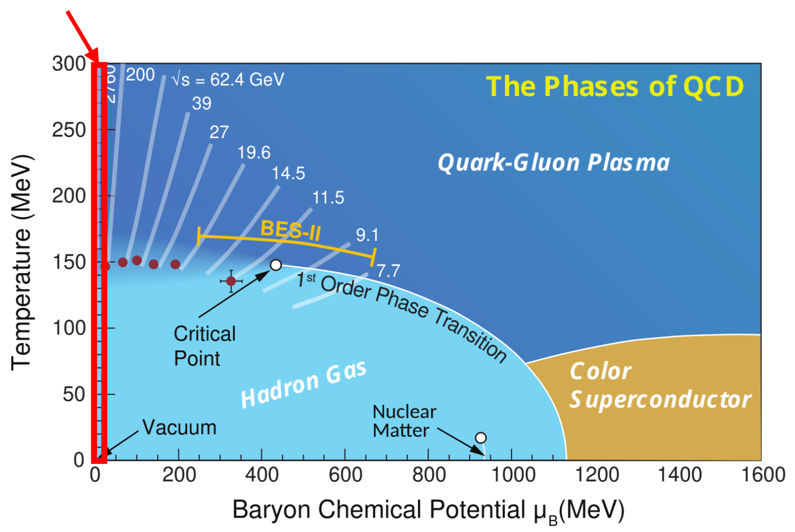


Figure from Bzdak et al., Phys. Rept. '20

- Analytic **crossover** at vanishing net baryon density at $k_B T_{pc} \approx 155$ MeV – a first-principle result
  [Y. Aoki et al., Nature 443, 675 (2006)]
- Finite baryon densities inaccessible due to the **sign problem**
- Many effective theories predict first-order phase transition and the **QCD critical point**

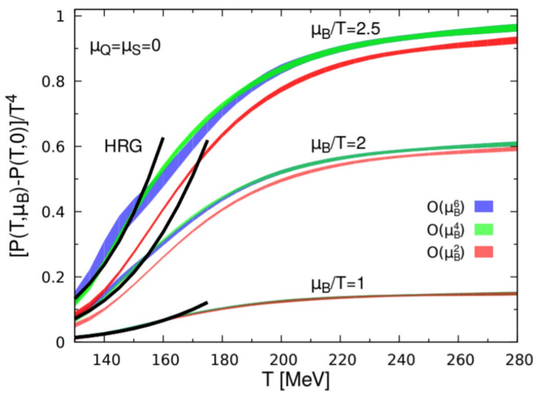
First-principle constraints on the QCD critical point

Indirect lattice QCD methods offer glimpse into small μ_B/T

- Taylor expansion around $\mu_B/T\!=\!0$

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T,0)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T,0)}{4!}(\mu_B/T)^4 + \dots$$

  No hints for the critical point at T $>$ 135 MeV
  Critical point $\mu_B/T < 3$ disfavored

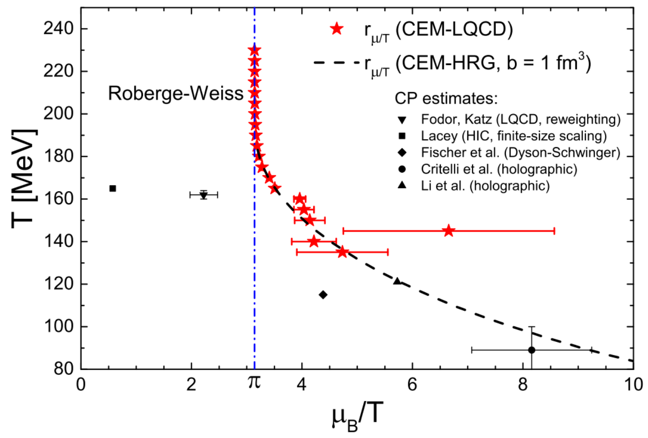


[HotQCD Collaboration, PRD 95, 054504 (2017)]

- Relativistic virial expansion in fugacities via analytic continuation from imaginary μ_B/T

$$\frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T) \cosh\left(\frac{k\,\mu_B}{T}\right)$$

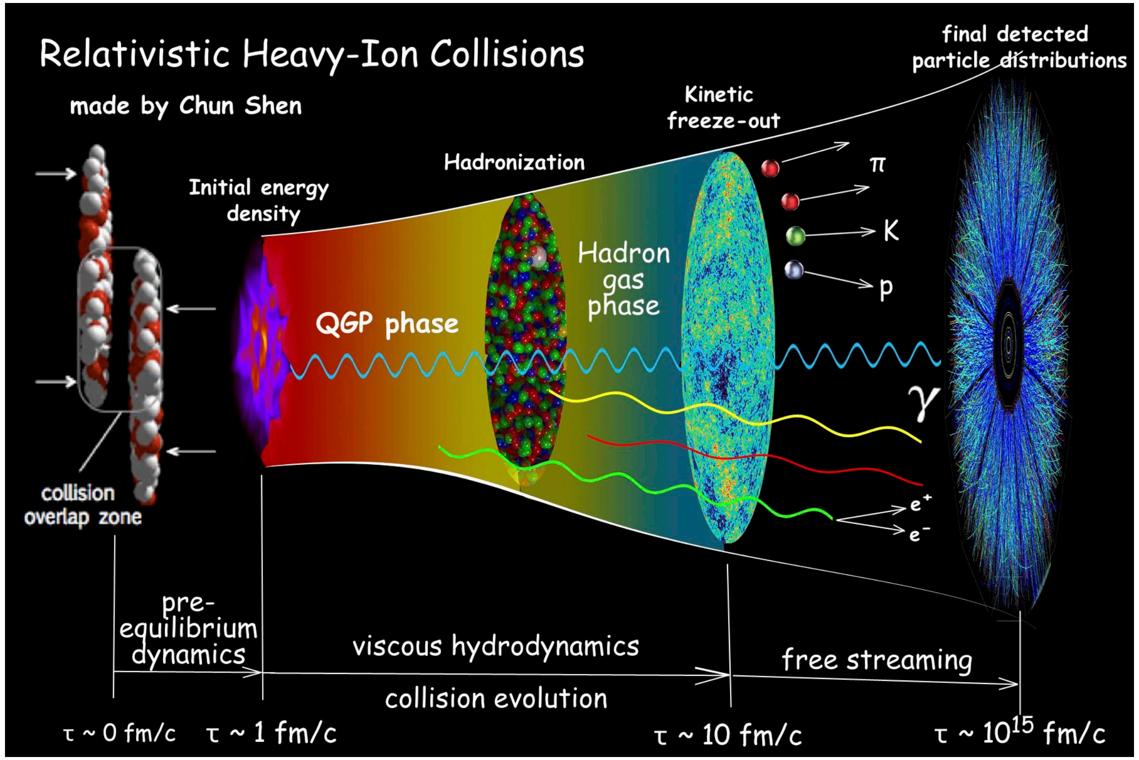
  Expansion sees singularity in the complex plane, Im $[\mu_B/T] = \pi$
  Critical point at $\mu_B/T < \pi$ disfavored



[V.V., Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)]

Critical point, if it exists, likely located beyond the reach of lattice methods

Relativistic heavy-ion collisions – "Little Bangs"



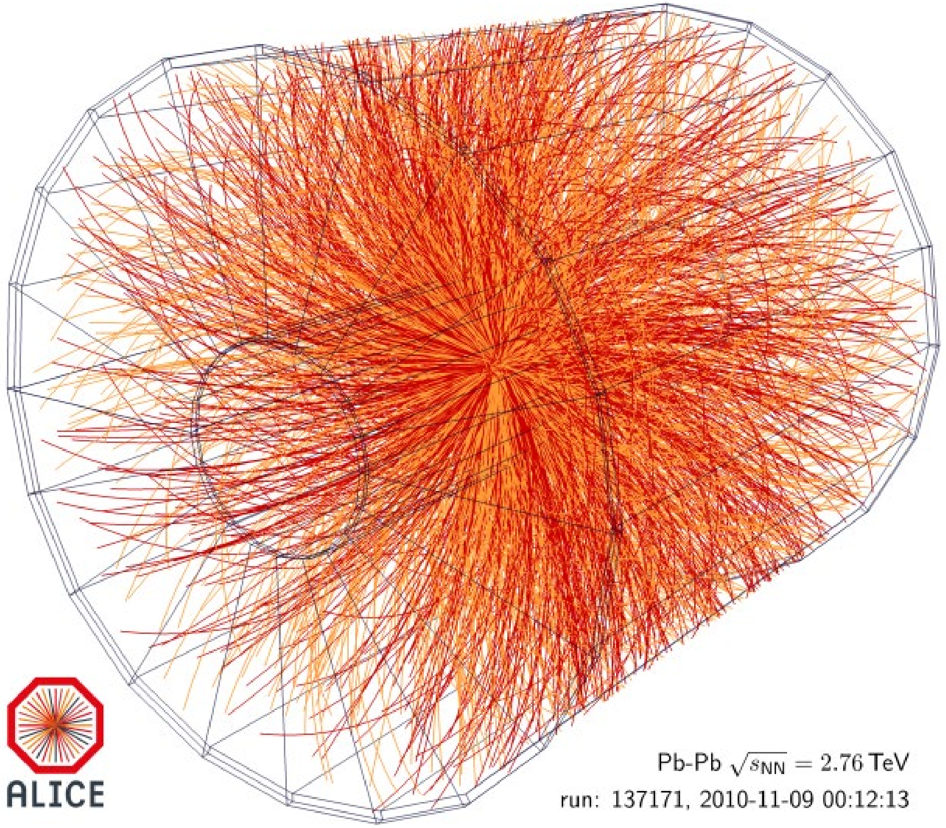
Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 – 5020$ GeV
- Size of the collision region

Measurements

- Final hadron abundances and momentum distributions

QCD phase diagram with heavy-ion collisions



Event display of a Pb-Pb collision in ALICE at the LHC

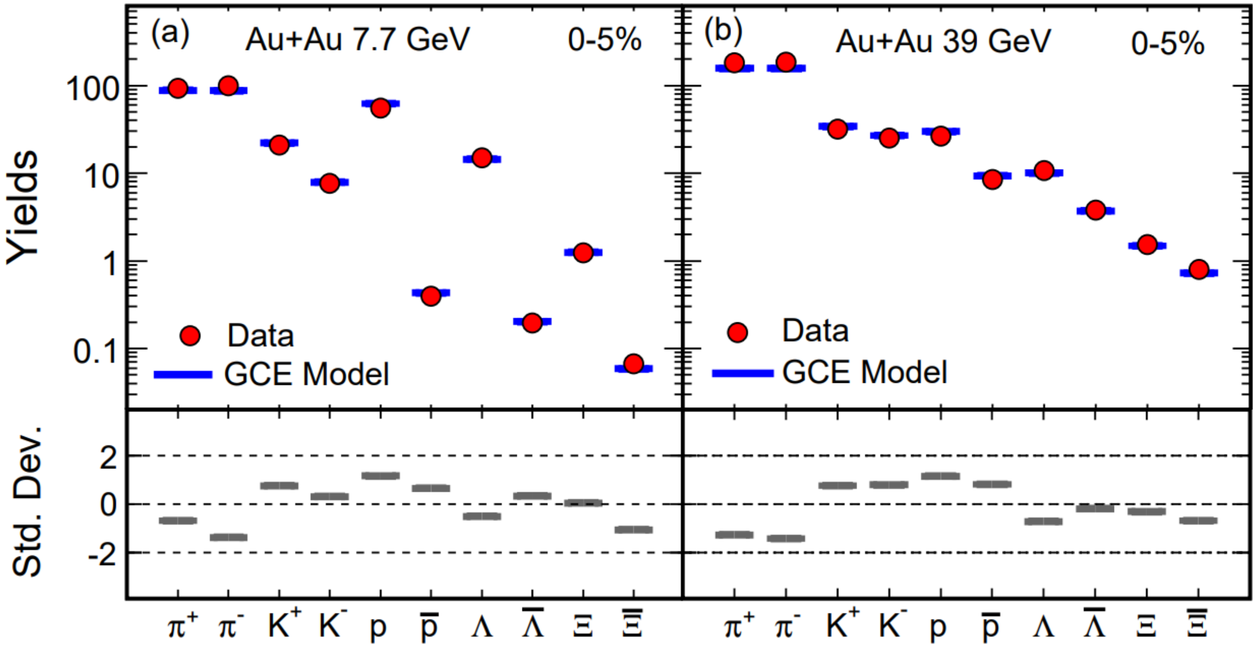
Thousands of particles created in relativistic heavy-ion collisions

⬇

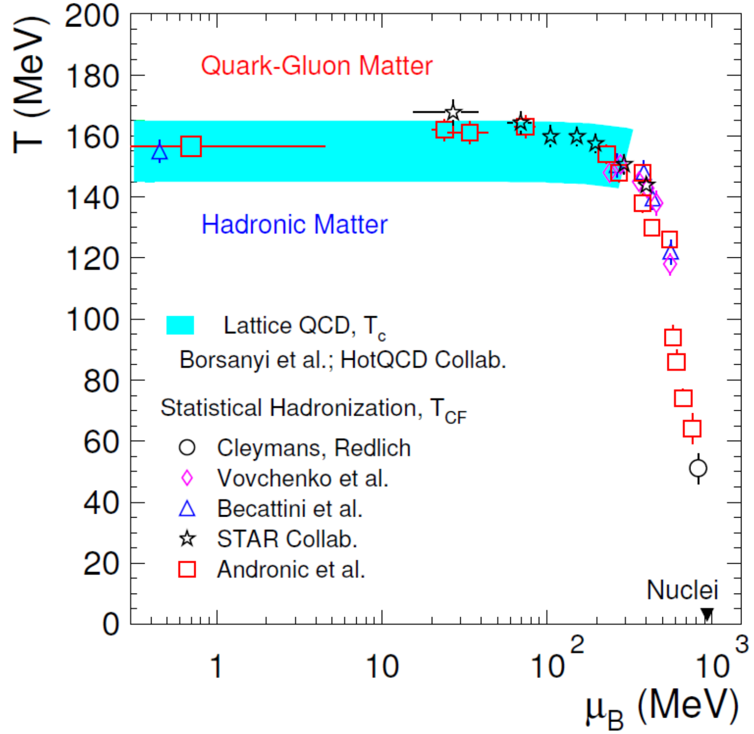
Apply concepts of statistical mechanics

Mapping heavy-ion collisions onto the QCD phase diagram

Fit hadron yields with the thermal model: $N_i = V \frac{d_i m_i^2 T}{2\pi^2} K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}} + \text{feedown}, \qquad \mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$



STAR Collaboration, PRC 96, 044904 (2017)



A. Andronic et al., Nature 561, 321 (2018)

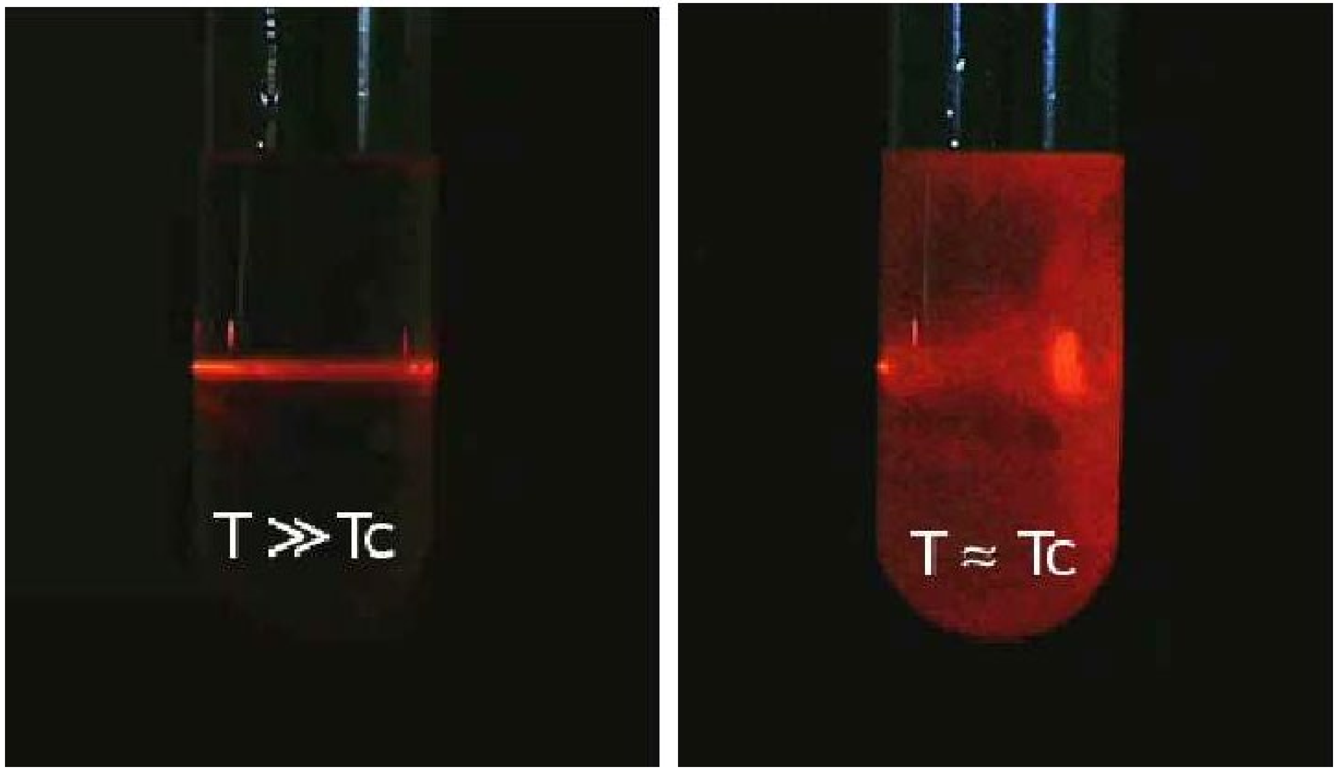
$\sqrt{s_{NN}} \searrow \quad \Rightarrow \quad \mu_B \nearrow$

For differential observables (spectra, flow, ...) use relativistic hydrodynamics

Critical point and fluctuations

Density fluctuations at macroscopic length scales

Critical opalescence



Unfortunately, we cannot do this in heavy-ion collisions

Event-by-event fluctuations and statistical mechanics

Consider a fluctuating number N

Cumulants: $G_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$

variance $\kappa_2 = \langle(\Delta N)^2\rangle = \sigma^2$  width

skewness $\kappa_3 = \langle(\Delta N)^3\rangle$  asymmetry

kurtosis $\kappa_4 = \langle(\Delta N)^4\rangle - 3\langle(\Delta N^2)\rangle^2$  peak shape

Experiment:

$$P(N) \sim \frac{N_{events}(N)}{N_{events}^{total}}$$

Statistical mechanics:

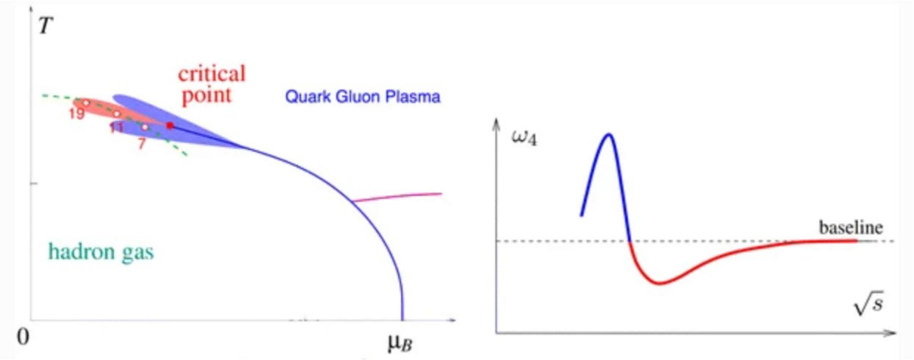
Grand partition function $\quad ln\, Z^{\mathrm{gce}}(T, V, \mu) = ln \left[\sum_N e^{\mu N} Z^{\mathrm{ce}}(T, V, N)\right],$

$$\boxed{\kappa_n \propto \frac{\partial^n(\ln\, Z^{\mathrm{gce}})}{\partial(\mu_N)^n}}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

Applications

- **(QCD) critical point** – large critical fluctuations of baryon (proton) number



M. Stephanov, Phys. Rev. Lett. (2011)

Correlation length $\xi \to \infty$ diverges at the critical point

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

Looking for non-monotonic dependence of κ_4 vs $\sqrt{s_{NN}}$

- Test of (lattice) QCD at $\mu_B \approx 0$

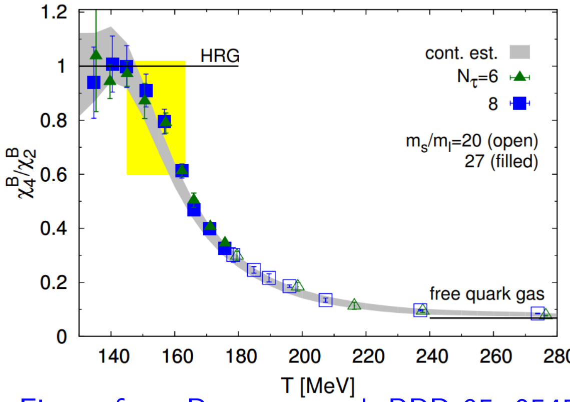
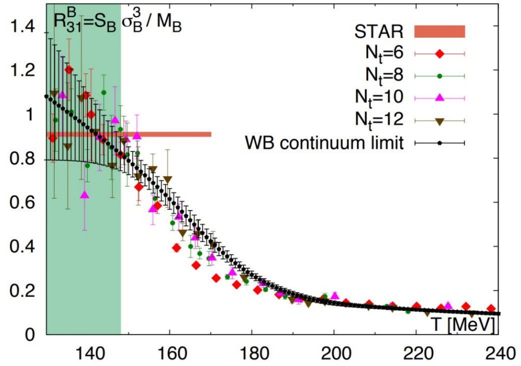
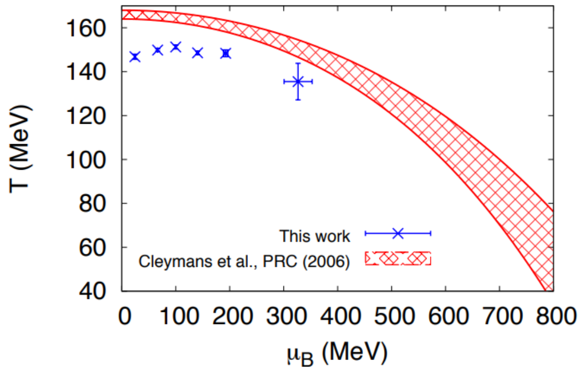


Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC

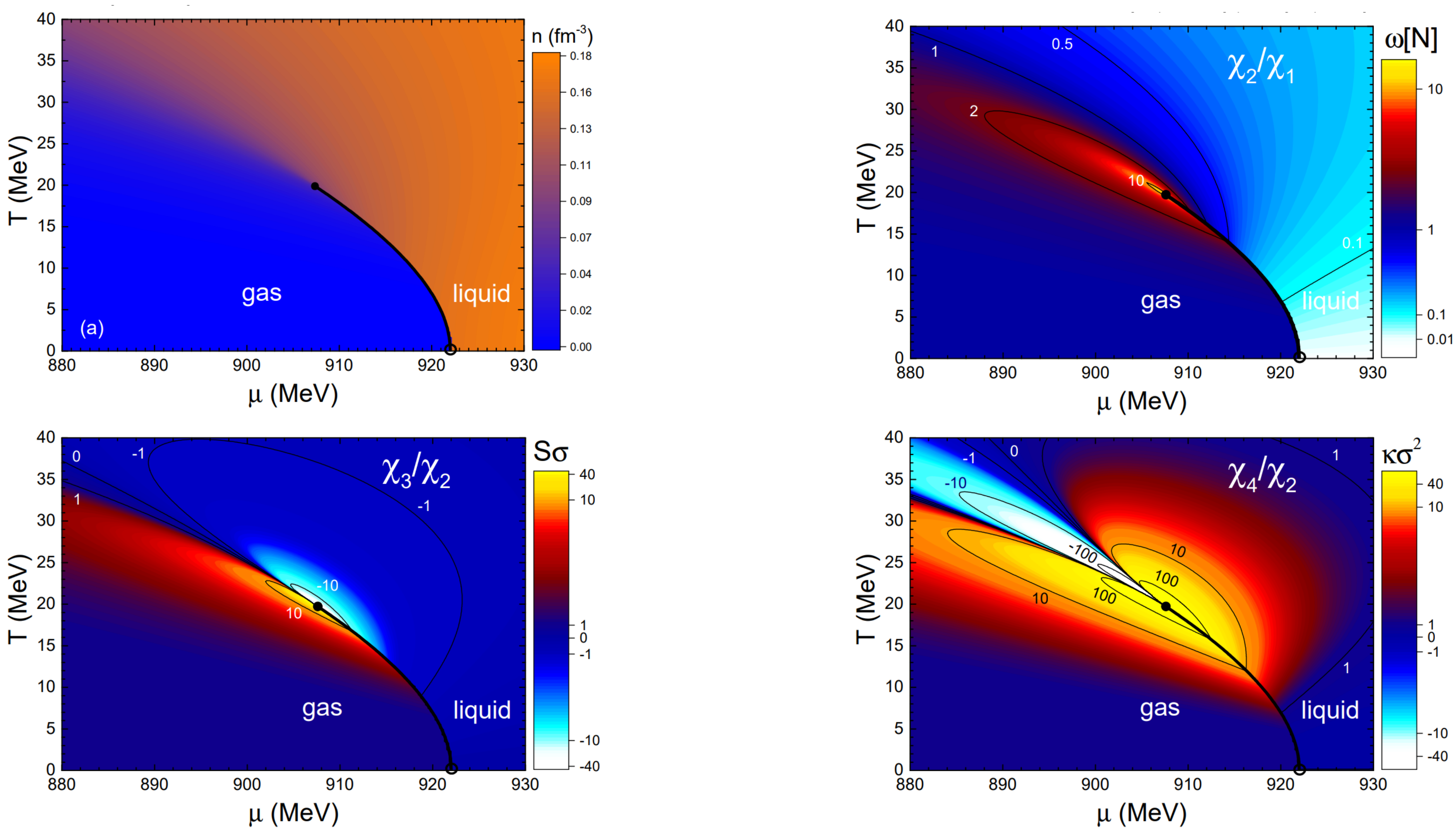
- Freeze-out from fluctuations





Borsanyi et al. PRL 113, 052301 (2014); Bazavov et al. PRL 109, 192302 (2012)

Example: Nuclear liquid-gas transition



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Example: Lennard-Jones fluid

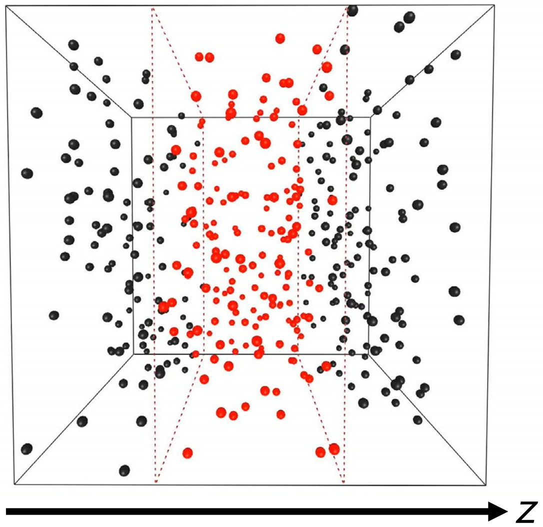
Kuznietsov, Savchuk, Gorenstein, Koch, VV, Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations* of a **Lennard-Jones fluid** along the (super)critical isotherm of the liquid-gas transition

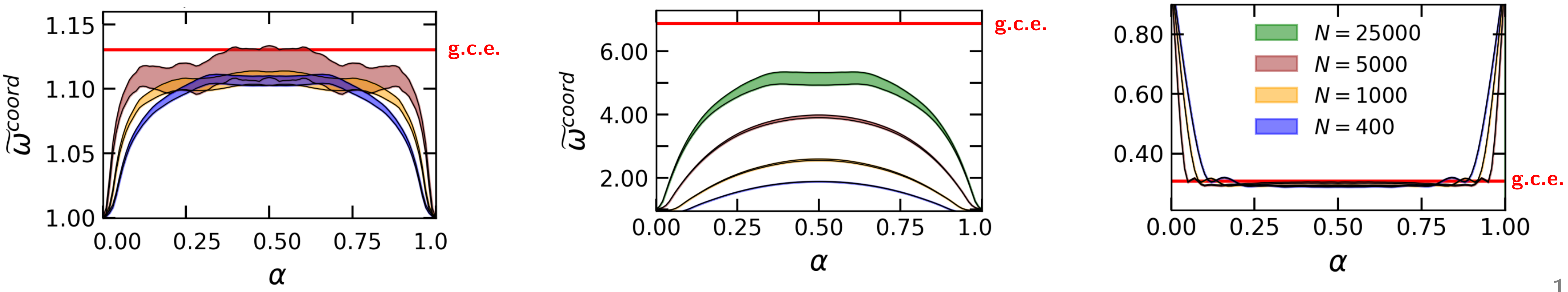
Microcanonical (const. *EVN*) ensemble with periodic boundary conditions

Variance of conserved particle number distribution inside coordinate space subvolume $|z| < z^{max}$ as time average

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



$n \approx 0.15\, n_c$ $\qquad$ $n \approx n_c$ $\qquad$ $n \approx 2 n_c$

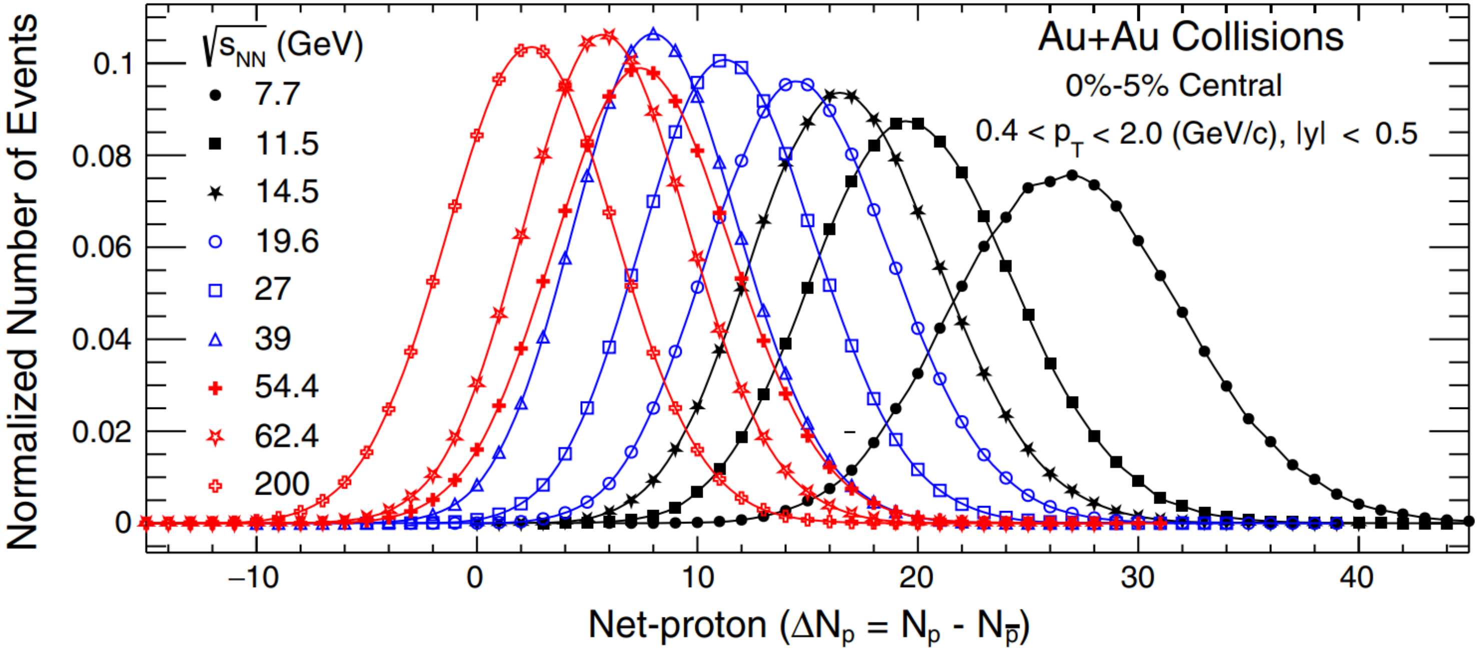


*Molecular dynamics code from https://github.com/vlvovch/lennard-jones-cuda

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $$P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{events}^{total}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)

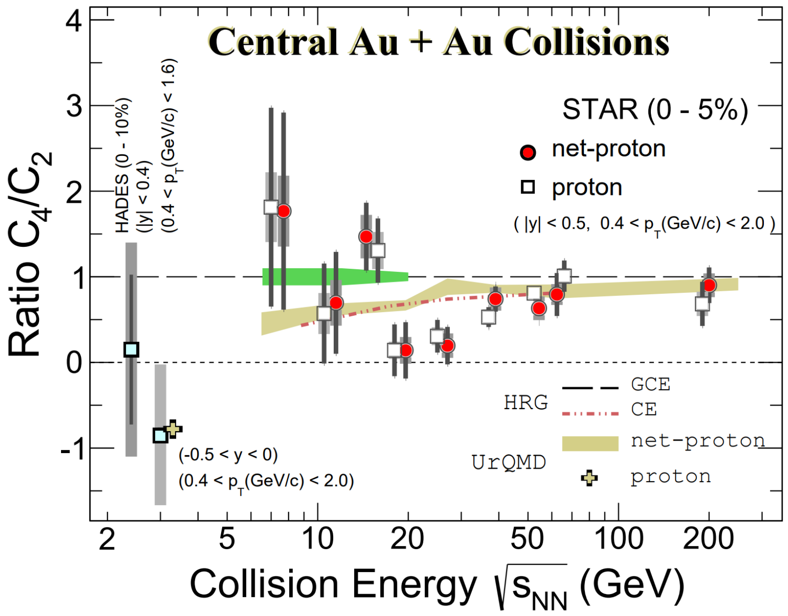


Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

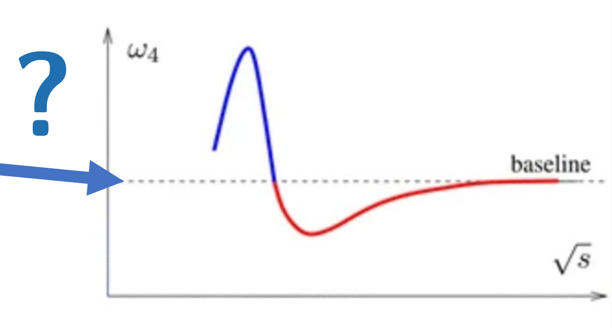
$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

Experimental measurements

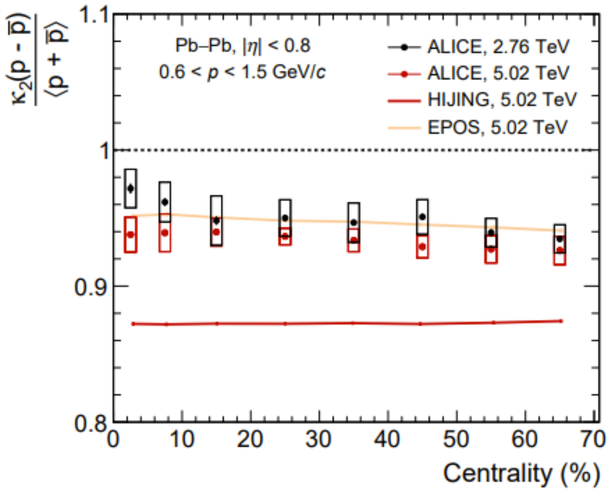
Beam energy scan in search for the critical point (STAR Coll.)



STAR Coll., Phys. Rev. Lett. 126, 092301 (2021); arXiv:2112.00240



M. Stephanov, Phys. Rev. Lett. (2011)



ALICE Coll., arXiv:2206.03343

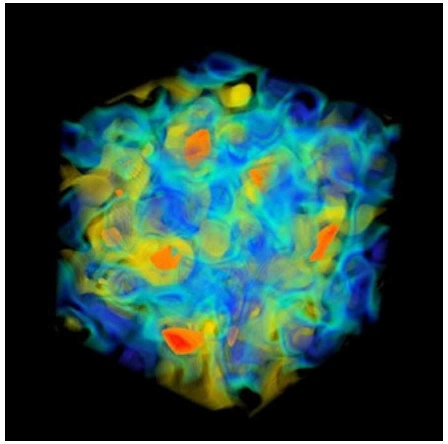
Reduced errors (better statistics), more energies, to come soon from RHIC-BES-II program, STAR-FXT etc.

Can we learn more from the more accurate data available for κ_2 and κ_3?

Other measurements: LHC-ALICE, GSI-HADES & CERN-NA61/SHINE Collaborations

Theory vs experiment: Challenges for fluctuations

Theory



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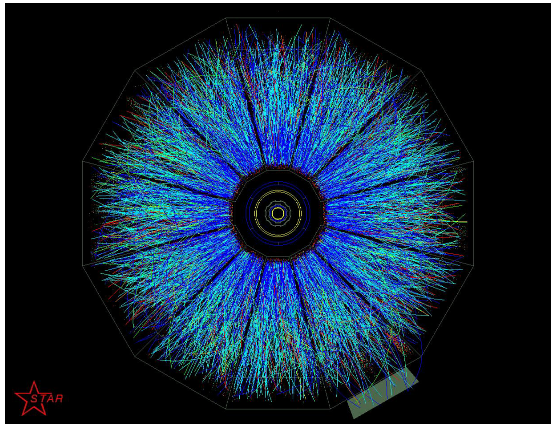
- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations
   - Fluctuating hydrodynamics
   - Equation of state with tunable critical point [P. Parotto et al, Phys. Rev. C 101, 034901 (2020)]

   Under development within the Beam Energy Scan Theory (BEST) Collaboration

   BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

2. Molecular dynamics with a critical point

   V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

3. **Deviations from precision calculations of non-critical fluctuations**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

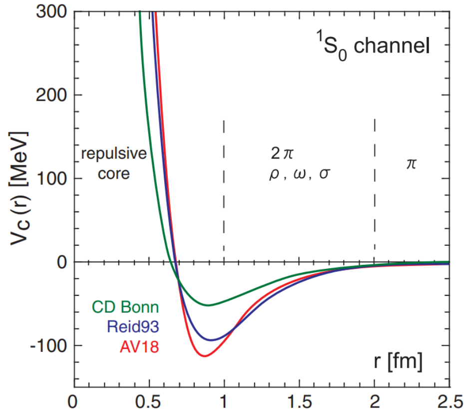


Figure from Ishii et al., PRL '07

Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via excluded volume

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

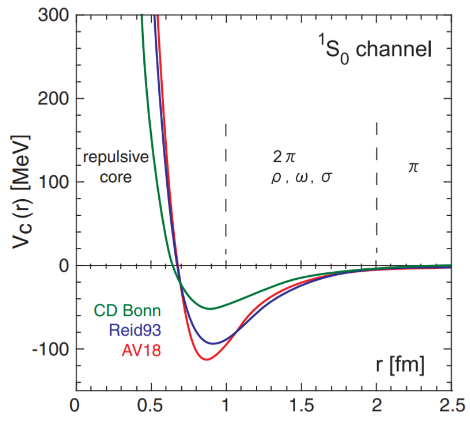
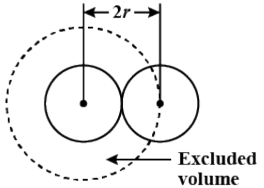


Figure from Ishii et al., PRL '07

Amounts to a van der Waals correction for baryons in the HRG model

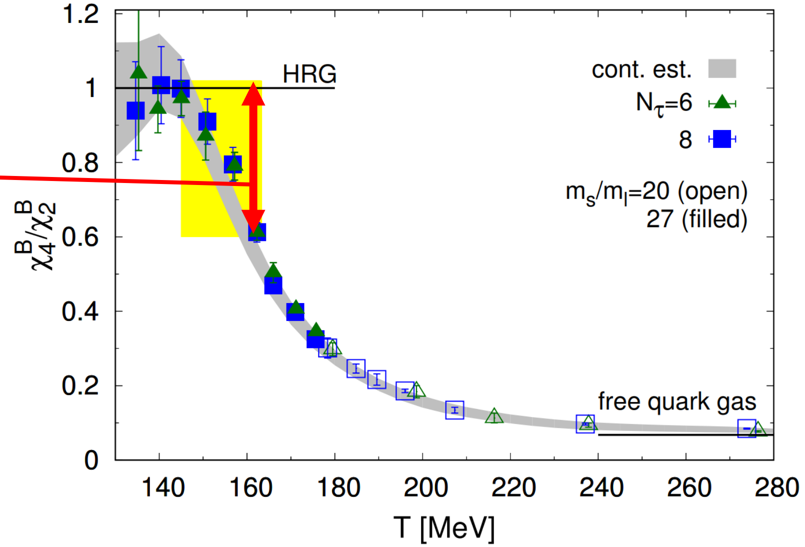
$$V \to V - bN$$



$$p^{\mathrm{ev}}_{B(\bar{B})} = p^{\mathrm{id}}_{B(\bar{B})}\, e^{-bp^{\mathrm{ev}}_{B(\bar{B})}/T}$$

- Net baryon kurtosis suppressed as in lattice QCD

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12b\phi_B(T) + O(b^2)$$



- Reproduces virial coefficients of baryon interaction from lattice QCD

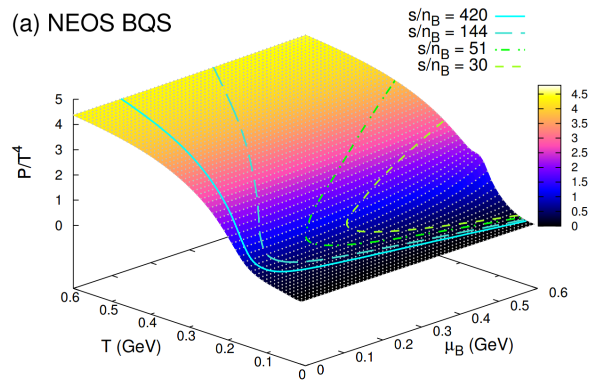
Excluded volume from lattice QCD: $\boxed{b \approx 1\ \mathrm{fm}^3}$

VV, A. Pasztor, S. Katz, Z. Fodor, H. Stoecker, Phys. Lett. B 755, 71 (2017)

Hydrodynamic description within non-critical physics

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

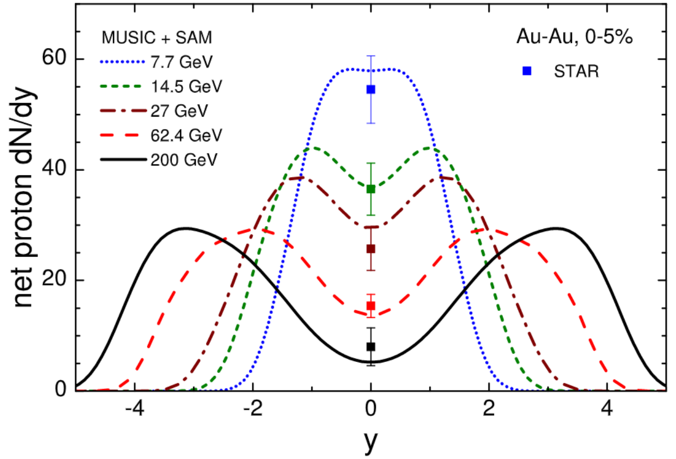
- Collision geometry based 3D initial state
  - Constrained to net proton distributions [Shen, Alzhrani, Phys. Rev. C '20]
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C '19]



- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \,\frac{d_j\, \lambda_j^{\mathrm{ev}}(x)}{(2\pi)^3}\, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization respects QCD-based baryon number distribution
  - Incorporated via baryon excluded volume b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]



- Incorporates exact global baryon conservation via a method SAM-2.0 [VV, Phys. Rev. C 105, 014903 (2022)]

Calculating cumulants from hydrodynamics

- Strategy:
  1. Calculate proton cumulants in the experimental acceptance in the grand-canonical limit
  2. Apply correction for the exact global baryon number conservation

First step:

- Sum contributions from each hypersurface element x_i at freeze-out
  - Cumulants of joint (anti)proton/(anti)baryon distribution

$$\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(\Delta p_{\mathrm{acc}}) = \sum_{i \in \sigma} \delta\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(x_i; \Delta p_{\mathrm{acc}})$$

$$p_{\mathrm{acc}}(x_i; \Delta p_{\mathrm{acc}}) = \frac{\int_{p \in \Delta p_{\mathrm{acc}}} \frac{d^3p}{\omega_p} \delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu; T(x_i), \mu_j(x_i)]}{\int \frac{d^3p}{\omega_p} \delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu; T(x_i), \mu_j(x_i)]}.$$

- To compute each contribution
  - GCE susceptibilities $\chi^{B^\pm}(x_i)$ define the distribution of the emitted (anti)baryons
  - Each baryon ends up in acceptance Δp_{acc} with binomial probability via the Cooper-Frye formula
  - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i)\rangle / \langle N_B(x_i)\rangle$

*For the Monte Carlo version of this procedure see [VV, Koch, Phys. Rev. C 103, 044903 (2021)]

Correcting for baryon number conservation with SAM-2.0

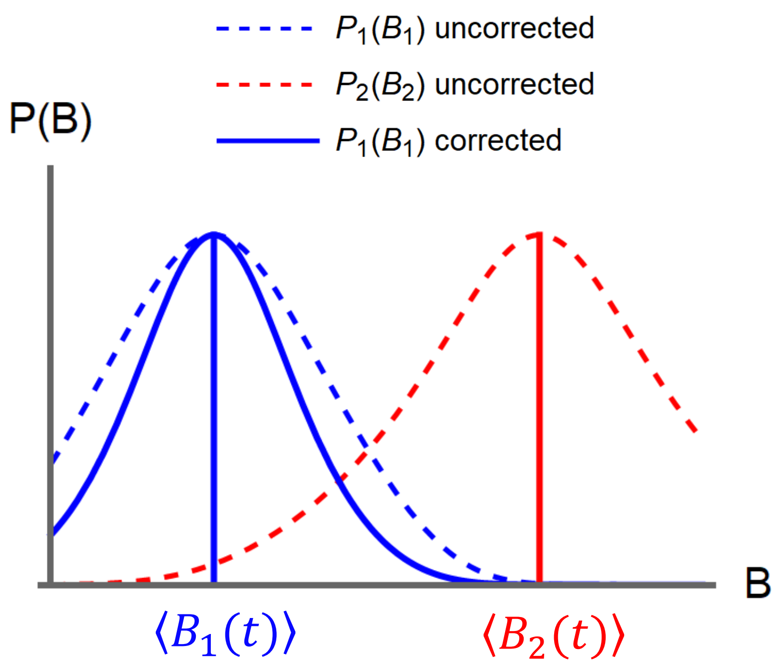
VV, Phys. Rev. C 105, 014903 (2022)

$$P_1^{\rm ce}(B_1) \propto \sum_{B_1,B_2} P_1^{\rm gce}(B_1) P_2^{\rm gce}(B_2) \times \delta_{B,B_1+B_2}$$

SAM-1.0: uniform thermal system and **coordinate** space

SAM-2.0: apply the correction for *arbitrary* distributions inside and outside the acceptance that are peaked at the mean

- Spatially inhomogeneous systems (e.g. RHIC)
- Momentum space
- Non-conserved quantities (e.g. proton number)
- Map "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance

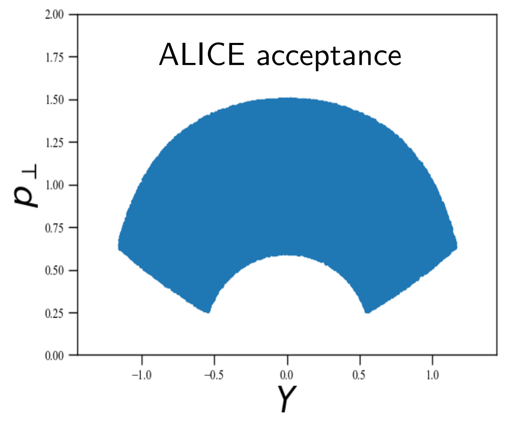


$$\boxed{\kappa_{p,B}^{\rm in,ce} = {\rm SAM}\left[\kappa_{p,B}^{\rm in,gce}, \kappa_{p,B}^{\rm out,gce}\right]}$$

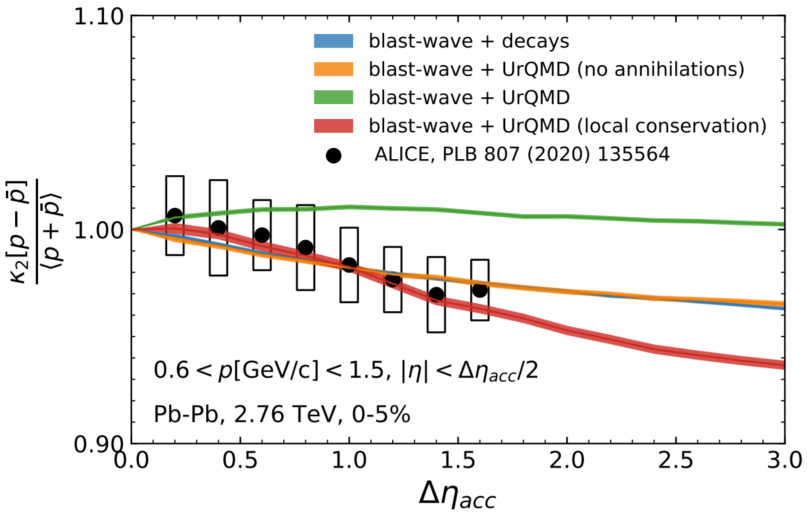
*Explicit expressions for any cumulant available via a Mathematica notebook at https://github.com/vlvovch/SAM

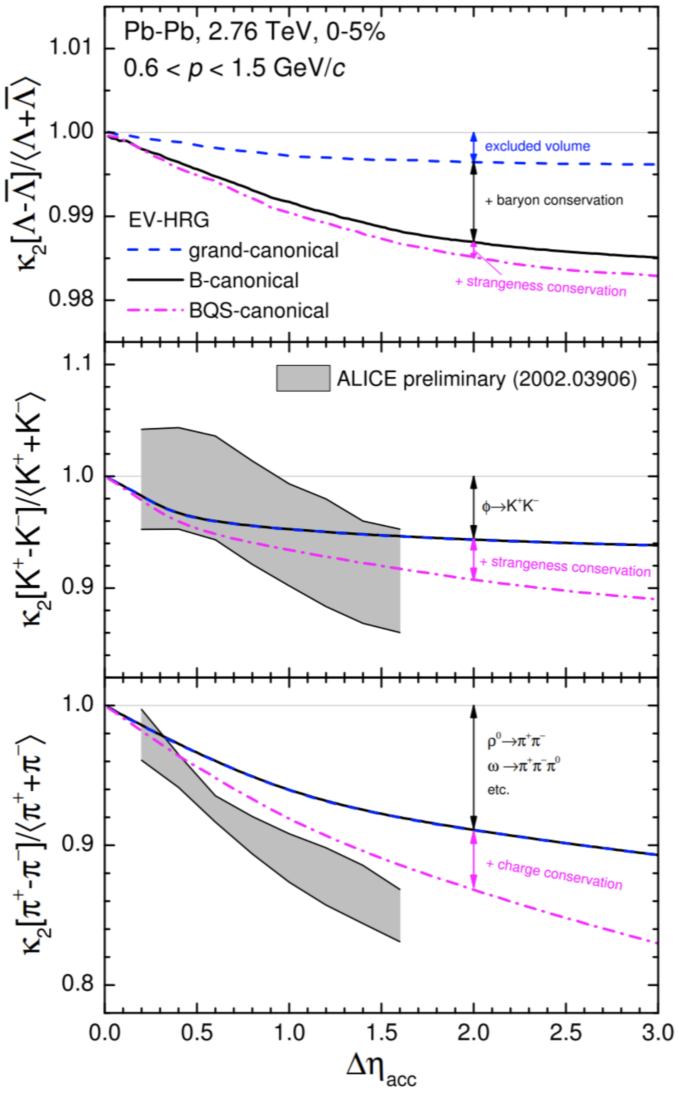
Net-particle fluctuations at the LHC (blast-wave)

- Net protons described within errors and consistent with either
  - global baryon conservation without $B\bar{B}$ annihilations
    see e.g. ALICE Coll. arXiv:2206.03343
  - or local baryon conservation with $B\bar{B}$ annihilations
    O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



$0.6 < p < 1.5$ GeV/c, $\Delta\eta_{acc} = 1.6$



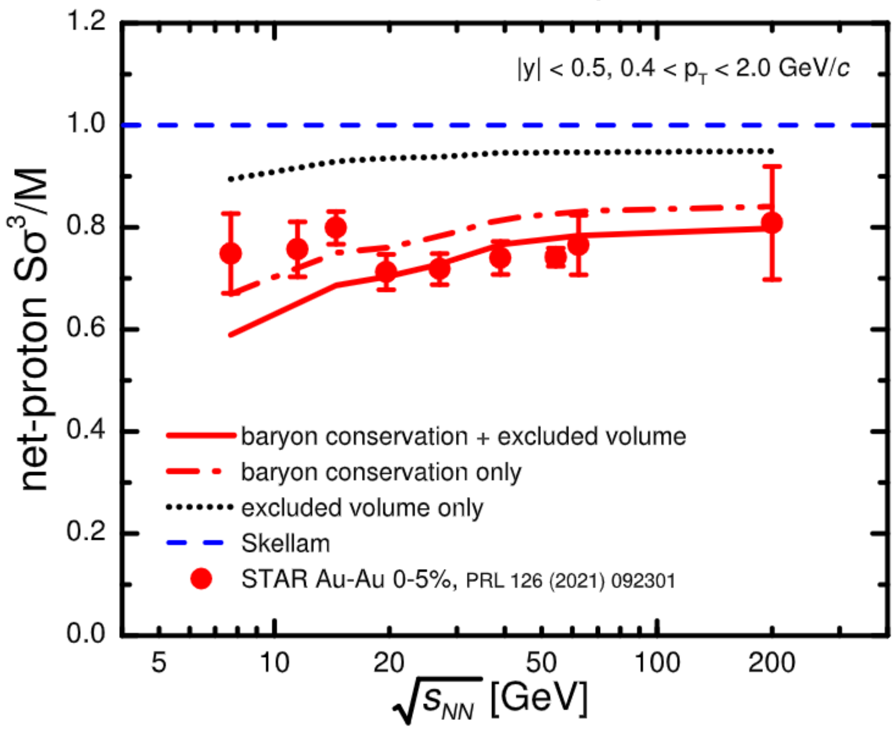


- Large effect from resonance decays for pions and kaons + exact conservation of electric charge/strangeness

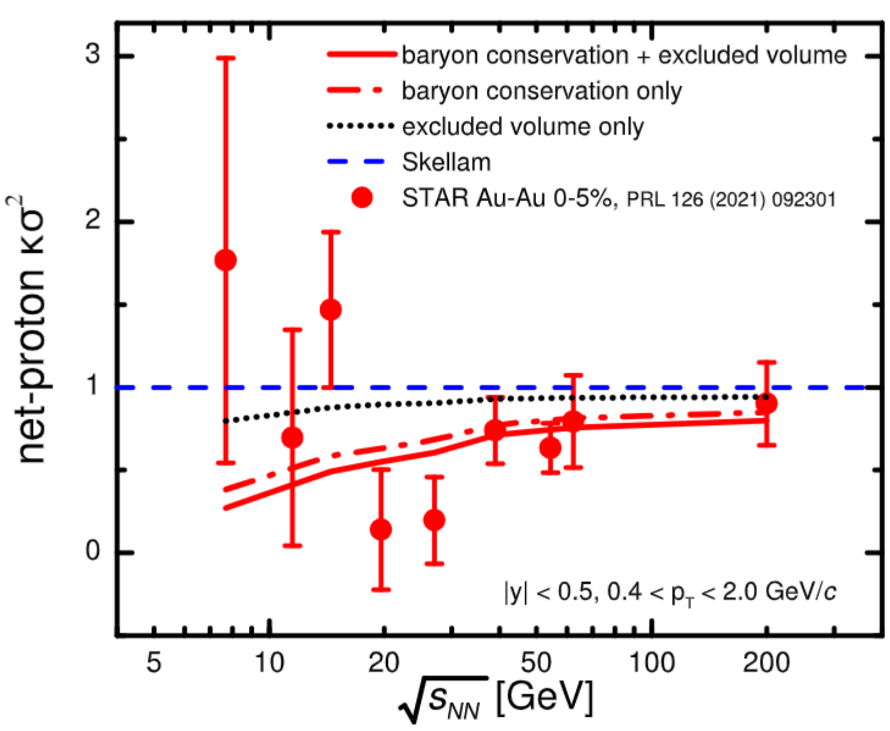
VV, Koch, Phys. Rev. C 103, 044903 (2021)

RHIC-BES: Net proton cumulant ratios (MUSIC)

κ_3/κ_1



κ_4/κ_2



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (baryon conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of skewness in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

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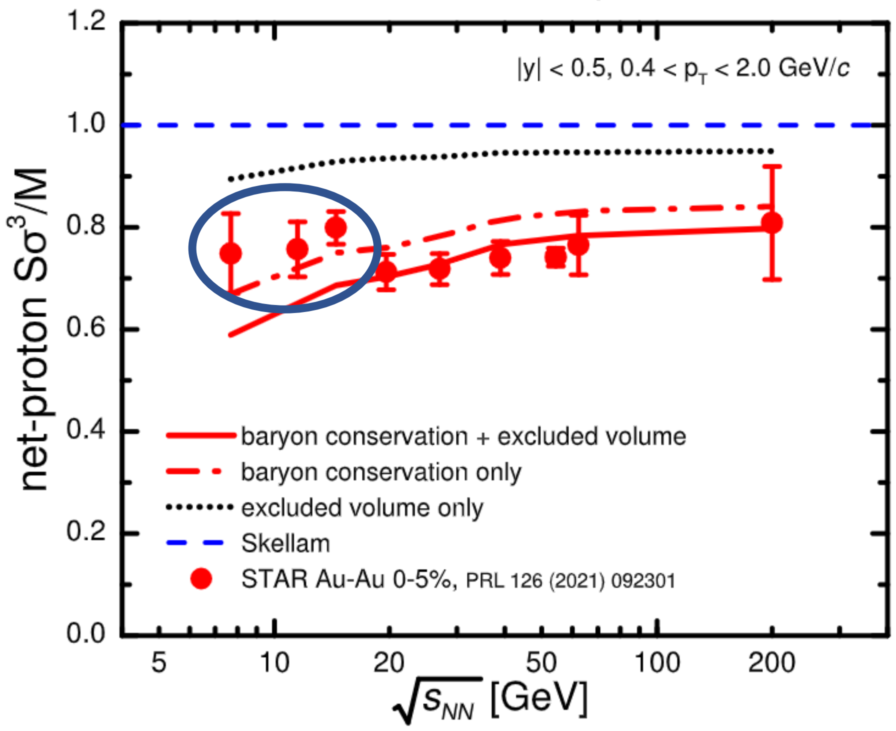
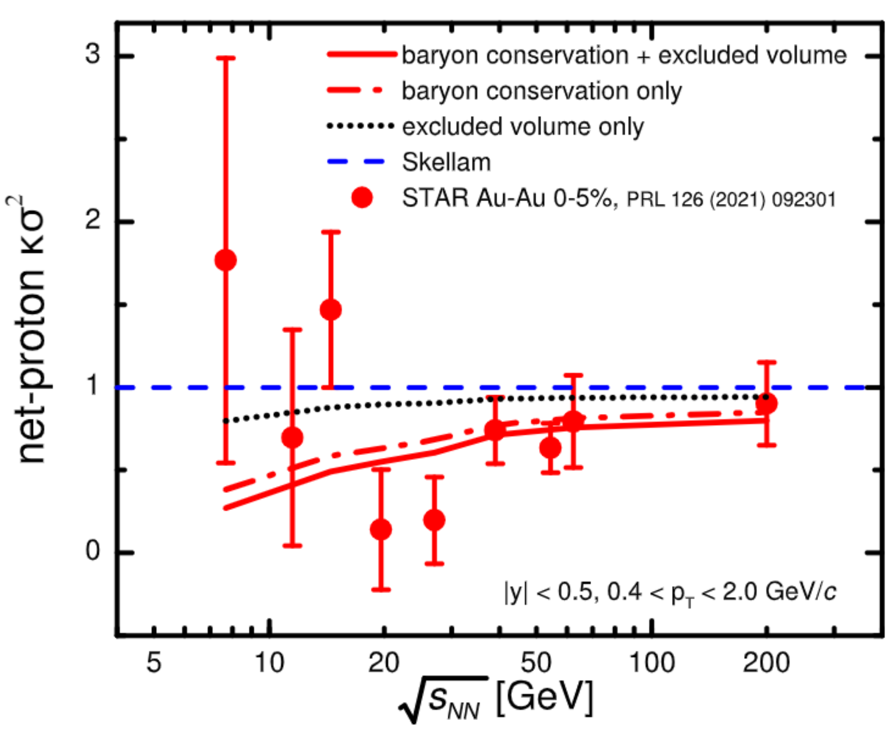


κ_4/κ_2



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VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Correlation Functions

- Analyze genuine multi-particle correlations via **factorial cumulants** $\hat{C}_n$
  [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

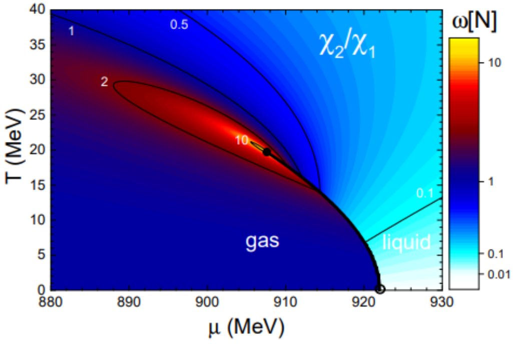
$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

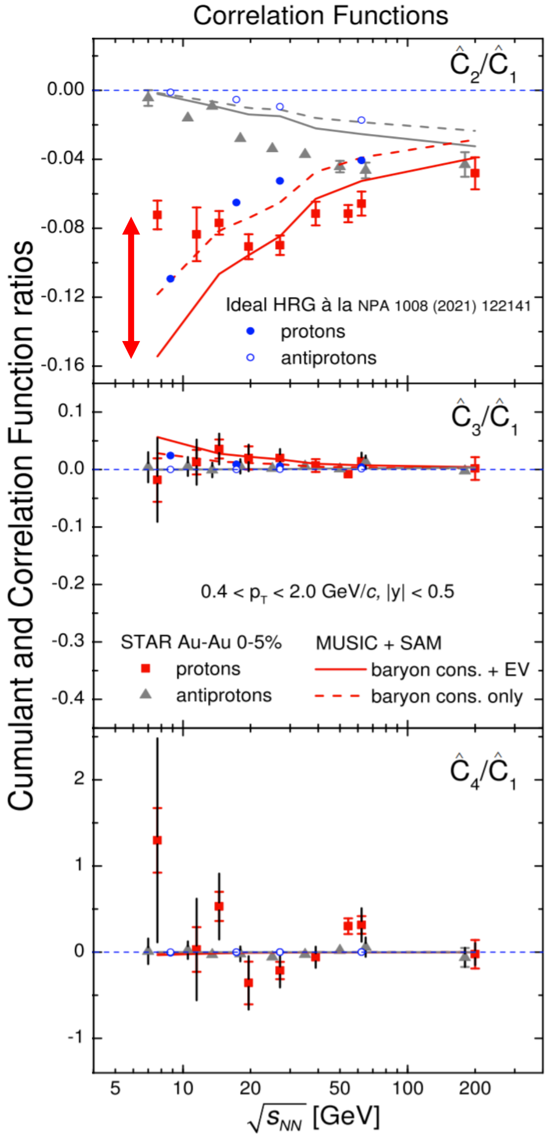
$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

- Three- and four-particle correlations are small without a CP
  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

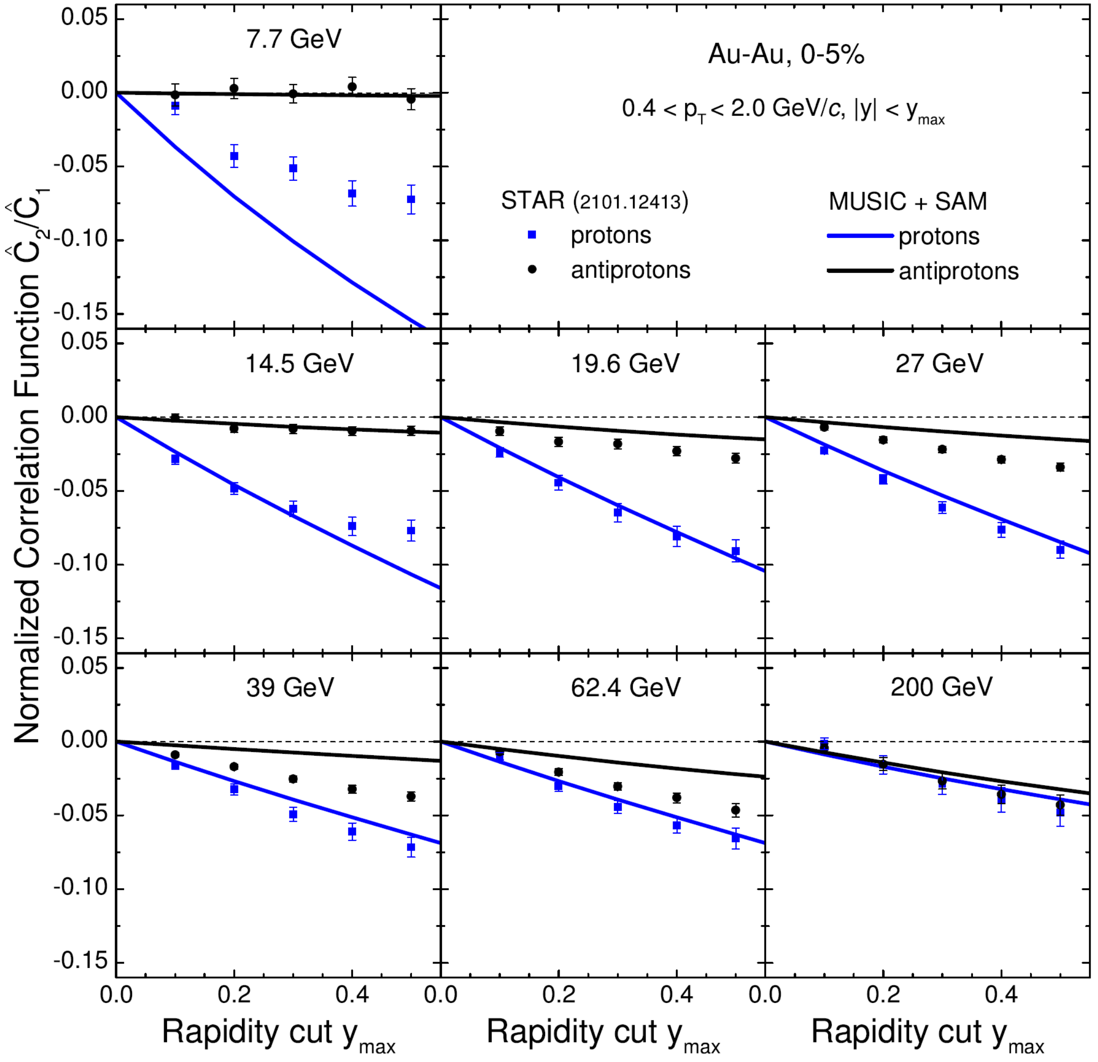
- Signals from the data at $\sqrt{s_{NN}} \leq 20$ GeV
  - Excess of two-proton correlations
  - Possibility of significant 4-proton correlations
  - ***Critical point?***





Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



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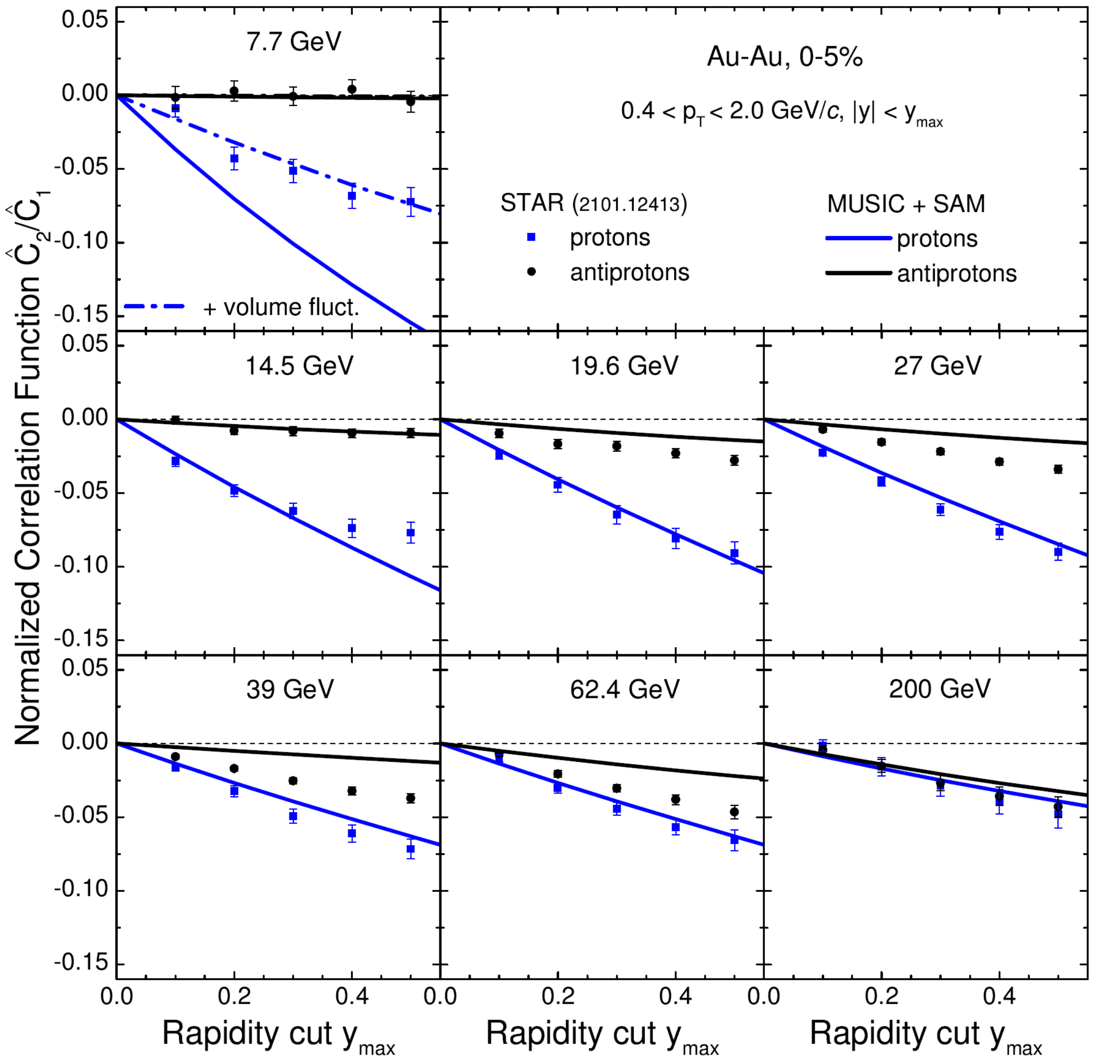
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$



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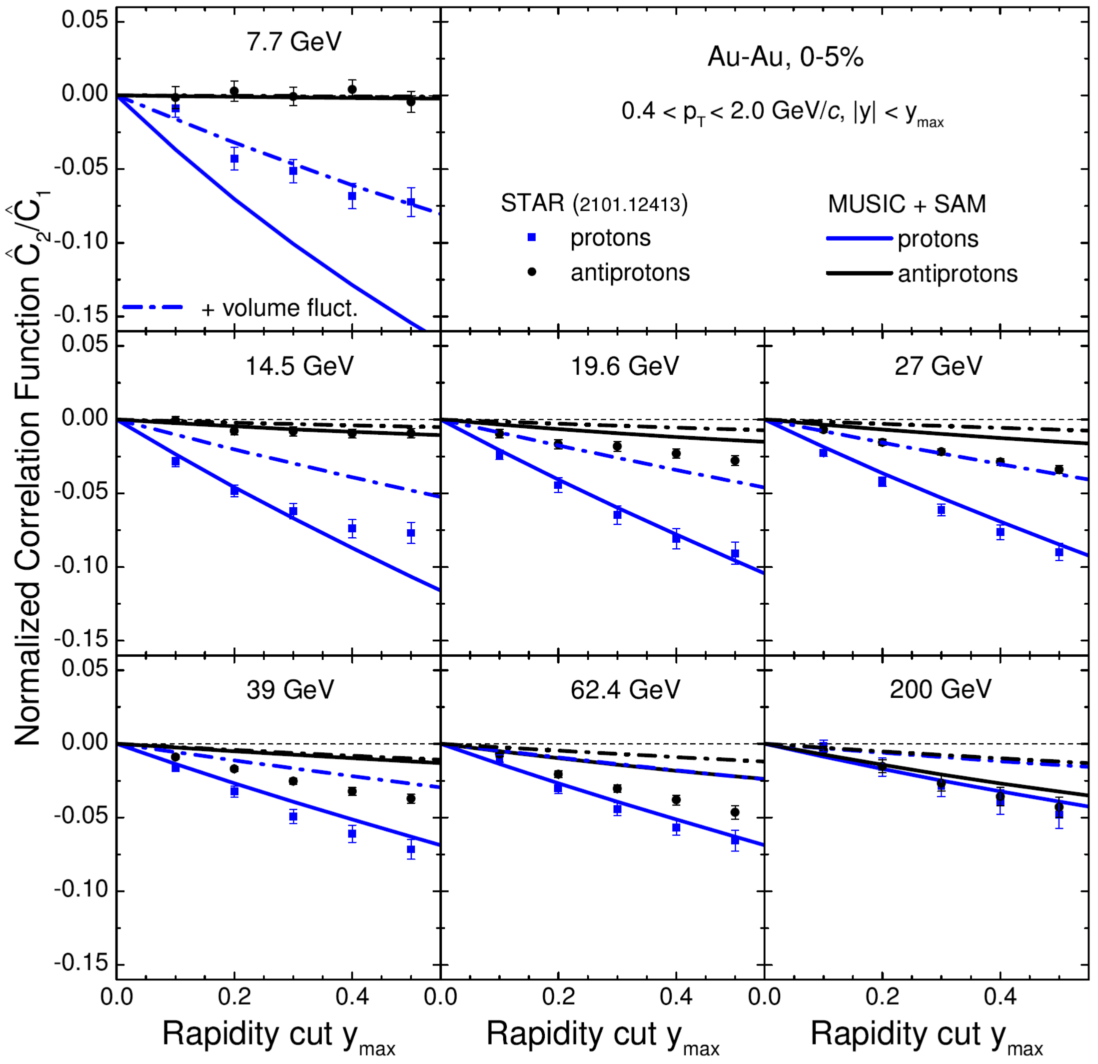

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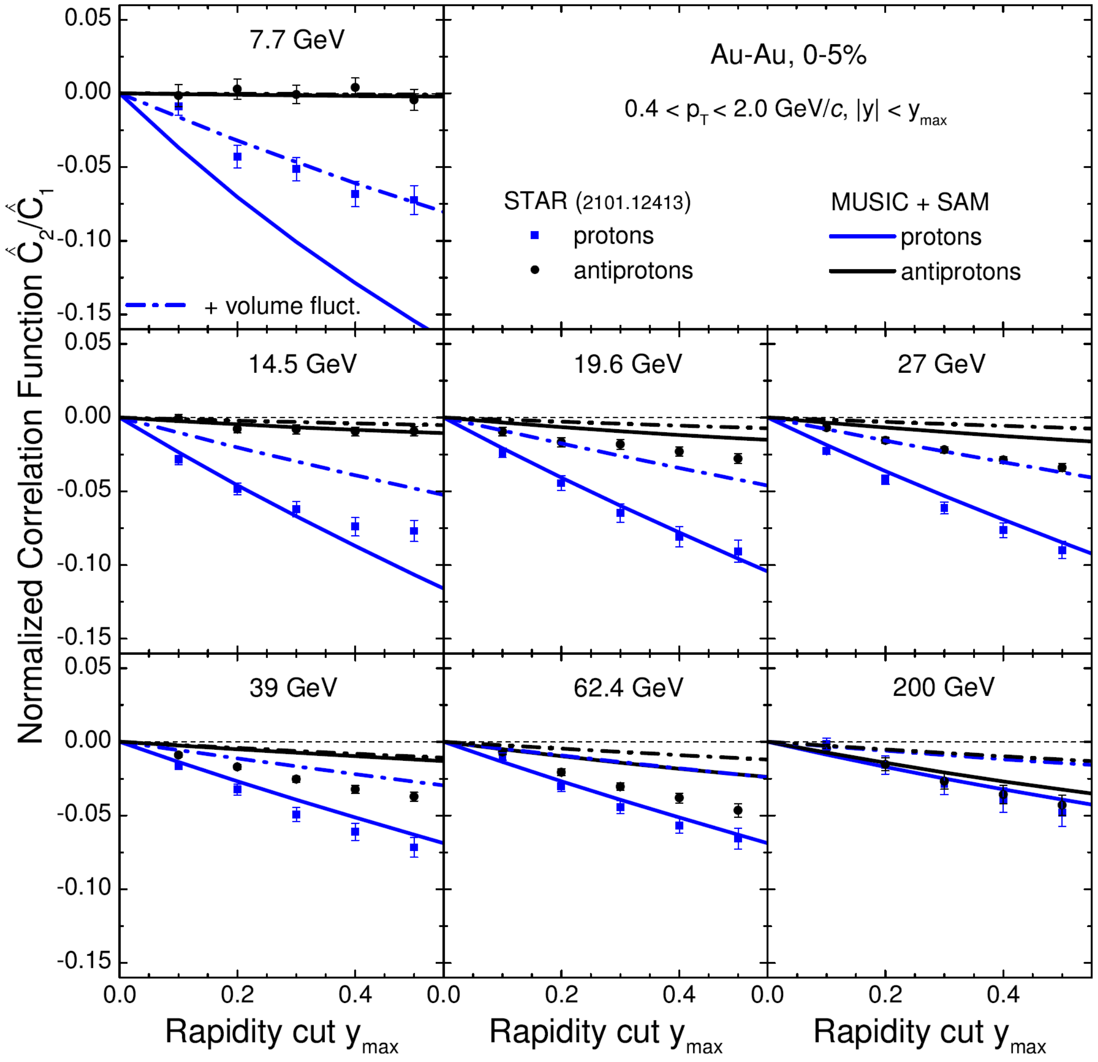
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected



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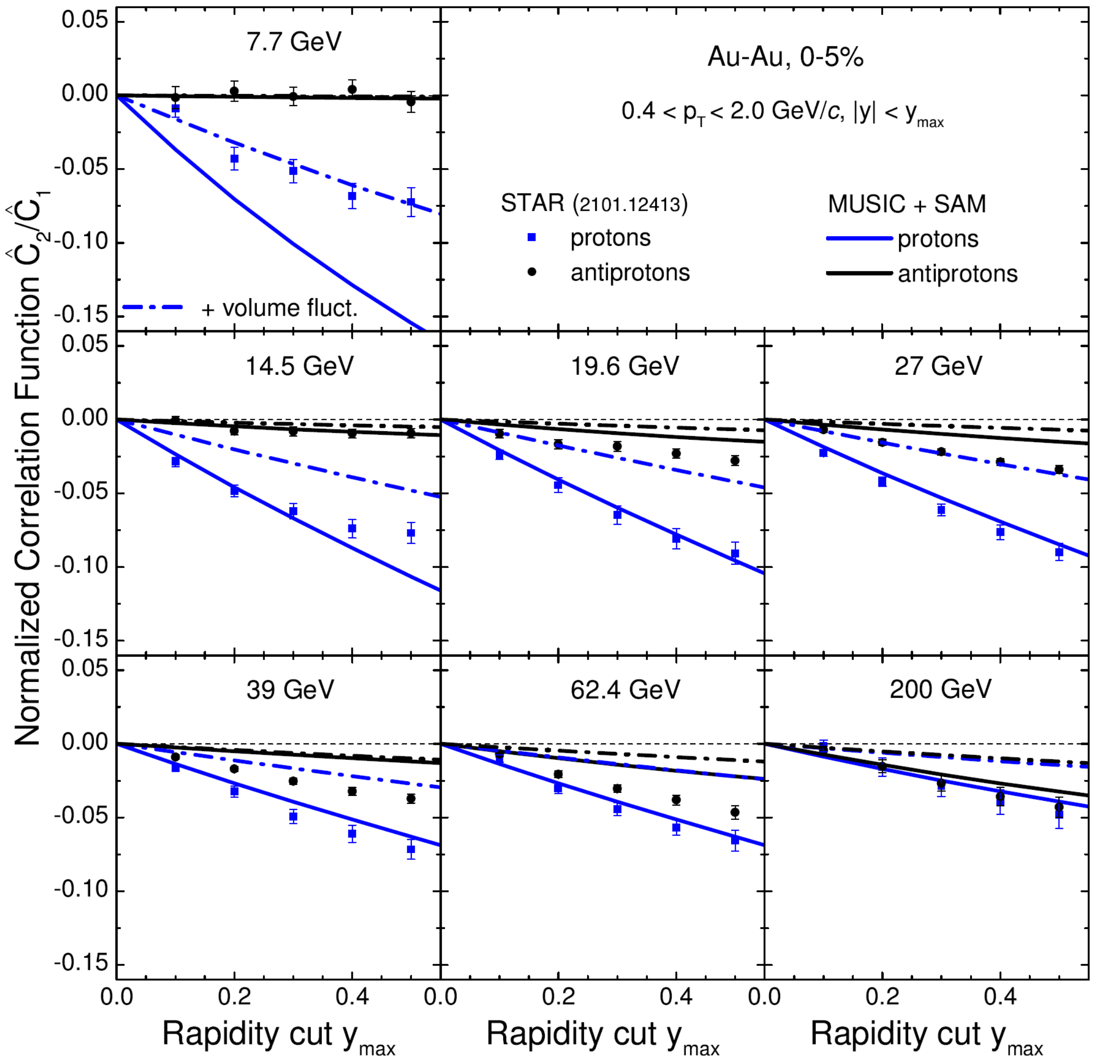
- Exact electric charge conservation?
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**



Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

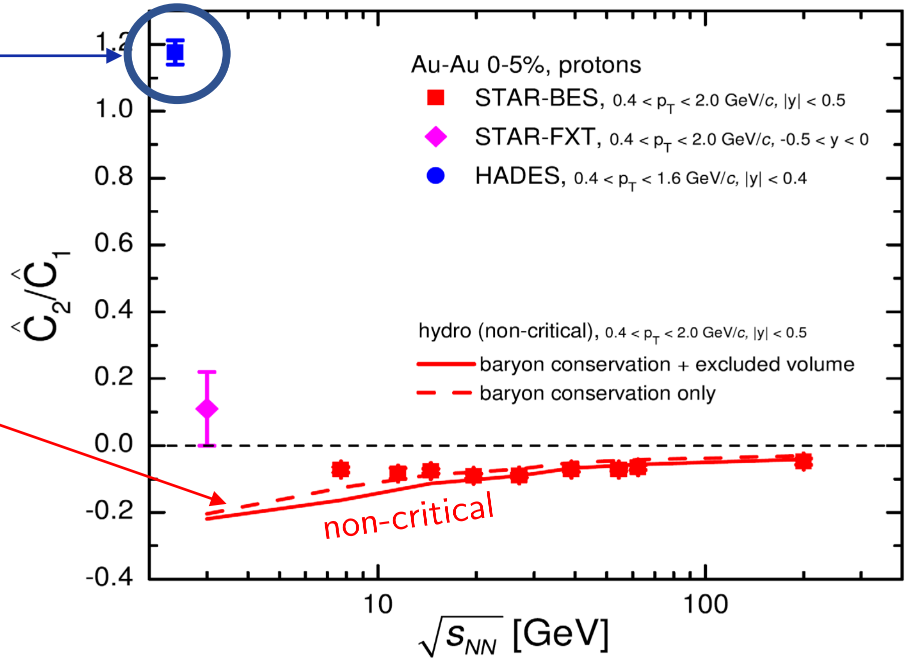
- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge excess of two-proton correlations!

  [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

- No change of trend in the non-critical reference

- Additional mechanisms:
  - Nuclear liquid-gas transition
  - Light nuclei formation

- Fill the gap with ongoing/future data from STAR-FXT (e.g. arXiv:2112.00240), future experiments like CBM-FAIR



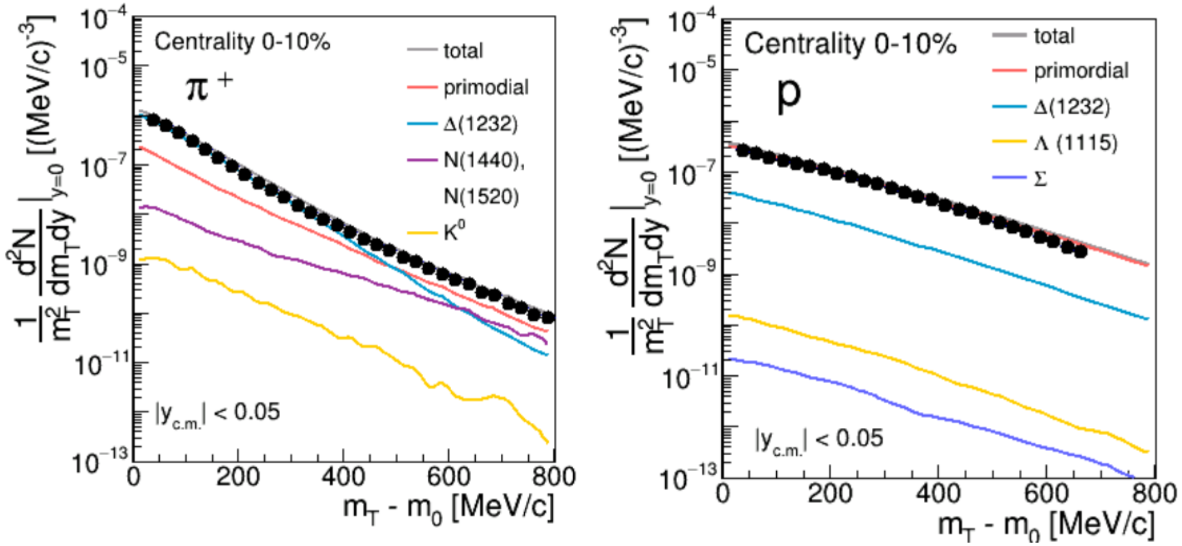
Take a closer look at the HADES data

Thermodynamic analysis of HADES data

VV, Koch, arXiv:2204.00137

- Single freeze-out scenario: Emission from Siemens-Rasmussen hypersurface with Hubble-like flow
  - $\rightarrow$ Pion and proton spectra o.k.

  [S. Harabasz et al., PRC 102, 054903 (2020)]



- Uniform $T \approx 70$ MeV, $\mu_B \approx 875$ MeV across the fireball [A. Motornenko et al., PLB 822, 136703 (2021)]

- Fluctuations:
  - Same as before but incorporate additional binomial filtering to account for protons bound in light nuclei
  - Uniform fireball $\rightarrow$ Final proton cumulants are linear combinations of baryon susceptibilities χ_n^B at freeze-out

$$\kappa_n^p = \sum_{m=1}^{n} \alpha_{n,m}\, \chi_m^B$$

Extract χ_n^B directly from experimental data

A closer look at the HADES data

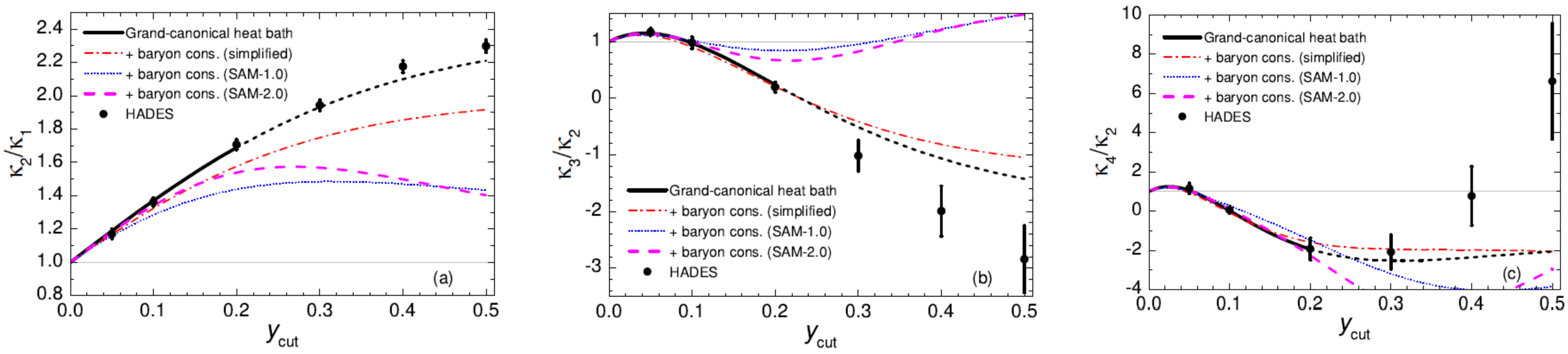
VV, Koch, arXiv:2204.00137

- Fit baryon susceptibilities to data within a fireball model (Siemens-Rasmussen*)

- In the grand-canonical limit (no baryon conservation, small y_{cut}) the data are described well with

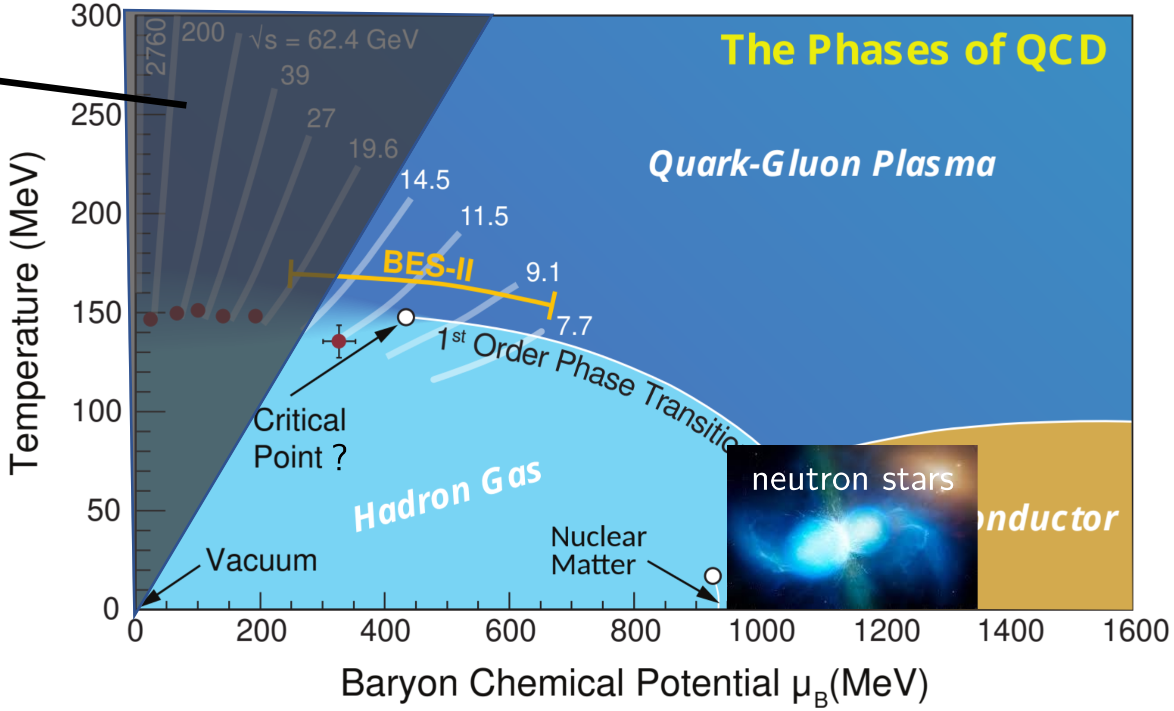
$$\frac{\chi_2^B}{\chi_1^B} \sim 9.17 \pm 0.21, \qquad \frac{\chi_3^B}{\chi_2^B} \sim -33.1 \pm 0.8, \qquad \frac{\chi_4^B}{\chi_2^B} \sim 691 \pm 50, \quad \text{i.e.} \quad \boxed{\chi_4^B \gg -\chi_3^B \gg \chi_2^B \gg \chi_1^B}$$

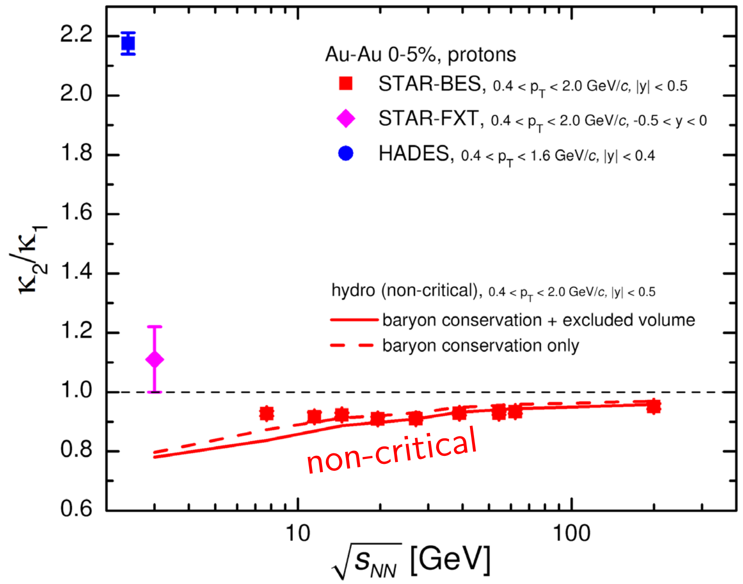
- Could be indicative of a *critical point* near the HADES freeze-out at $T{\sim}70$ MeV, $\mu_B{\sim}875$ MeV
- However, the results for $y_{cut} > 0.2$ are challenging to describe with baryon conservation included



Summary: What we learned so far from fluctuations

Critical point disfavored





- Heavy-ion data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with "non-critical" physics
  - Disfavors QCD critical point at μ_B/T<2-3, consistent with what we know from lattice QCD
- Interesting indications for (multi)-proton correlations at $\sqrt{s_{NN}} \leq 7.7$ GeV

Thanks for your attention!