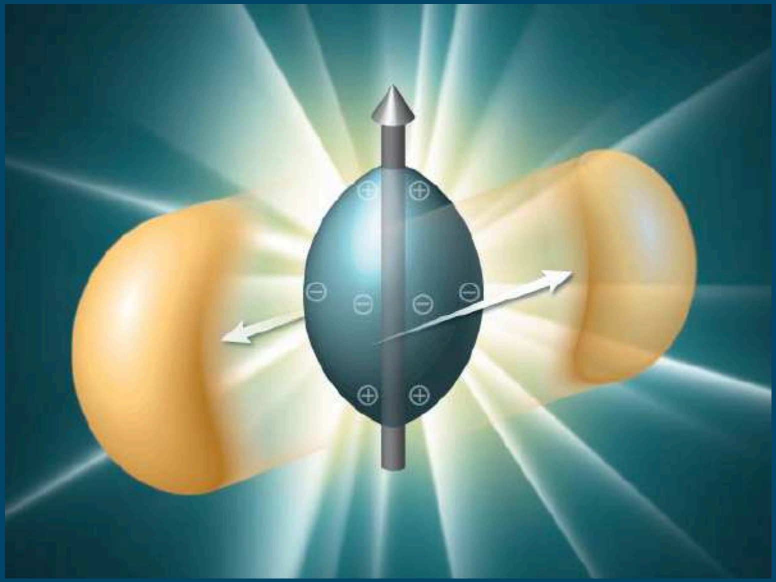
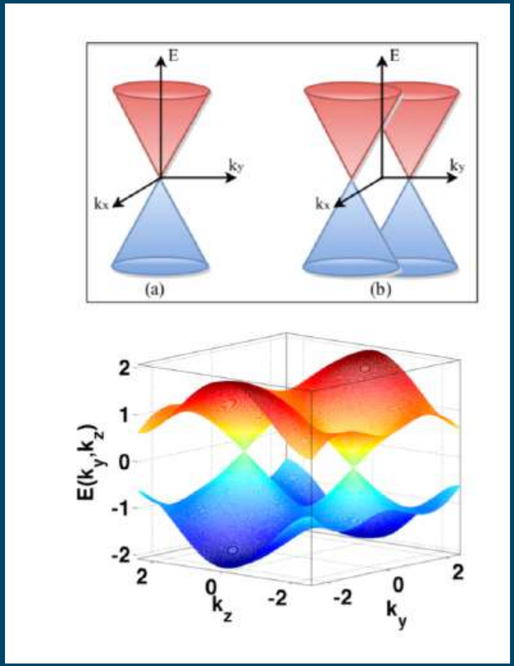
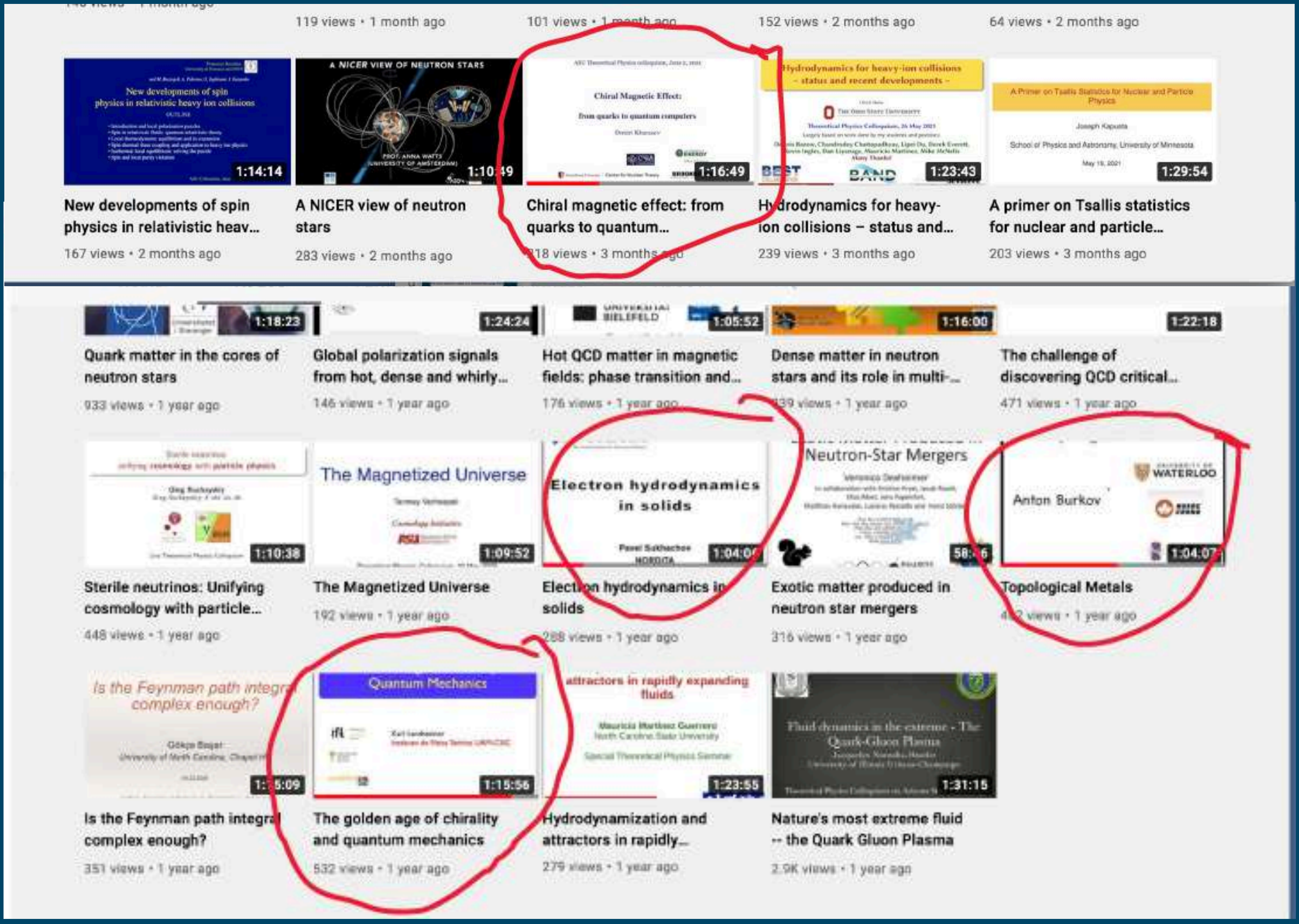
Geometry and anomalies in Dirac matter





María A. H. Vozmediano
Instituto de Ciencia de Materiales de Madrid, CSIC

admin note: text overlap with

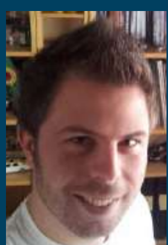


Condensed matter collaborators

A. Cortijo

F. de Juan

A. Grushin



Y. Ferreiros

V. Arjona

O. Pozo

High energy collaborators

M. Chernodub

K. Landsteiner

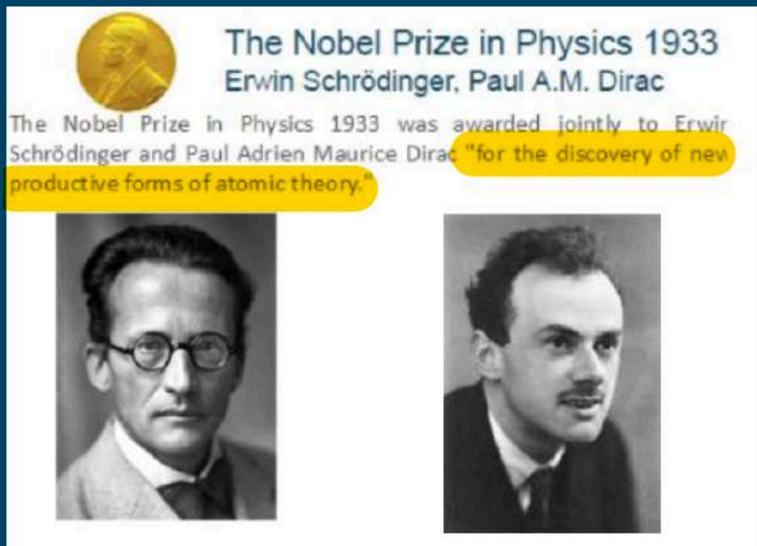
D. Kharzeev

M. Baggioli

https://wp.icmm.csic.es/field-theories-in-condensed-matter-physics/

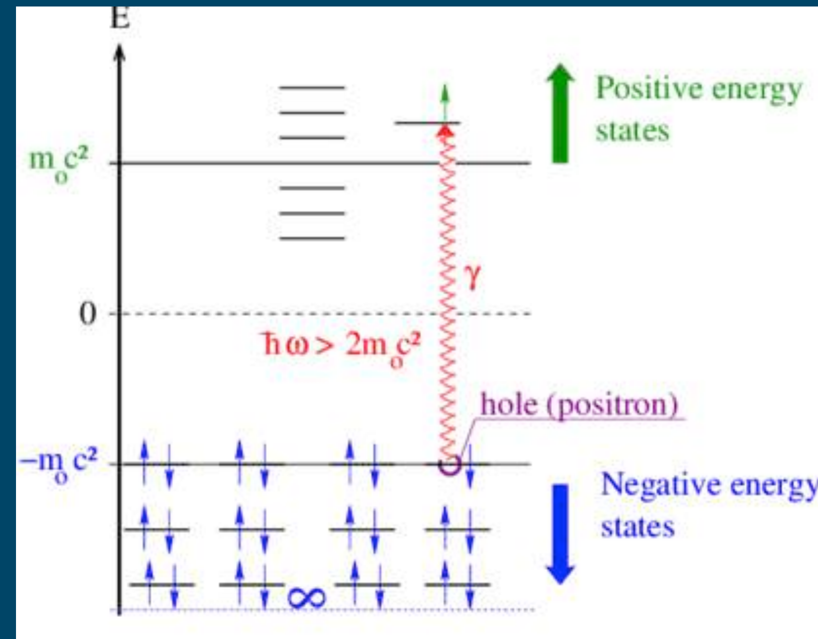
1. Dirac physics: HEP vs CM

High school QFT fermions



$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$(i\gamma\cdot\partial - m)\psi = 0$$



Negative energy solutions:

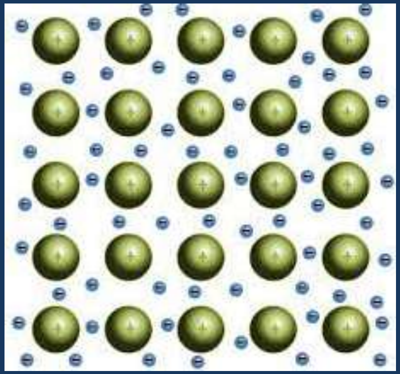
Vacuum: the infinite, filled, Dirac sea.

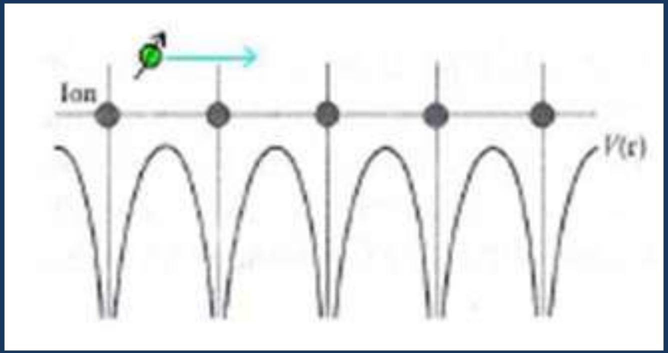
Particles: quantum excitations of the Dirac sea

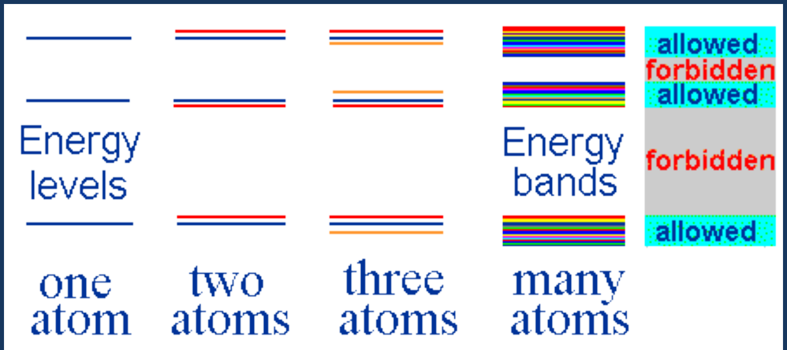
Antiparticles

High school solids

Crystal: many atoms in a periodic lattice. Band theory.

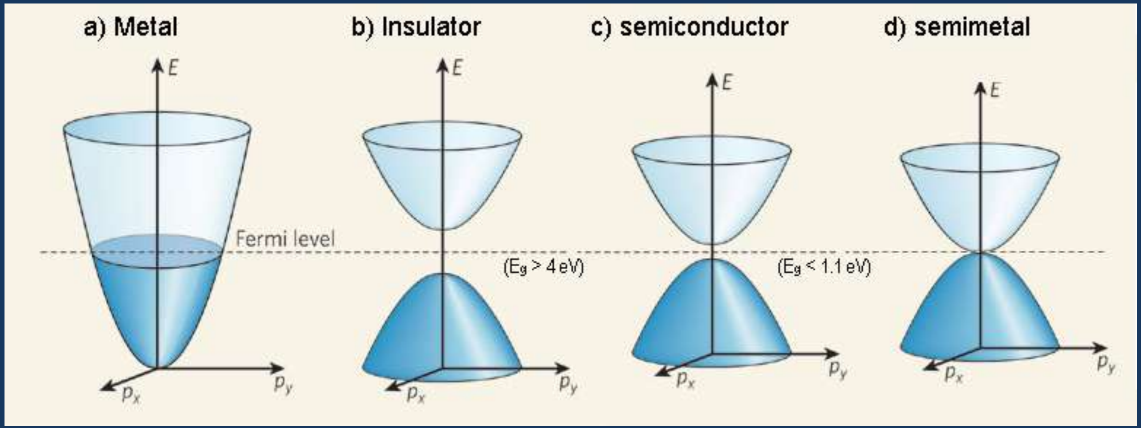






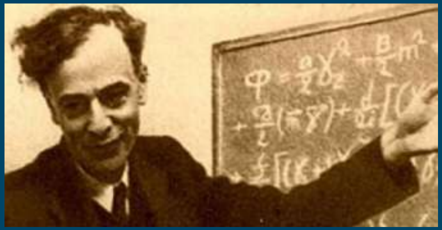
Electrons: Pauli exclusion principle

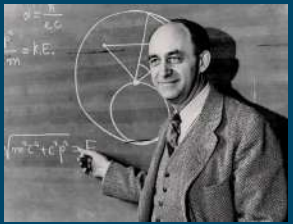
The Fermi level:
Last occupied energy



The standard model of condensed matter

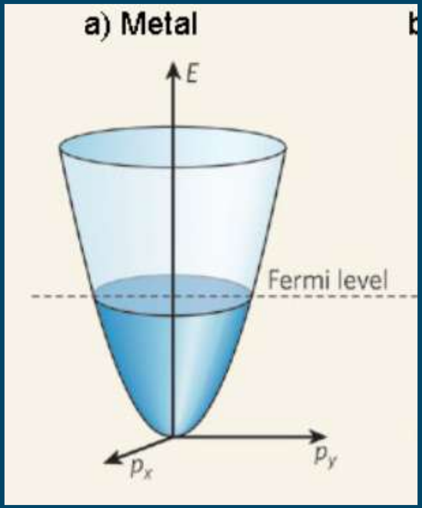
The Landau Fermi liquid for metals*



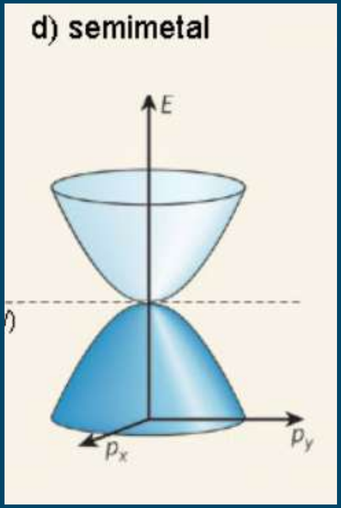


Vacuum: the Fermi sea

$$\varepsilon(k) = k^2 / 2m^*$$



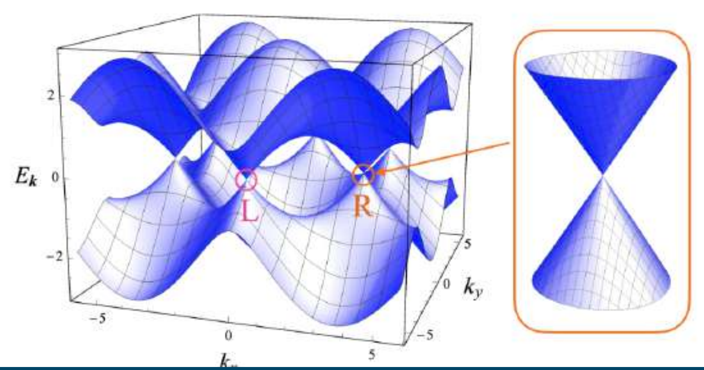
$$\varepsilon(k) = k^2 / 2m^*$$
$$m^* = \partial^2 \varepsilon(k) / \partial k^2 \big|_{kF}$$



$$\varepsilon(k) = \pm v|k|$$

?

Dirac or Weyl semimetal



Quasiparticles: quantum excitations of the (filled) Fermi sea.

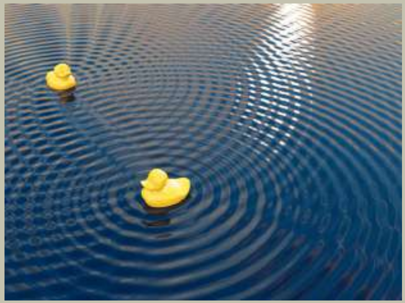
*HEP ref.: J. Polchinski, Effective Field Theory and the Fermi Surface, hep-th/9210046

Particles and quasiparticles

Compact

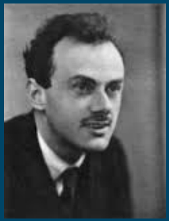
Dirac and Fermi seas

Particles / Quasiparticles {are elementary excitations of the {Dirac / Fermi seas

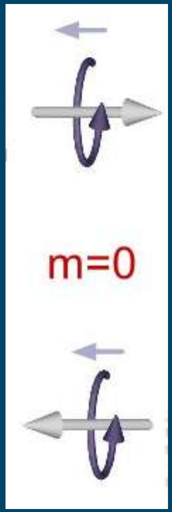


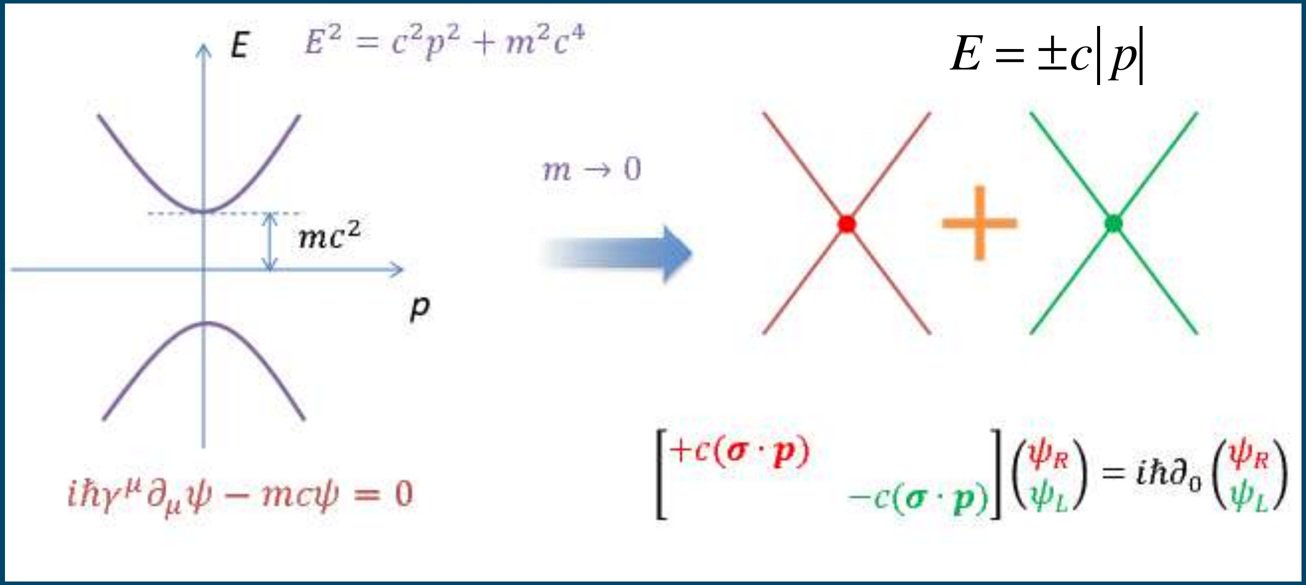
Same equations, same solutions!

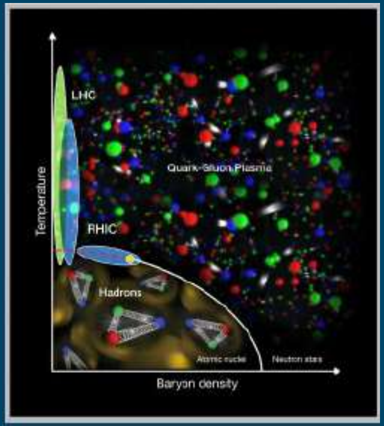
2. Weylfermions in nature



Dirac and Weyl fermions







Massless fermions **in (3+1) dimensions** are 4 component Dirac spinors which split into two 2 component **Weyl fermions** of well defined **chirality.**

$$H = \pm v_F \vec{\sigma}.\vec{p}$$

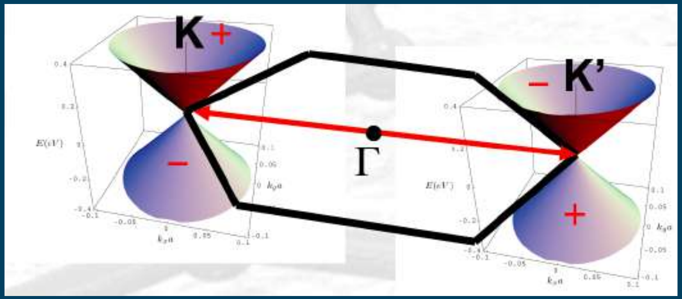
High energy labs: Quark-gluon plasma in heavy ion collisions
Early universe

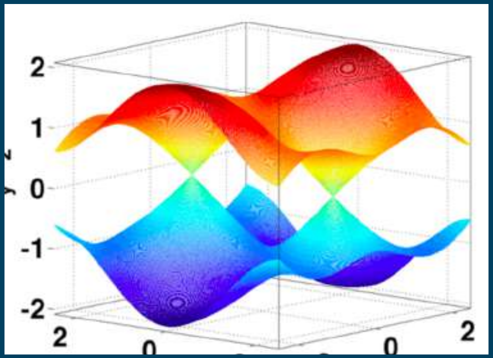
Obstructions to the existence of Weyl fermions

1. Nielsen-Ninomiya

Absence of neutrinos on a lattice, NPB'81

Chiral fermions in a lattice come in pairs of opposite **chirality**



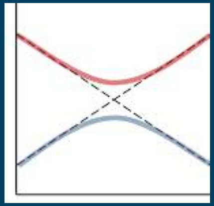


2. Krammers degeneracy

Time reversal invariance -> pairs of oposite spin.

3. Level repulsion

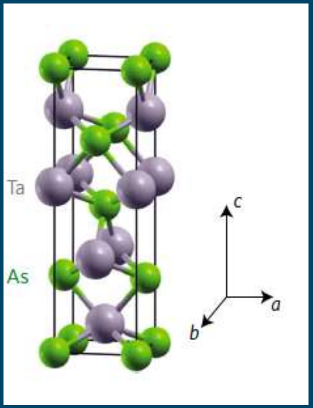
Bands avoid crossings

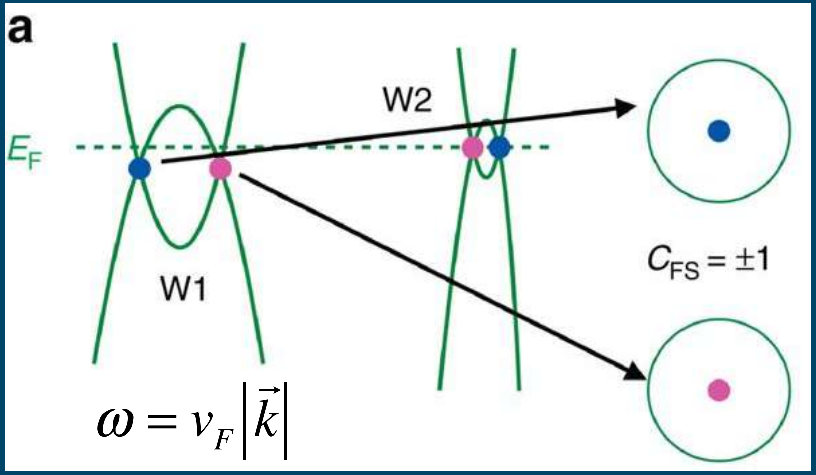


E. Witten, Three lectures on topological phases of matter, arXiv:1510.07698

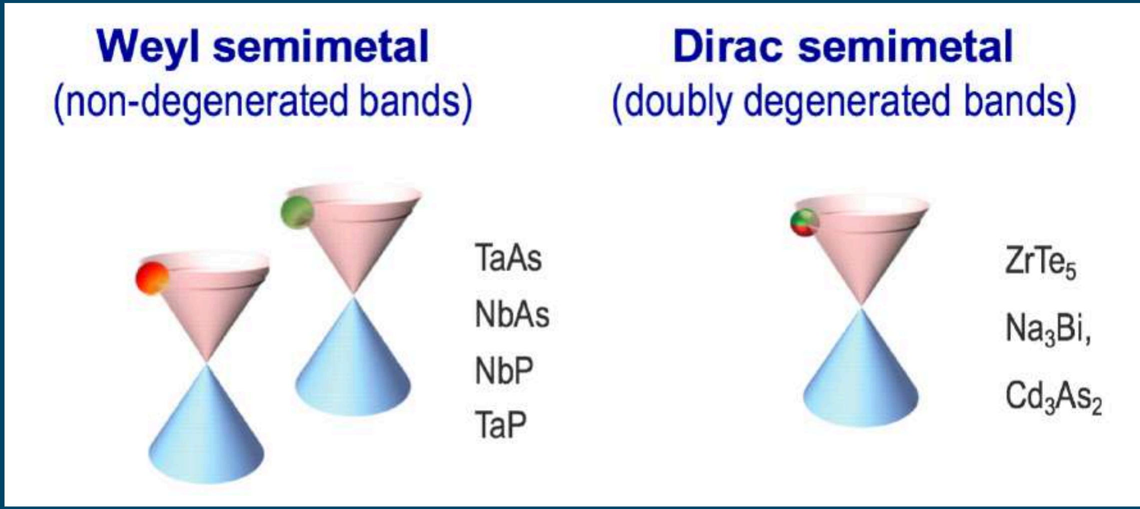
Dirac fermions in condensed matter

Effective low energy description around band crossings in 3D crystals.





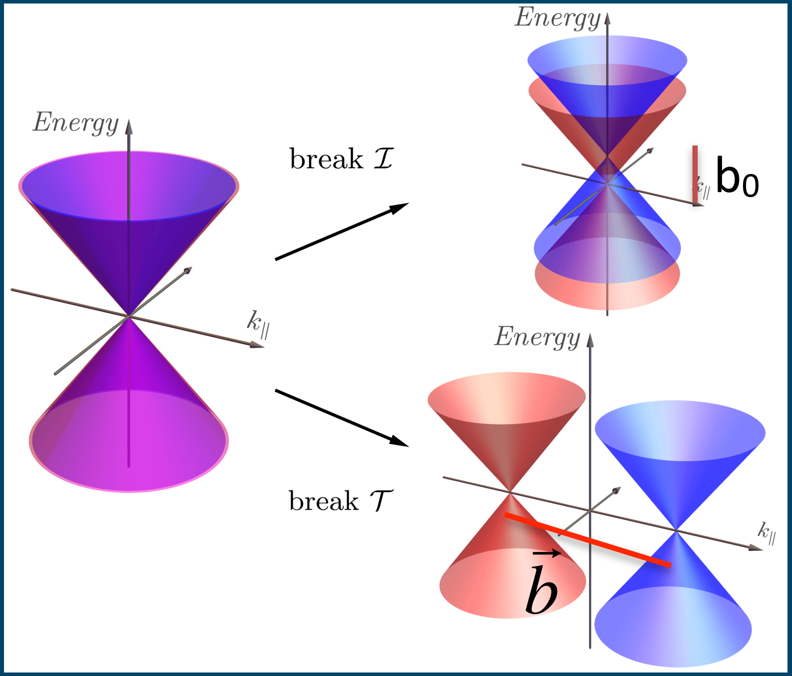
They exist!



Special issue
NATURE MATERIALS | VOL 15 | NOVEMBER 2016

Weyl from Dirac in 3D matter

Dirac: doubly degenerate Weyl



I broken crystal lattices. Nodes protected by crystal symmetries

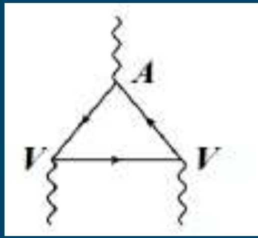
Magnetic materials or pair of nodes at T conjugate points of the BZ.

$$H = \begin{pmatrix} -\vec{\sigma}(\vec{k} - \frac{\vec{b}}{2}) & 0 \\ 0 & \vec{\sigma}(\vec{k} + \frac{\vec{b}}{2}) \end{pmatrix}.$$

$$\mathcal{L} = \bar{\Psi}\gamma^\mu(i\partial_\mu + \gamma^5 b_\mu)\Psi,$$

The nodes separation couples as a (constant) axial gauge field (more to come)

The chiral anomaly



Chiral anomaly

$$S = \int d^4x \, \bar{\Psi}\gamma^\mu \partial_\mu \Psi$$

Action classically invariant under independent L and R rotations: L and R currents and charges.

$$\Psi \rightarrow e^{i\alpha\gamma}\Psi$$

Electric (vector) current.

$$\Psi \rightarrow e^{i\alpha\gamma_5}\Psi$$

Axial current.

$$j^\mu = \bar{\Psi}\gamma^\mu\Psi \;, \; \mathrm{Q} = \int \mathrm{j}^0(x)d(x)$$

Electric charge: $N_L + N_R$

$$j_5^\mu = \bar{\Psi}\gamma_5\gamma^\mu\Psi \;, \; \mathrm{Q}_5 = \int \mathrm{j}_5^{\,0}(x)d(x)$$

Axial charge: $N_L - N_R$.

$$J_V = J_L + J_R \;, J_5 = J_L - J_R$$

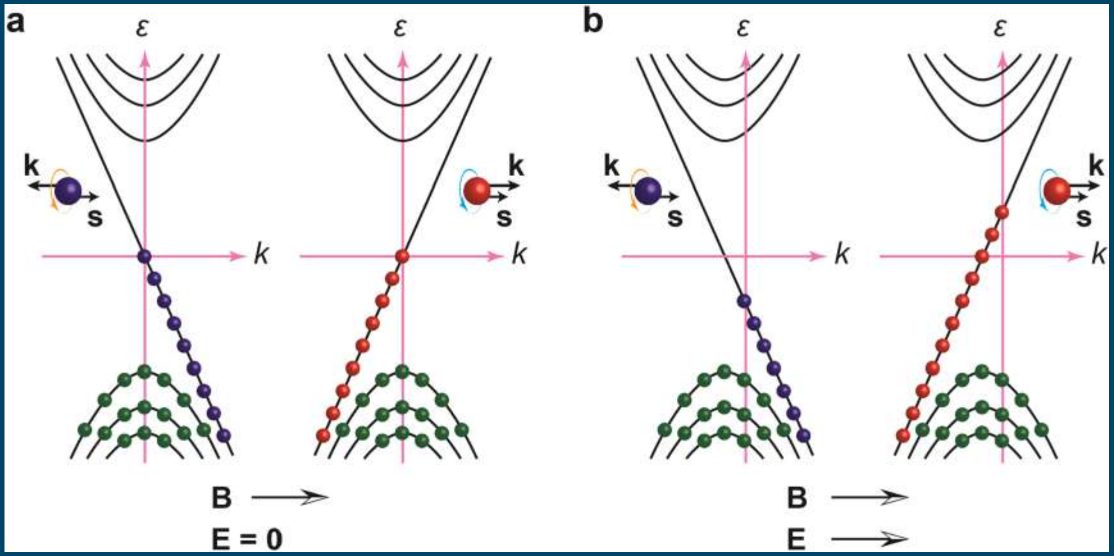
The quantum system can not keep both: axial charge not conserved at the quantum level.

$$\partial_\mu J^A_\mu = \frac{1}{8\pi^2}\vec{E}.\vec{B}$$

Physical implications: Nielsen-Ninomiya

The Adler-Bell-Jackiw anomaly and Weyl fermions in a crystal, Phys. Lett. B'83

$$\partial_\mu J_5^\mu = \frac{1}{8\pi^2}\vec{E}.\vec{B} \Rightarrow$$ **Charge is pumped from L to R points**

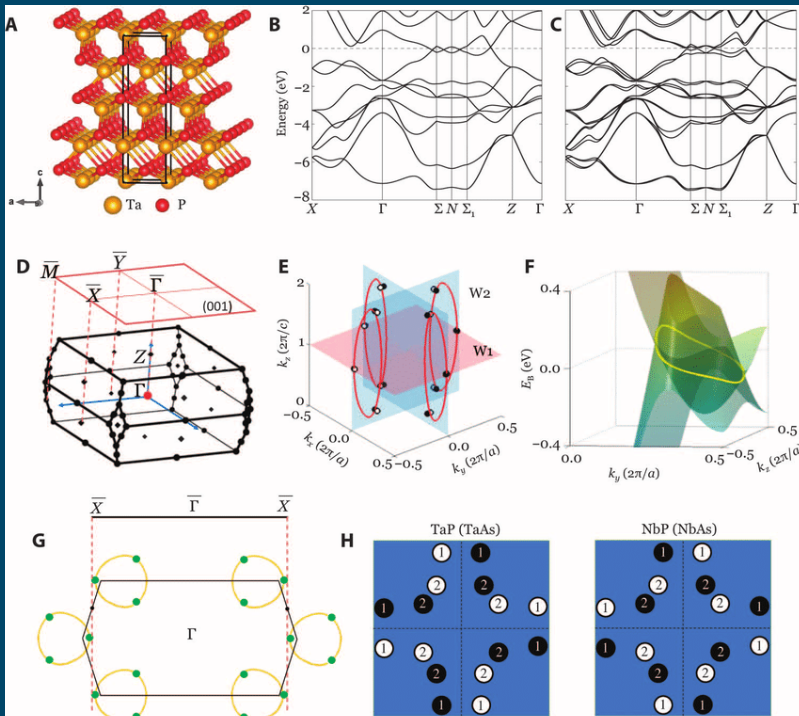


$$\frac{dQ_5}{dt} = \frac{e^2}{4\pi^2}\vec{E}\cdot\vec{B}$$

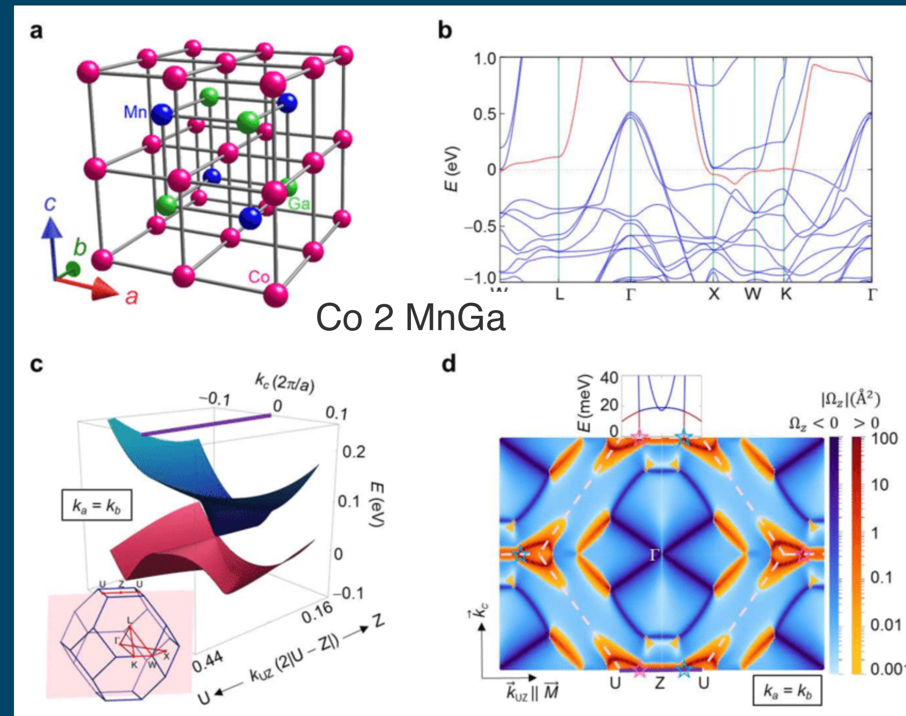
Observed?

It generates **large negative MR** locked to the B field.

Real materials complications



Science Advances 1, e1501092 (2015)



Nature Physics 14, 1119 (2018)

- Many pairs of Weyl nodes (12)
- Very anisotropic cones
- Surface states (Fermi arcs)
- E_F not at the Dirac point
- Other trivial bands cross E_F

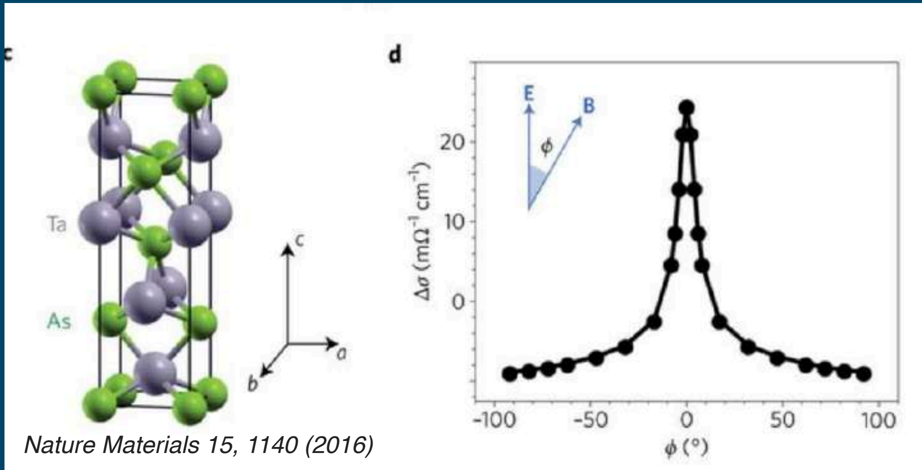
Still..

Experimental observation?

Science 23 Oct 2015:

Evidence for the chiral anomaly in the Dirac semimetal Na_3Bi

Jun Xiong[1], Satya K. Kushwaha[2], Tian Liang[1], Jason W. Krizan[2], Max Hirschberger[1], Wudi Wang[1], R. J. Cava[2], N. P. Ong[1,*]



Nature Materials 15, 1140 (2016)

PHYSICAL REVIEW X

Observation of the Chiral-Anomaly-Induced Negative Magnetoresistance in 3D Weyl Semimetal TaAs, Xiaochun Huang, et al, PRX5, 031023 (2015)

nature COMMUNICATIONS

Article | OPEN | Published: 25 February 2016

Signatures of the Adler–Bell–Jackiw chiral anomaly in a Weyl fermion semimetal

Cheng-Long Zhang, Su-Yang Xu, Ilya Belopolski, Zhujun Yuan, Ziquan Lin, Bingbing Tong, Guang Bian, Nasser Alidoust, Chi-Cheng Lee, Shin-Ming Huang, Tay-Rong Chang, Guoqing Chang, Chuang-Han Hsu, Horng-Tay Jeng, Madhab Neupane, Daniel S. Sanchez, Hao Zheng, Junfeng Wang, Hsin Lin, Chi Zhang, Hai-Zhou Lu, Shun-Qing Shen, Titus Neupert, M. Zahid Hasan & Shuang Jia

Nature Communications **7**, Article number: 10735 (2016) | Download Citation

nature materials

Letter | Published: 27 June 2016

The chiral anomaly and thermopower of Weyl fermions in the half-Heusler GdPtBi

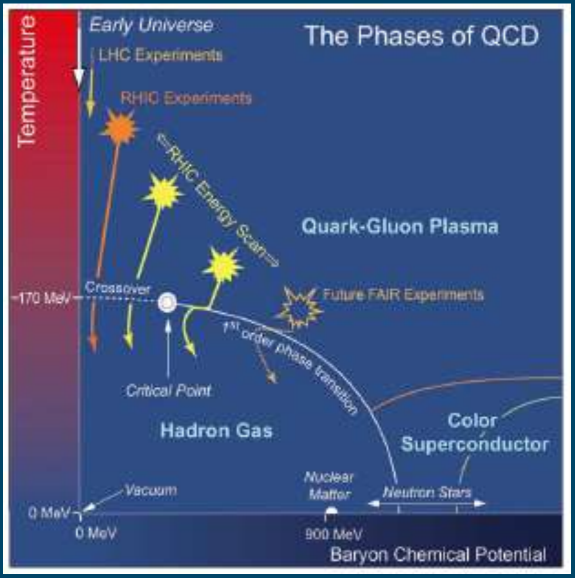
Max Hirschberger, Satya Kushwaha, Zhijun Wang, Quinn Gibson, Sihang Liang, Carina A. Belvin, B. A. Bernevig, R. J. Cava & N. P. Ong

Nature Materials **15**, 1161–1165 (2016) | Download Citation

A good discussion on the experimental situation in the review
Review of experiments on the chiral anomaly in Dirac-Weyl semimetals
N. P. Ong, Sihang Liang, arXiv2010.08564

Anomaly related transport

In the presence of a chiral imbalance μ_5 or an axial field B5



- Chiral magnetic effect: $\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$
- Axial magnetic effect: $\vec{J}_\varepsilon = \frac{\mu^2 + \mu^2{}_5}{4\pi^2} B_5$
- Chiral vortical effect: $\vec{J} = \frac{\mu_5 \mu}{\pi^2}\vec{\omega}$

Coefficients (conductivities) determined by the anomaly.
Topologically protected.
Difficult to observe.
No axial magnetic fields in HEP.

LETTERS
PUBLISHED ONLINE: 8 FEBRUARY 2016 | DOI: 10.1038/NPHYS3648
nature physics

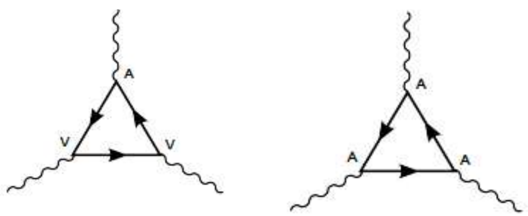
Chiral magnetic effect in $ZrTe_5$

Qiang Li[1]*, Dmitri E. Kharzeev[2,3]*, Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1]*

3. New effects arising in matter: Axial gauge fields

Chiral anomaly with axial gauge fields*

$$S = \int d^4x \bar{\psi}\gamma^\mu(i\partial_\mu + eA_\mu + b_\mu\gamma^5)\psi.$$



The AAA triangle anomaly

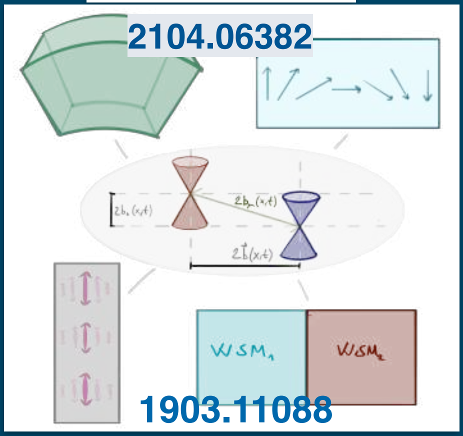
$$\partial_\mu J^\mu_5 = \frac{1}{2\pi^2}\left(\vec{E}\vec{B} + \vec{E}_5\vec{B}_5\right)$$

No axial fields in particle physics

The nodes separation b in WSMs acts as a (constant) axial gauge field

How to promote b to a dynamical field?
1. Elastic deformations of the lattice
2. Boundaries
3. Non-uniform magnetization

Two reviews

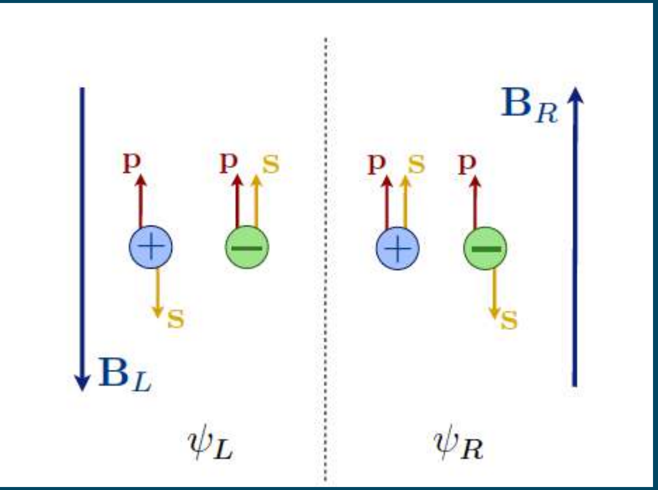


*K. Landsteiner, Y. Liu, arXiv:1703.01944

Axial magnetic effect

Generation of an energy current in the direction of an axial magnetic field.

(B5From boundaries)

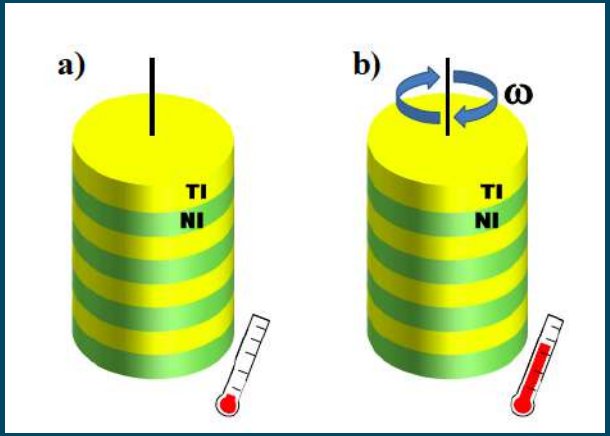


$$S = \int d^4k \bar{\Psi}_k (\gamma^\mu k_\mu - b_\mu \gamma^\mu \gamma^5) \Psi_k$$

b_0: Energy separation

b_i: k separation. b(x) → B_5(x).

$$\mathbf{b} = \hat{z} b_z \Theta(a - |r|). \quad B_\theta \sim b_z \delta(|r| - a).$$



$$T^{0i} = J^i_\epsilon = \sigma_{AME} B^i_5.$$

$$\sigma_{AME} = \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12},$$

$$L_z = \frac{N_f}{6} T^2 b_z \mathcal{V},$$

Experimental confirmation of the gravitational anomaly?

Cortijo, Grushin, Landsteiner, MAHV, PRB 89, 081407R (2014)

Axial gauge fields from strain

(Effective action approach)

Totalitarian principle: "Everything not forbidden is compulsory"

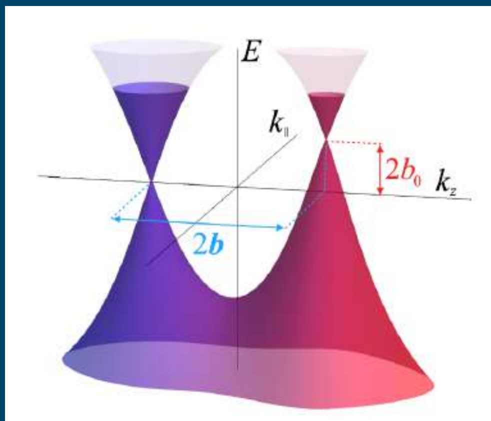


Fig. from M. Chernodub

Elasticity: Strain tensor $u_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$

Building blocks $\sigma_i, k_j, u_{ij}, \text{b}_i$

Elastic axial gauge field

$$A_i^5(x) \approx u_{ij}(x) b_j \quad A_0^5(x) \sim \beta b_0 tr(u(x))$$

Fluctuations of the axial vector b induced by:
- Static inhomogeneous deformations
- Time dependent deformations

A tight binding derivation provides values for the coupling[1]

In the presence of b_0 get an A_0 component[2]

1. A. Cortijo, Y. Ferreiros, K. Landsteiner, MAHV, Elastic gauge fields in Weyl semimetals PRL (2015).

2. A. Cortijo, D. Kharzeev, K. Landsteiner, MAHV. *Strain-induced CME in Weyl semimetals*, PRR (2016).

Mixed axial-em gauge fields

$$S=\int d^4k\bar{\Psi}_k\gamma^\mu(\partial_\mu-ib_\mu\gamma^5-ieA_\mu)\Psi_k$$

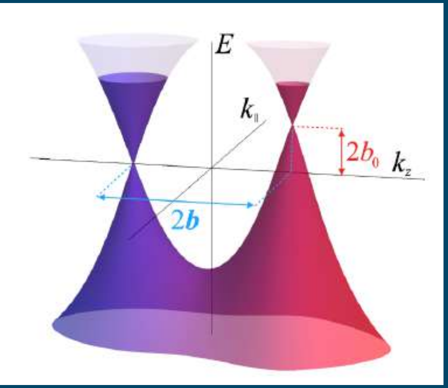


Fig. from M. Chernodub

Effective action:

$$S_{CS}=\frac{e^2}{16\pi^2}\int d^4x\ \mathrm{b}_\mu\varepsilon_{\mu\nu\rho\sigma}A_\nu\,\partial_\rho A_\sigma$$

- Elastic contribution to all anomaly related responses!
- New response functions.
- Mixed em-elastic responses (piezoelectric-like).

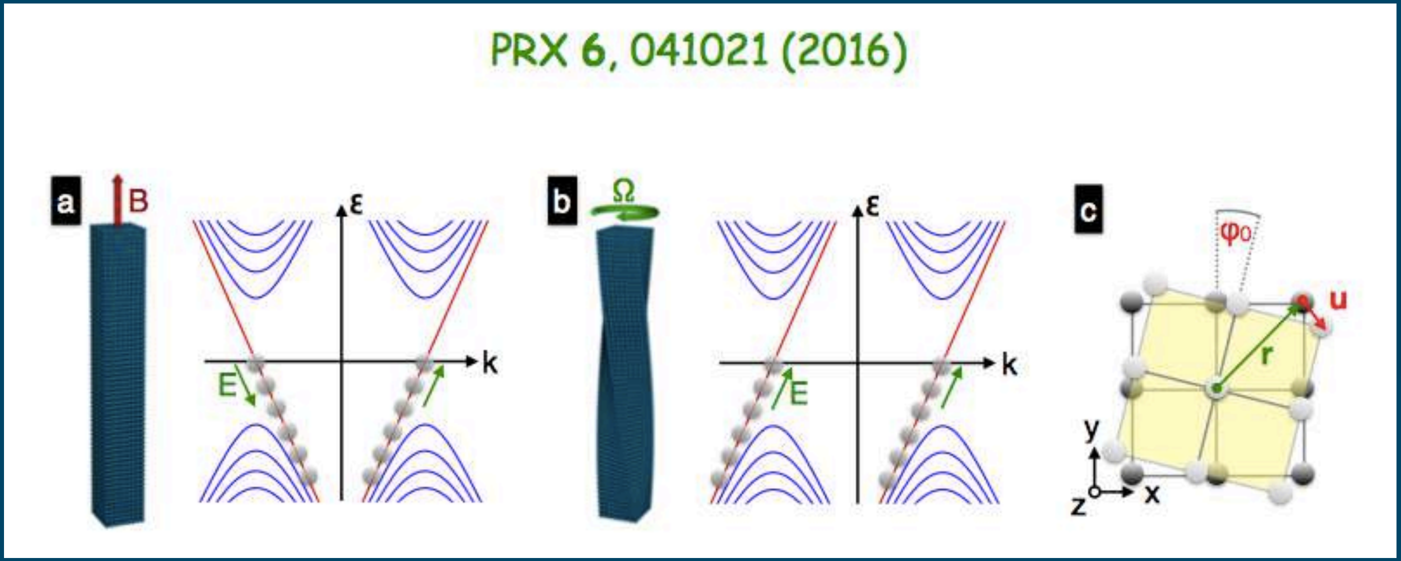
Hall viscosity from Hall conductivity

Chiral magnetic effect in equilibrium

1. A. Cortijo, Y. Ferreiros, K. Landsteiner, MAHV, Elastic gauge fields in Weyl semimetals PRL (2015).

2. A. Cortijo, D. Kharzeev, K. Landsteiner, MAHV. *Strain-induced CME in Weyl semimetals*, PRR (2016).

WSM straintronics (theory)



$$\vec{B}^5 = \vec{\nabla} \wedge \vec{A}^5$$
$$\vec{E}^5 = \partial_t \vec{A}^5 + \vec{\nabla} A_0^5$$

Pikulin, D. I., Chen, A. & Franz, M. Chiral anomaly from strain-induced gauge fields in dirac and weyl semimetals. *Phys. Rev. X* **6**, 041021 (2016).

Grushin, A. G., Venderbos, J. W. F., Vishwanath, A. & Ilan, R. Inhomogeneous weyl and dirac semimetals: Transport in axial magnetic fields and fermi arc surface states from pseudo-landau levels. *Phys. Rev. X* **6**, 041046 (2016).

Liu, T., Pikulin, D. I. & Franz, M. Quantum oscillations without magnetic field. *Phys. Rev. B* **95**, 041201 (2017).

Arjona, V., Castro, E. V. & Vozmediano, M. A. H. Collapse of landau levels in weyl semimetals. *Phys. Rev. B* **96**, 081110 (2017). arXiv:1703.05399.

Gorbar, E. V., Miransky, V. A., Shovkovy, I. A. & Sukhachov, P. O. Pseudomagnetic lens as chirality spectrometer in weyl materials. *Phys. Rev. B* **95**, 241114(R) (2017).

Gorbar, E. V., Miransky, V. A., Shovkovy, I. A. & Sukhachov, P. O. Pseudomagnetic helicons. *Phys. Rev. B* **95**, 115422 (2017).

Gorbar, E. V., Miransky, V. A., Shovkovy, I. A. & Sukhachov, P. O. Chiral response in lattice models of weyl materials. *arXiv:1706.04919* (2017).

Zabolotskiy, A. D. & Lozovik, Y. E. Strain-induced pseudomagnetic and scalar fields in symmetry-enforced dirac nodes. *arXiv:1707.02781* (2017).

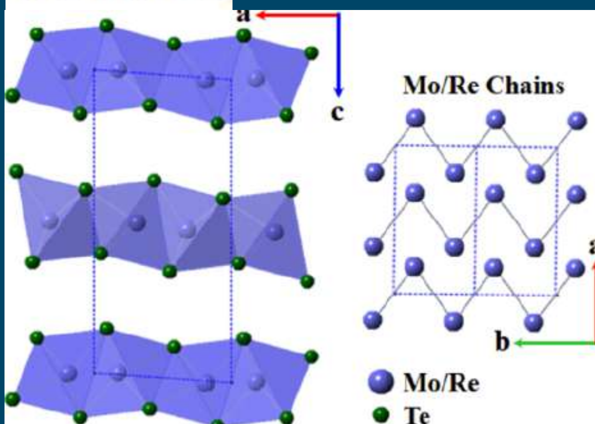
Gao, Kaushik, Kharzeev, Philip, Chiral kinetic theory of anomalous transport induced by torsion, arXiv: 2010.07123. ……

Experimental evidence?

Generation of strain-induced pseudo-magnetic field in a doped type-II Weyl semimetal

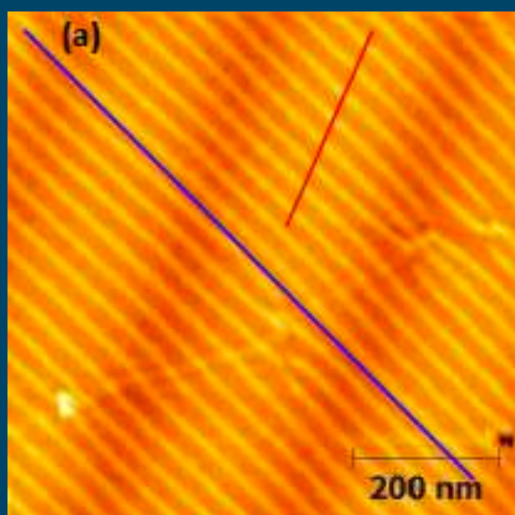
Suman Kamboj[1], Partha Sarathi Rana[2], Anshu Sirohi[1], Aastha Vasdev[1], Manasi arXiv:1903.06224

Re-$MoTe_2$

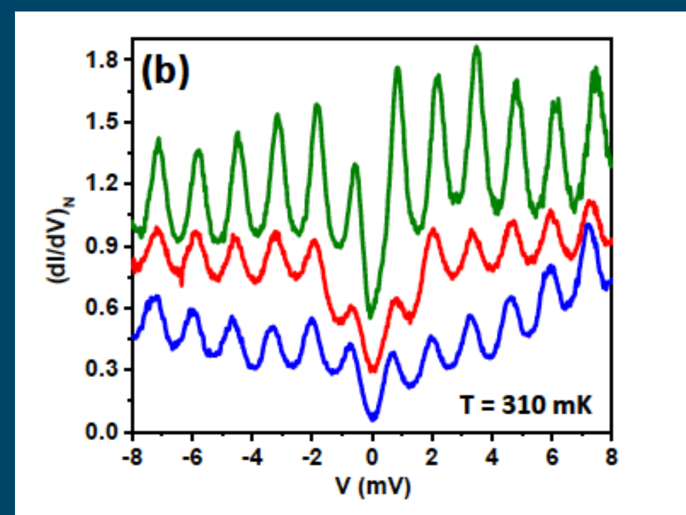


Landau levels from strain

$$\vec{B}_5 = \vec{\nabla} \times \vec{A}_5$$



Stripes λ~35 nm



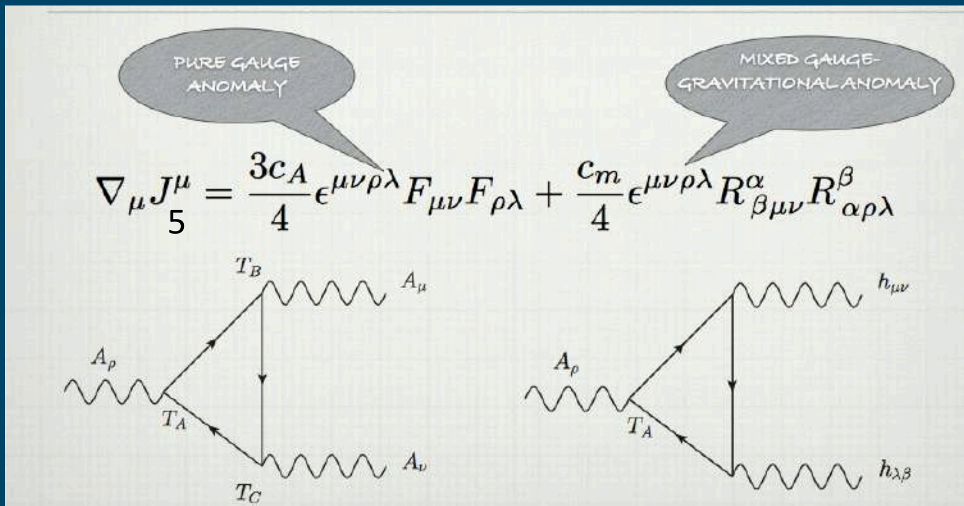
$B \approx 3T$, 300 mK,

4. Heat and geometry: Geometrical anomalies

Thermal transport, geometry, and anomalies

Maxim N. Chernodub,[1,2] Yago Ferreiros,[3] Adolfo G. Grushin,[4] Karl Landsteiner,[5] and María A. H. Vozmediano[6]

Gravitational anomalies



QFT in curved space

Contribution to the axial current from curvature.

$$S = \int d^4x\sqrt{g}\,\bar{\Psi}\gamma^\mu(\partial_\mu + \Gamma_\mu + ieA_\mu)\Psi$$

nature
International journal of science

Letter Published: 20 July 2017

Experimental signatures of the mixed axial–gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth, Anna C. Niemann, Tobias Meng, Adolfo G. Grushin, Karl Landsteiner, Bernd Gotsmann, Fabian Menges, Marcus Schmidt, Chandra Shekhar, Vicky Süß, Ruben Hühne, Bernd Rellinghaus, Claudia Felser, Binghai Yan & Kornelius Nielsch

nature materials
International journal of science

Article
Published: 07 June 2021

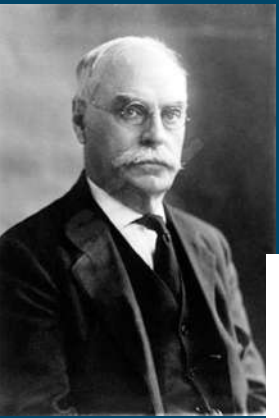
Thermal chiral anomaly in the magnetic-field-induced ideal Weyl phase of Bi1−xSb

Dung Vu, Wenjuan Zhang, Cüneyt Şahin, Michael E Flatté, Nandini Trivedi & Joseph P. Heremans

Thermoelectrical transport in a magnetic field: positive-magnetothermoelectric conductance

Curvature, thermal? What the hell?

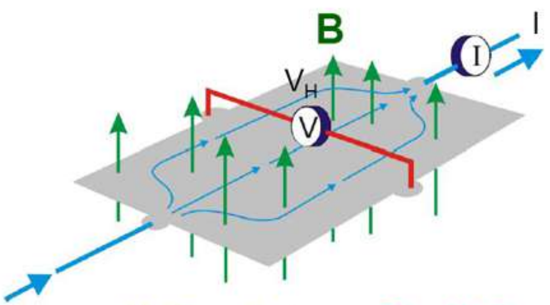
Thermoelectric responses



General: $\vec{J} = \hat{\sigma}\vec{E} - \hat{\alpha}\,\vec{\nabla}T$

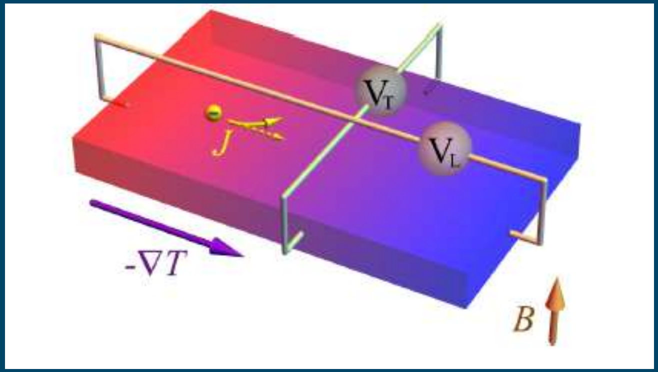
Hall Effect

Edwin H. Hall (1879)



Hall resistance $R_H = V_H / I$

Nernst effect



$$J^i = \sigma_H^{ij} \nabla_j V$$

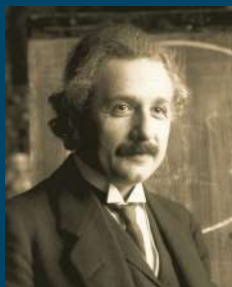
$$J^i = \alpha^{ij} \nabla_j T$$

$$\sigma^{ij} \sim \langle J^i J^j \rangle$$

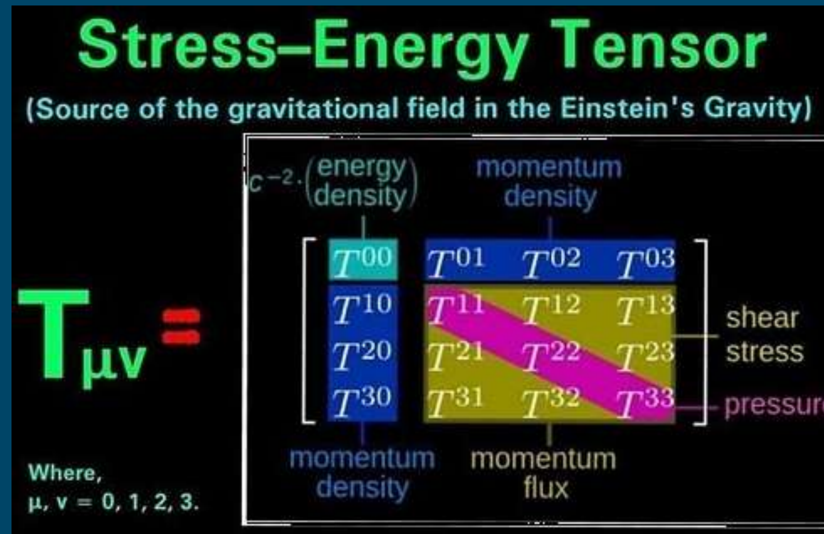
??

How can we get Kubo formulas for thermal transport coefficients?

Relating gravity to thermal transport



$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

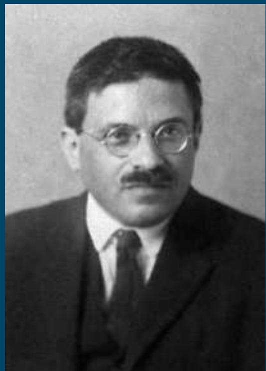


In a curved space formalism, variations of the metric generates variations in the energy-momentum tensor.

$$S = S_{flat} - \frac{1}{2}\int d^d x\, h_{\mu\nu} T^{\mu\nu}$$

$$T^{\mu\nu} = -2\frac{\delta S}{\delta h_{\mu\nu}}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



Tolman-Ehrenfest effect

Precursors

APRIL 15, 1930 — PHYSICAL REVIEW — VOLUME 35

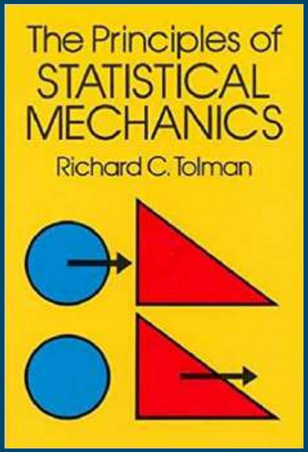
ON THE WEIGHT OF HEAT AND THERMAL EQUILIBRIUM IN GENERAL RELATIVITY

By Richard C. Tolman

DECEMBER 15, 1930 — PHYSICAL REVIEW — VOLUME 36

TEMPERATURE EQUILIBRIUM IN A STATIC GRAVITATIONAL FIELD

By Richard C. Tolman and Paul Ehrenfest



The problem: thermodynamic equilibrium in gravitational fields.

heat has weight

$$\frac{1}{T}\nabla T = -\frac{1}{c^2}\nabla\Phi,$$

Variations of gravitational potentials induce variations of temperature

PHYSICAL REVIEW VOLUME 135, NUMBER 6A 14 SEPTEMBER 1964

Theory of Thermal Transport Coefficients*

J. M. LUTTINGER

Just as the space- and time-varying external electric potential produced electric currents and density variations, so a varying gravitational field will produce, in principle,[7] energy flows and temperature fluctuations.

Gravitational potential Φ as a local source of thermal (energy) currents

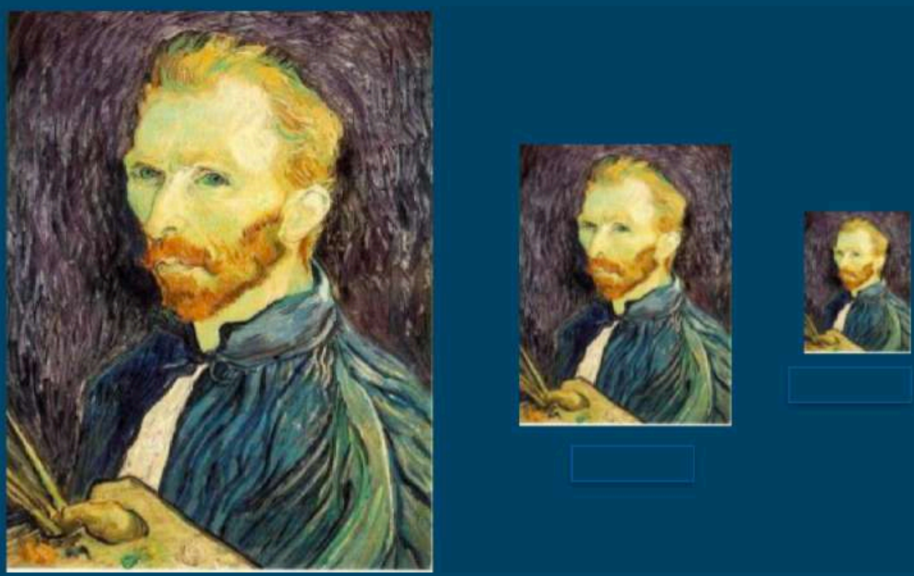
$$\frac{1}{T}\nabla T = -\frac{1}{c^2}\nabla\Phi,$$

Kubo formulas for thermal transport coefficients

7 calling these interesting references to my attention.) Although the effect is very small, in practice we are only interested in questions of principle, and an arbitrarily small effect is just as good as a large one. In fact, if the gravitational field didn't exist, one could invent one for the purposes of this paper.

Conformal anomaly

Scale invariance*: $x \to \lambda^{-1} x,$ Noether: $j_D^\mu = T^{\mu\nu} x_\nu$ $(T^\mu_\mu)_{\rm cl} \equiv 0.$



Dirac action is scale invariant

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} i \not{D} \psi,$$

And has an anomaly when coupled to **A**

$$\langle T^\alpha{}_\alpha(x) \rangle = \frac{\beta(e)}{2e} F^{\mu\nu}(x) F_{\mu\nu}(x),$$

Which gives rise to a Scale Magnetic effect:**

Scale transformation (Weyl)

$$g_{\mu\nu}(x) = e^{2\tau(x)} \eta_{\mu\nu},$$

$$S \to S_\tau = S + \int d^4x\, \tau(x) T^\alpha_\alpha(x)$$

Conformal factor

$$\boldsymbol{J} = -\frac{2\beta(e)}{e} \boldsymbol{\nabla}\tau(x) \times \boldsymbol{B}(x).$$

* Scale+Lorentz = conformal

** M. N. Chernodub, Anomalous Transport due to the Conformal Anomaly, Phys. Rev. Lett. 117, 141601 (2016).

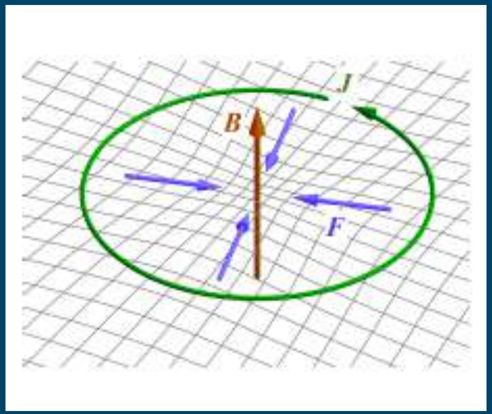
Generation of a Nernst Current from the Conformal Anomaly in Dirac and Weyl Semimetals

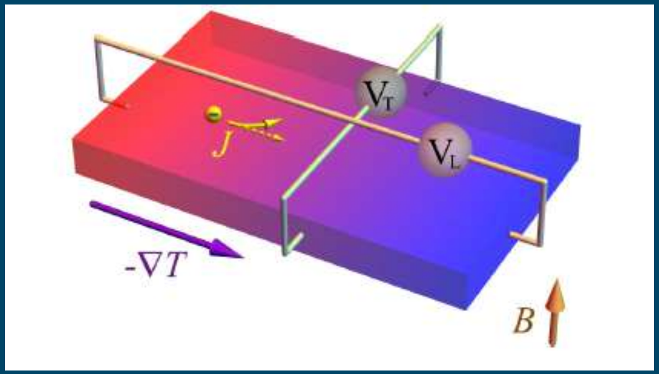
M. N. Chernodub,[1,2] Alberto Cortijo,[3] and María A. H. Vozmediano[3]

PHYSICAL REVIEW LETTERS **120**, 206601 (2018)

Scale magnetic effect

Nernst effect





$$\boldsymbol{J} = -\frac{2\beta(e)}{e}\boldsymbol{\nabla}\tau(x) \times \boldsymbol{B}(x).$$

$$\nabla\phi \leftrightarrow \frac{\nabla T}{T}$$

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T\hbar}\boldsymbol{B} \times \boldsymbol{\nabla} T\,.$$

$$\beta_{\rm QED}^{\rm 1loop} = \frac{e^3}{12\pi^2}.$$

Newtonian limit: $\Phi \sim g_{00} \sim \tau$

Kubo calculation: Arjona, Chernodub , MV, PRB**99** , 235123 (2019).

Fingerprints of the conformal anomaly in the thermoelectric transport in Dirac and Weyl semimetals

Vicente Arjona,[1,*] Maxim N. Chernodub,[2,3,†] and María A. H. Vozmediano[1,‡]

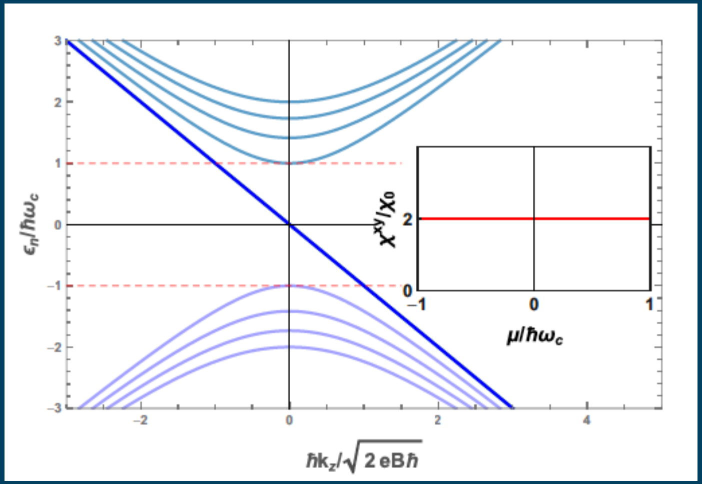
PRB99,235123(2019)

Kubo calculation:

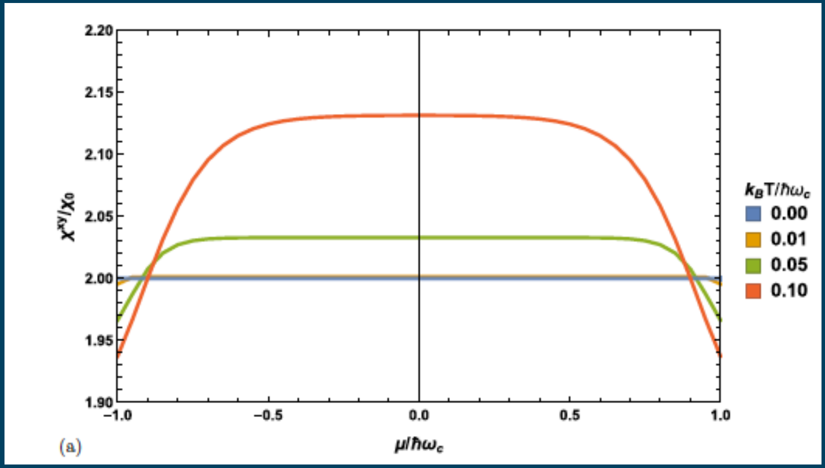
$$J^x = \sigma^{xy} E_y - \alpha^{xy} \nabla_i T$$

$$\alpha^{xy}(T=\mu=0) = \frac{e^2 v_F B_z}{18\pi^2}$$

$$\chi^{ij}(\mathbf{r}-\mathbf{r}',\tau) = -i\Theta(\tau)\langle[\hat{J}^i(\mathbf{r},\tau),\hat{T}^{0j}(\mathbf{r}')]\rangle_0$$



LL structure



Finite μ

$$\chi^{xy} = \frac{\alpha^{xy}}{T}$$

The thermoelectric coefficient remains constant for $\mu \in [-h\omega_c, h\omega_c]$

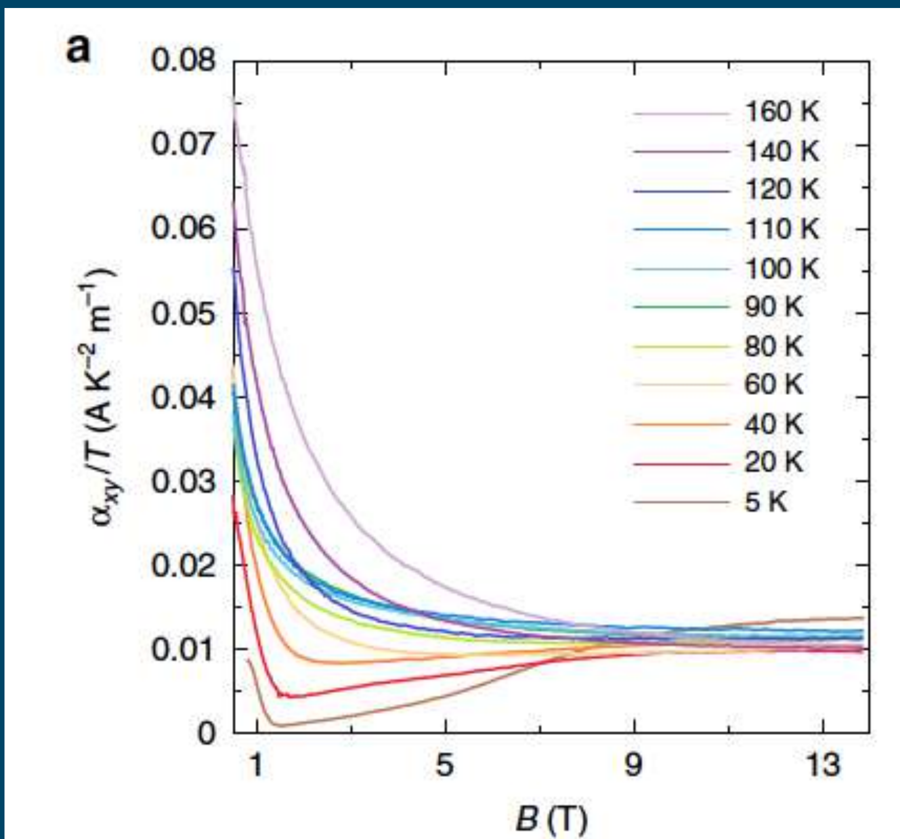
Experimental situation?

Observation of a thermoelectric Hall plateau in the extreme quantum limit

Wenjie Zhang[1,9], Peipei Wang[2,9], Brian Skinner[3,4,9], Ran Bi[1], Vladyslav Kozii[3,5,6], Chang-Woo Cho[2], Ruidan Zhong[7], John Schneeloch[7], Dapeng Yu[2], Genda Gu[7], Liang Fu[3]✉, Xiaosong Wu[1,8]✉ & Liyuan Zhang[2]✉

Thermoelectric Hall conductivity:

$$\mathbf{J} = \hat{\sigma}\mathbf{E} - \hat{\alpha}\nabla T$$



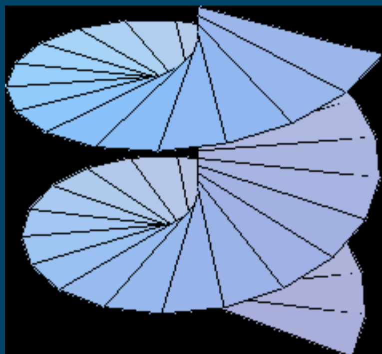
NATURE COMMUNICATIONS | (2020)11:1046 $ZrTe_5$

5. Torsional fields in matter

Thermal transport, geometry, and anomalies

Maxim N. Chernodub,[1,2] Yago Ferreiros,[3] Adolfo G. Grushin,[4] Karl Landsteiner,[5] and María A. H. Vozmediano[6]

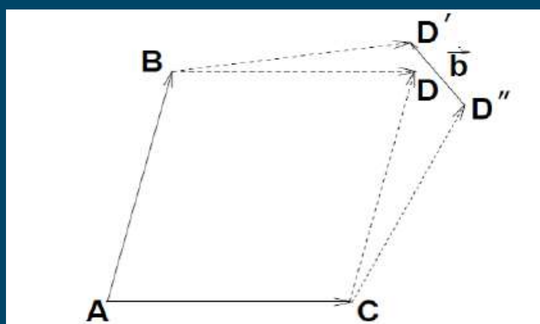
Torsion in differential geometry



Basic buildings of geometry in manifolds:
Metric tensor & Covariant derivative (connection)

$$D_\mu v^\nu = (\partial_\mu + \Gamma^{\ \nu}_{\mu\rho}) v^\rho$$

$$\Gamma^{\ \rho}_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} + \Theta^{\ \rho}_{\mu\nu}$$



In spaces with torsion infinitesimal parallelograms do not close. Parallel transport in different directions do not commute:

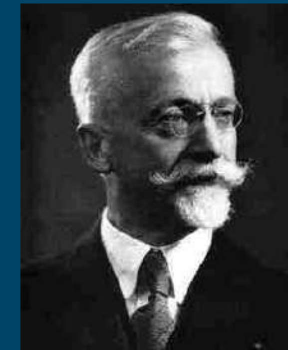
$$[\nabla_\mu, \nabla_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma + \theta^\sigma_{\mu\nu} \nabla_\sigma V^\rho .$$

Curvature (Riemann) tensor
Rotation

Torsion tensor
Displacement

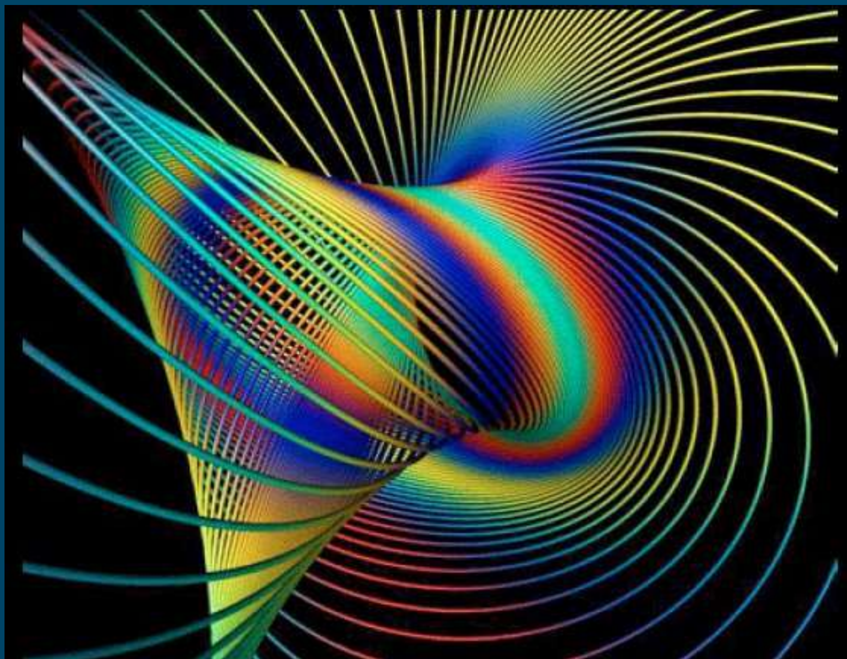
Friedrich W. Hehl, Yuri N. Obukhov
Elie Cartan's torsion in geometry and in field theory, an essay, arXiv:0711.1535

Torsion in General relativity



E. Cartan

Einstein-Cartan relativity



Torsion Field: Einstein's Metric Torsion Tensor allows a spin-field to twist spacetime.

$$G^{ij} = R^{ij} - \frac{1}{2} g^{ij} R^k_k \;\; = \kappa \Sigma^{ij},$$ [1]

The energy-momentum tensor of a spinor field has an antisymmetric part

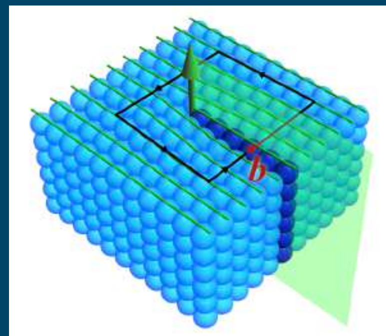
$$\Sigma^{ij} = -\frac{\hbar c}{2}[(\nabla^i \bar{\Psi})\gamma^j \Psi - \bar{\Psi}\gamma^j \nabla^i \Psi],$$

and can not be included in [1]

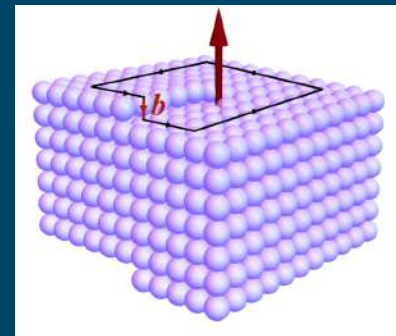
Include a general connection with antisymmetric part (torsion).
Curvature couples to mass and torsion to spin.

Dislocations in crystals

Characterized by the Burgers vector **b**



Edge $\vec{b} \perp \vec{t}$



Screw $\vec{b} \parallel \vec{t}$

Images from M. Chernodub

Geometrical description:

Gauge theory of linear defecs in solids: metric approach to elasticity
M.O. Katanaev and I.V. Volovich, Ann. Phys. (N.Y.) 216 (1992) 1

Elastic deformations	$R_{\mu\nu}{}^{ij} = 0$	$T_{\mu\nu}{}^{i} = 0$
Dislocations	$R_{\mu\nu}{}^{ij} = 0$	$T_{\mu\nu}{}^{i} \neq 0$
Disclinations	$R_{\mu\nu}{}^{ij} \neq 0$	$T_{\mu\nu}{}^{i} = 0$
Dislocations and disclinations	$R_{\mu\nu}{}^{ij} \neq 0$	$T_{\mu\nu}{}^{i} \neq 0$

H. Kleinert, Gauge fields in condensed matter, vol 2.

Dislocations can be described in the covariant formalism by adding torsion to the spacetime connection.

Weyl fermions in curved space with torsion

$$L=\overline{\Psi}\gamma^{\mu}\left(\partial_{\mu}-\Omega_{\mu}(x))\Psi\right)+ig_{v}\overline{\Psi}\gamma^{\mu}T_{\mu}\Psi+g_{A}\overline{\Psi}\gamma_{5}\gamma^{\mu}S_{\mu}\Psi$$

$$\Omega_{\mu}=\frac{1}{4}\gamma^{a}\gamma^{b}e^{\nu}{}_{a;\mu}e_{b\nu}$$

$$T_{\rho}=g^{\nu}_{\mu}\theta^{\mu}{}_{\nu\rho}\quad,\quad S_{\sigma}=\varepsilon_{\mu\nu\rho\sigma}\theta^{\mu\nu\rho}$$

The trace part of the torsion is associated to edge dislocations –change in volume element - and couples as a usual vector gauge field.

The axial part couples as an axial vector field and generates a contribution to the axial anomaly:

The Nieh-Yan term

$$\nabla_{\mu}J^{\mu}_{5}-\theta^{\lambda}_{\lambda\mu}J^{\mu}_{5}=C_{N}\ \epsilon^{\mu\nu\rho\lambda}(\eta_{ab}\theta^{a}_{\mu\nu}\theta^{b}_{\rho\lambda}-R_{ab\mu\nu}e^{a}_{\rho}e^{b}_{\lambda}).$$

The coefficient C_N has dimensions of ℓ^{-2}

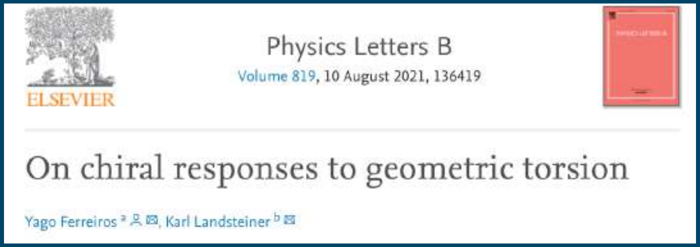
Torsional anomaly?

New anomaly-induced transport?

- Huang, Li, Zhou, Zhang, "Torsional response and liouville anomaly in Weyl semimetals with dislocations," PRB 2019.
- Huang, Han, Stone, "Nieh-Yan anomaly: Torsional Landau levels, central charge, and anomalous thermal Hall effect," arXiv:1911.00174.
- Nissinen ,Volovik, "On **thermal Nieh–Yan anomaly** in topological Weyl materials," arXiv:1911.03382, 1909.08936.
- Huang , Han, "**Torsional Anomalies** and Bulk-Dislocation Correspondence in Weyl Systems," arXiv:2003.04853; 2003.04853.
- Laurila, Nissinen, "Torsional Landau levels and geometric anomalies in condensed matter Weyl systems," arXiv: 2007.10682; 2007.10682 .
- Huang, Han, Stone, "Hamiltonian approach to the **torsional anomalies** and its dimensional ladder," PRB 2020.
- Liang , Ojanen, "Topological **magnetotorsional effect** in Weyl semimetals," PRR 2020.
- Khaidukov, Zubkov, "**Chiral torsional effect**," Jetp Lett. 2018.
- Imaki ,Yamamoto, "Lattice field theory with torsion," PRD 2019.
- Imaki, Qiu, "**Chiral torsional effec**t with finite temperature, density, and curvature," PRD2020.

…….

The Nieh-Yan term is a total derivative.
It can be removed by a local counterterm


ELSEVIER
Physics Letters B
Volume 819, 10 August 2021, 136419
On chiral responses to geometric torsion
Yago Ferreiros [a], Karl Landsteiner [b]

$$S^{NY}_{eff} = C_N \int d^4x\, e\, \epsilon^{\mu\nu\rho\lambda} \eta_{ab} A^5_\mu e^a_\nu \theta^b_{\rho\lambda}$$

No dissipationless chiral electric transport as a response to torsion.
Manifestation of the chiral vortical effect.
Open, debated question.

Other interesting developments

Hydro aspects of Dirac matter

$\tau_{ee} \gg \tau_{any}$ Fermi liquids in ultrapure crystals

JHEP March 24, 2015 — **Negative magnetoresistivity in chiral fluids and holography** — Karl Landsteiner, Yan Liu and Ya-Wen Sun

Negative local resistance caused by viscous electron backflow in graphene — SCIENCE 4 MARCH 2016

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]

Evidence for hydrodynamic electron flow in $PdCoO_2$ — SCIENCE 4 MARCH 2016

Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3] Burkhard Schmidt,[3] Andrew P. Mackenzie[3,4*]

ELECTRON TRANSPORT

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene — SCIENCE 4 MARCH 2016

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5*]

ARTICLE — NATURE COMMUNICATIONS | (2018) — DOI: 10.1038/s41467-018-06688-y — OPEN

Thermal and electrical signatures of a hydrodynamic electron fluid in tungsten diphosphide

J. Gooth[1,2], F. Menges[1,4], N. Kumar[2], V. Süß[2], C. Shekhar[2], Y. Sun[2], U. Drechsler[1], R. Zierold[3], C. Felser[2] & B. Gotsmann[1]

PNAS — **Hydrodynamic theory of thermoelectric transport and negative magnetoresistance in Weyl semimetals** — August 23, 2016

Andrew Lucas[a,1], Richard A. Davison[a,1], and Subir Sachdev[a,b,1]

The holographic Weyl semi-metal

Karl Landsteiner Yan Liu

May 2015 · Physics Letters B 753(C)

Featured in Physics | Open Access

Hear the Sound of Weyl Fermions

Zhida Song and Xi Dai

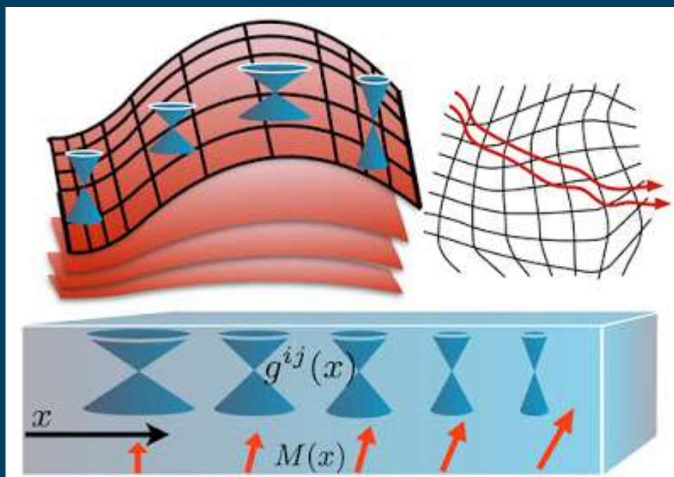
Phys. Rev. X **9**, 021053 – Published 17 June 2019

nature physics

TOPOLOGICAL SEMIMETALS

Sound of Weyl

Nature Physics **15**, 522 (2019) Yun Li



Designer Curved-Space Geometry for Relativistic Fermions in Weyl Metamaterials

Alex Westström and Teemu Ojanen* | PHYSICAL REVIEW X 7, 041026 (2017)

PHYSICAL REVIEW LETTERS

Signatures of the Chiral Anomaly in Phonon Dynamics

P. Rinkel, P. L. S. Lopes, and Ion Garate

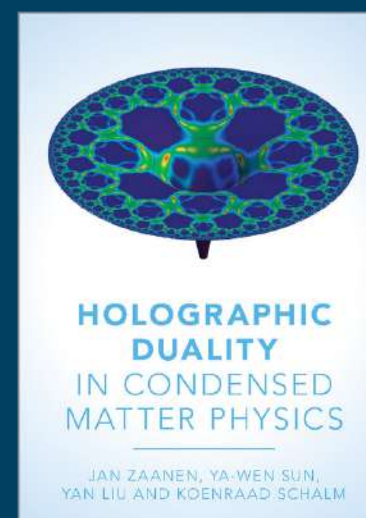
Phys. Rev. Lett. **119**, 107401 – Published 8 September 2017

The Chiral Qubit: quantum computing with chiral anomaly

Dmitri E. Kharzeev* Qiang Li†

arXiv:1903.07133

Quantum information



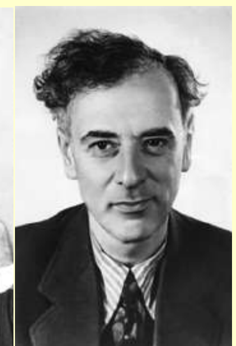
DIRAC MATTER
grand unification

Quantum field theory

particle physics

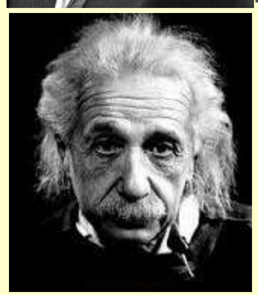
Condensed matter

solid state+stat. physics



Plasma physics

Elasticity



Hydrodynamics

Relativity and cosmology

String theory (holography) ADS/CMT