# Spin, Vortices, Anomaly and Hydrodynamics

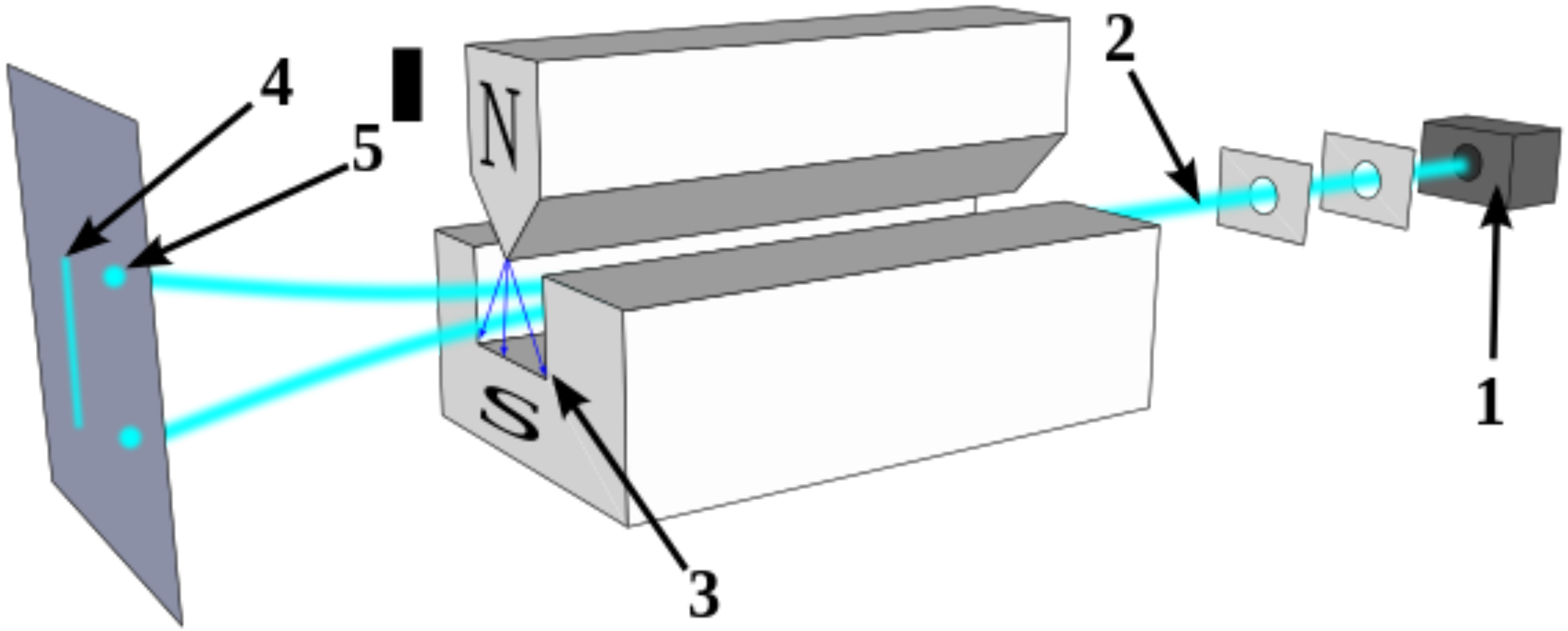
Theoretical Physics Colloquium
Arizona State University

March 10, 2021

Ho-Ung Yee (UIC)

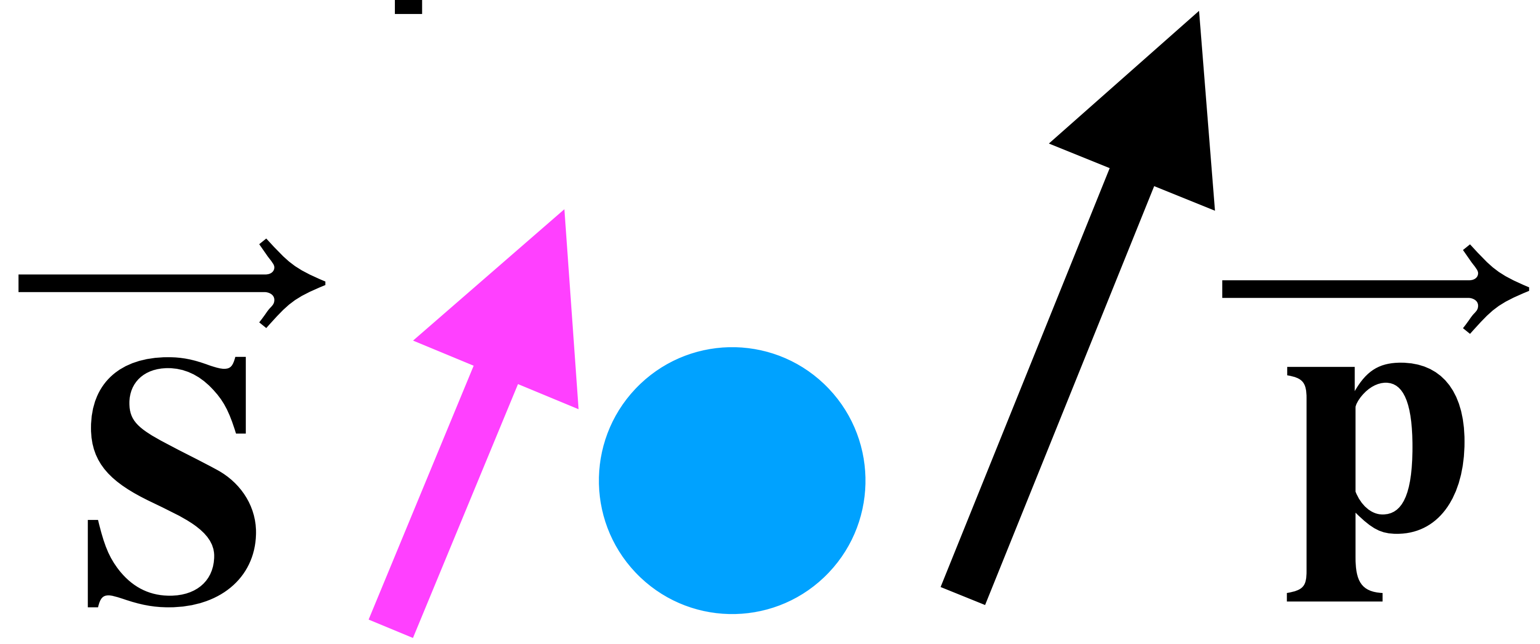
# Spin

- Most quantum object in Nature: Spin $\frac{1}{2}$ has two basis $\{|\uparrow\rangle, |\downarrow\rangle\}$




**Stern-Gelach Experiment with Ag atoms (1922)**

# Relativistic particles: Helicity




- Relativistic massless particles: $\overrightarrow{\mathbf{S}} = h\frac{\overrightarrow{\mathbf{p}}}{|\overrightarrow{\mathbf{p}}|}$, where $h$ is called helicity

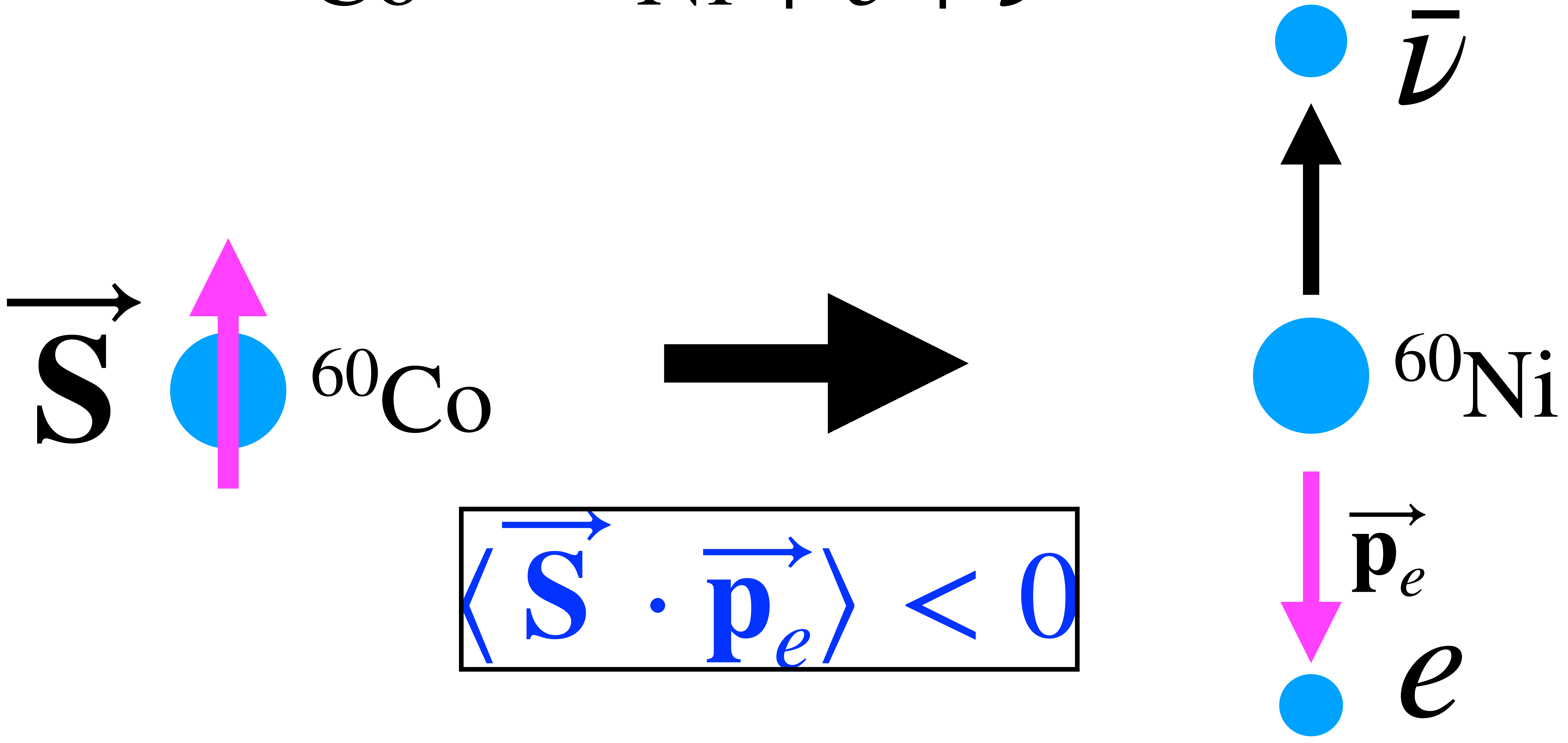
- Under the parity $\overrightarrow{x} \rightarrow -\overrightarrow{x}$ transformation (P), $\overrightarrow{\mathbf{S}} \rightarrow \overrightarrow{\mathbf{S}}$ and $\overrightarrow{\mathbf{p}} \rightarrow -\overrightarrow{\mathbf{p}}$, and helicity flips sign under P

- Any observable that correlates $\overrightarrow{\mathbf{S}}$ and $\overrightarrow{\mathbf{p}}$ breaks Parity symmetry !

# Parity breaking in Electro-Weak Theory

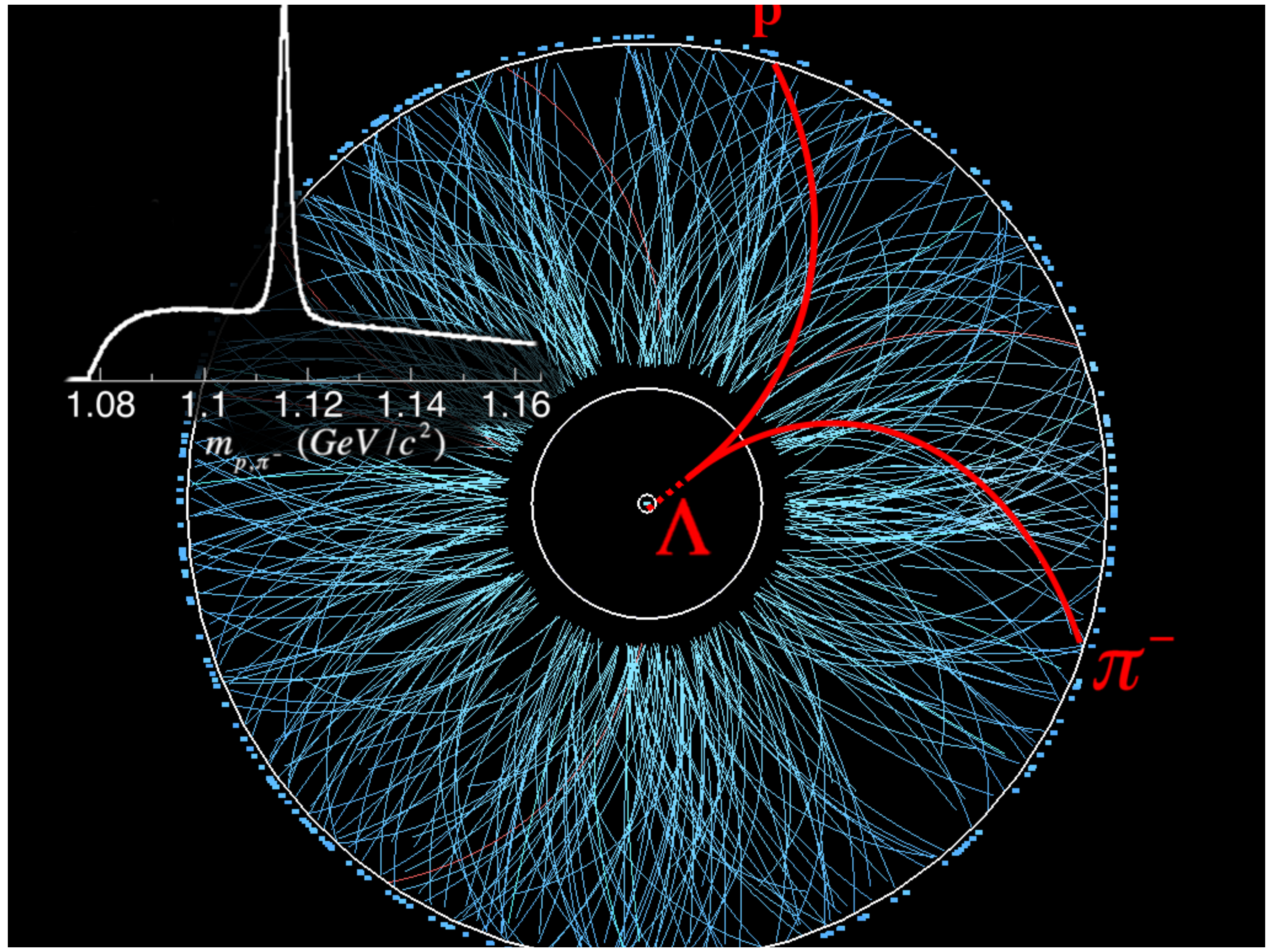
- Lee-Yang's proposal (1956) and the Wu's experiment (1956)

$$^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni} + e + \bar{\nu}$$




$$\langle \vec{\mathbf{S}} \cdot \vec{\mathbf{p}}_e \rangle < 0$$

# Λ-polarization in RHIC

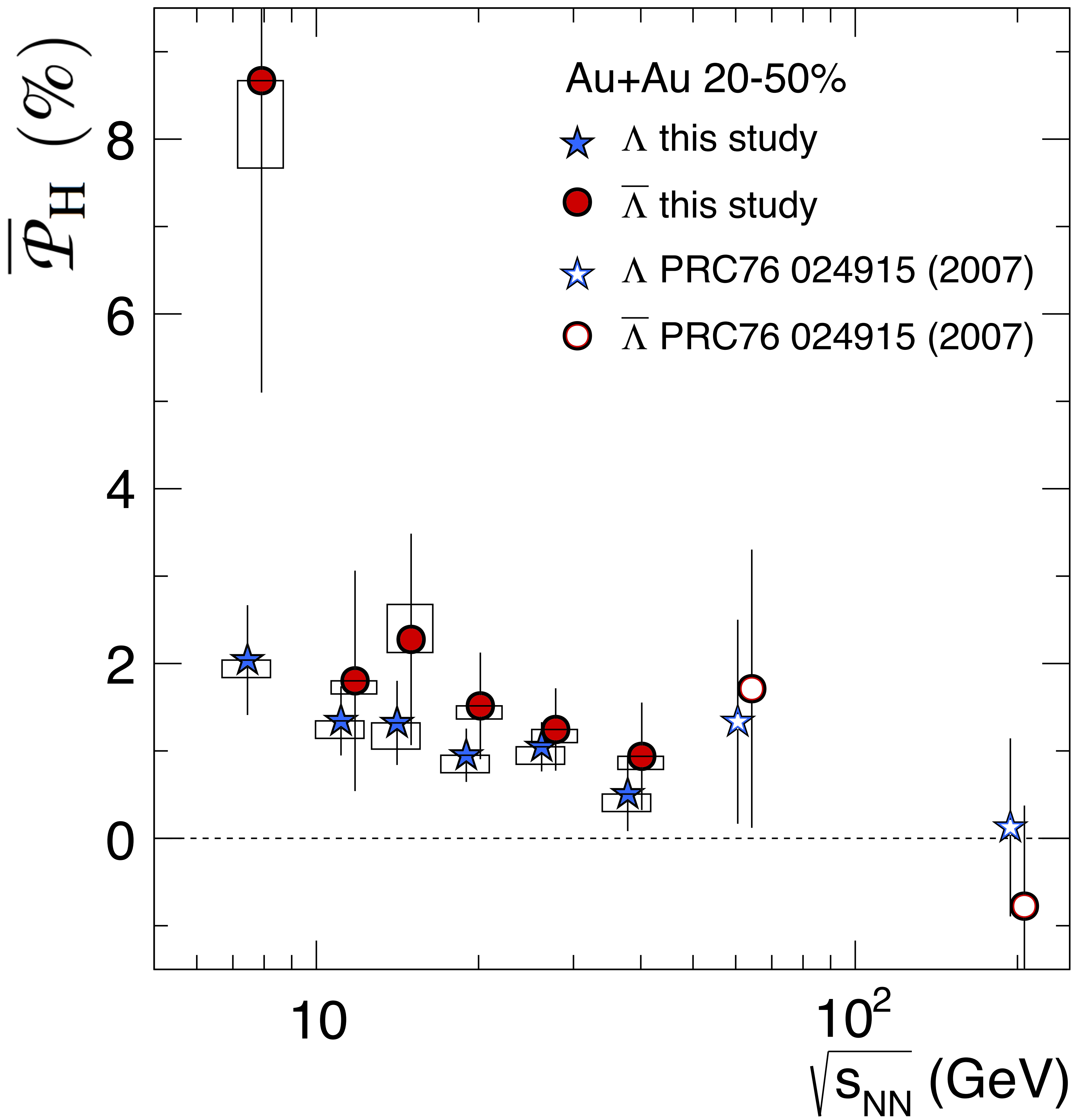



Λ (uds)-baryon self-analyzes its spin direction (1950)

$$\frac{dW}{d\Omega} = \frac{1}{4\pi}(1 + \alpha\cos\theta)\,, \quad \alpha = 0.642$$

Life-time $\tau \sim 10^{-10}$s with weak decay to $p + \pi^-$

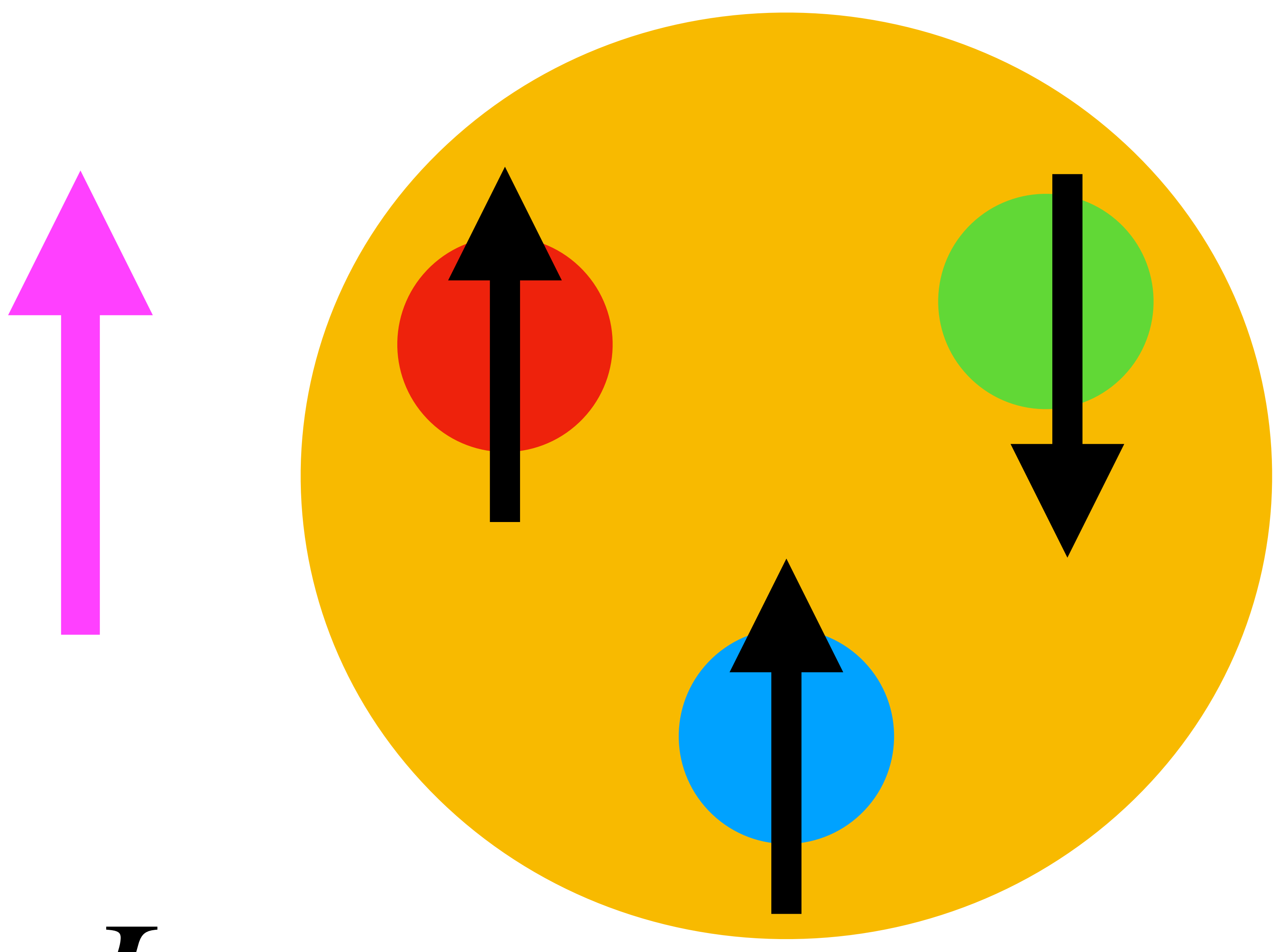
N.B. $\alpha = -0.642$ for $\bar{\Lambda}$ due to CP-conservation




STAR measurement of global Λ polarization. Figure from Nature 548, 62-65(2017) (STAR)

# Proton Spin Puzzle

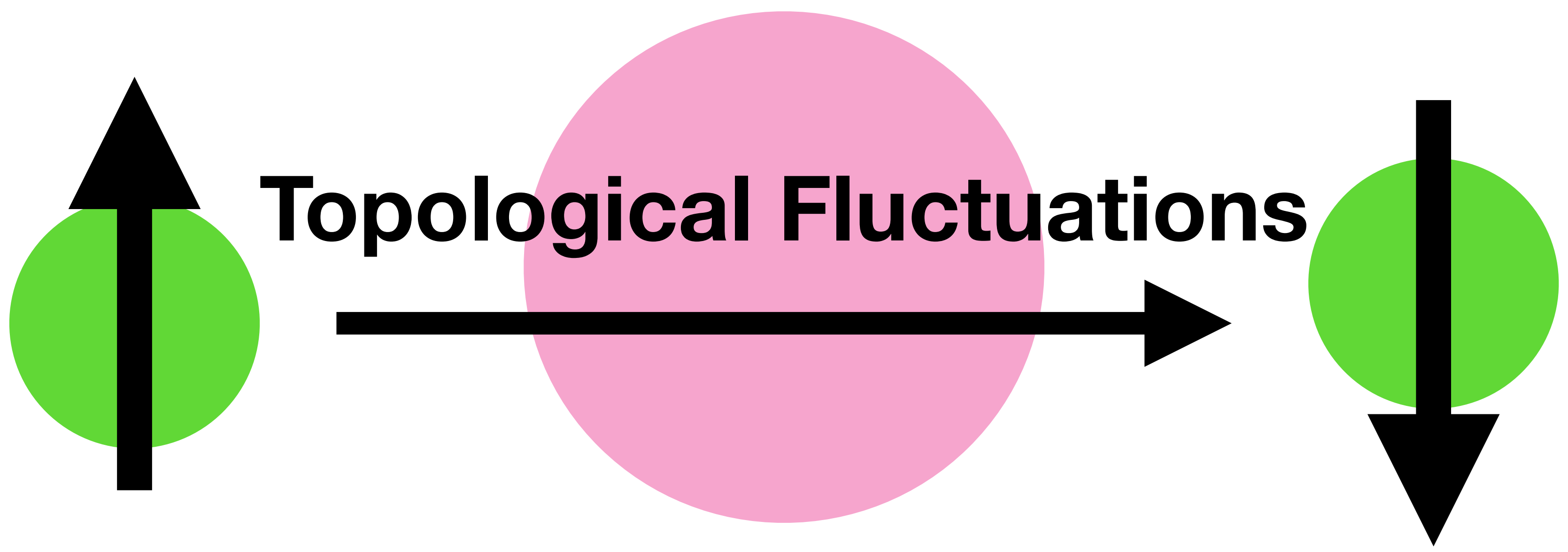
$$\vec{S}_{\text{total}} = \frac{\hbar}{2}$$



Topological Fluctuations flipping Quark Helicity

(Tarasov-Venugopalan)

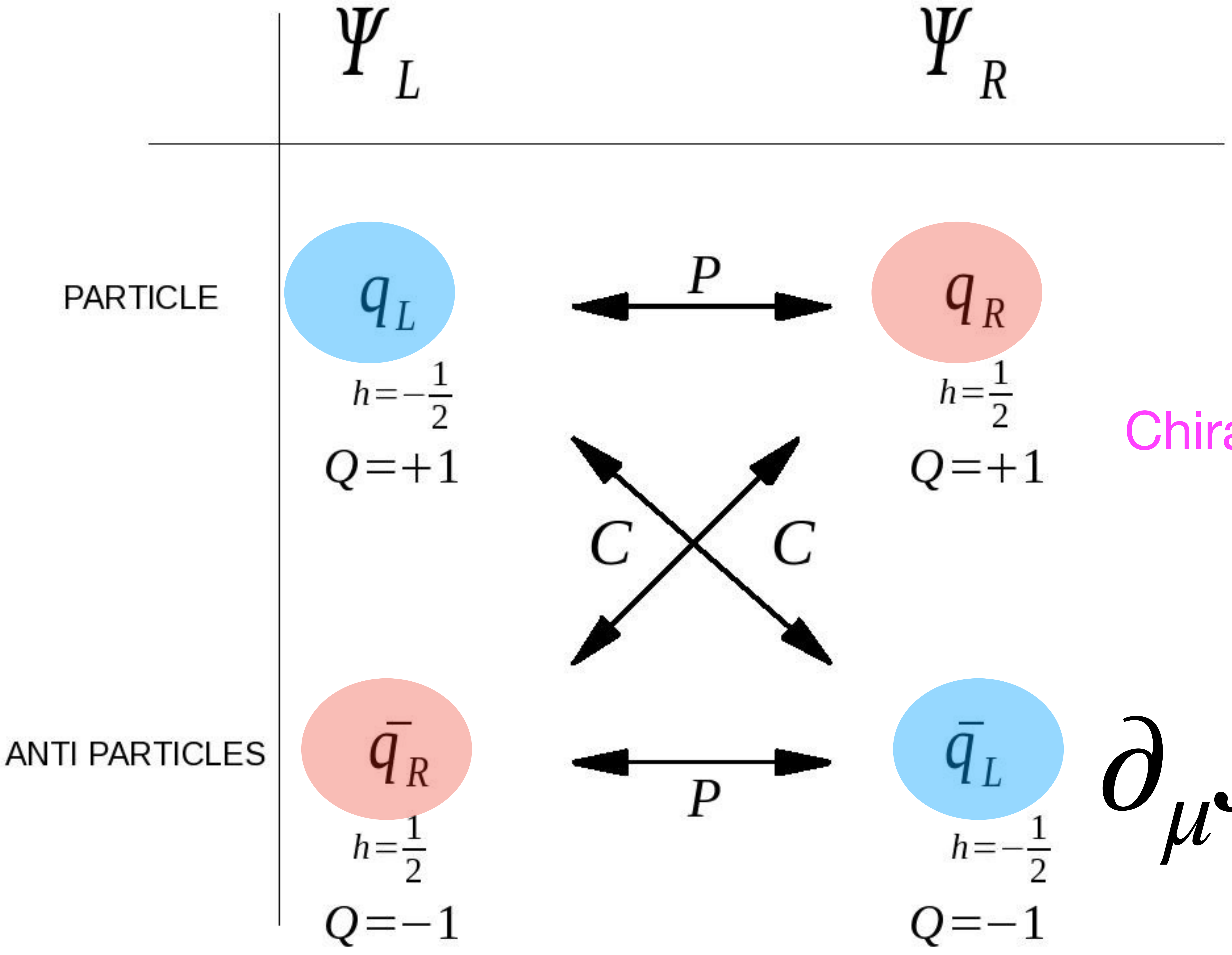
$$\frac{\hbar}{2} = L_{\text{quark}} + L_{\text{gluon}}$$




$$L_{\text{quark}} < L_{\text{gluon}} \;\;!!!$$

Q: Where does the angular momentum go?

# Chiral Anomaly



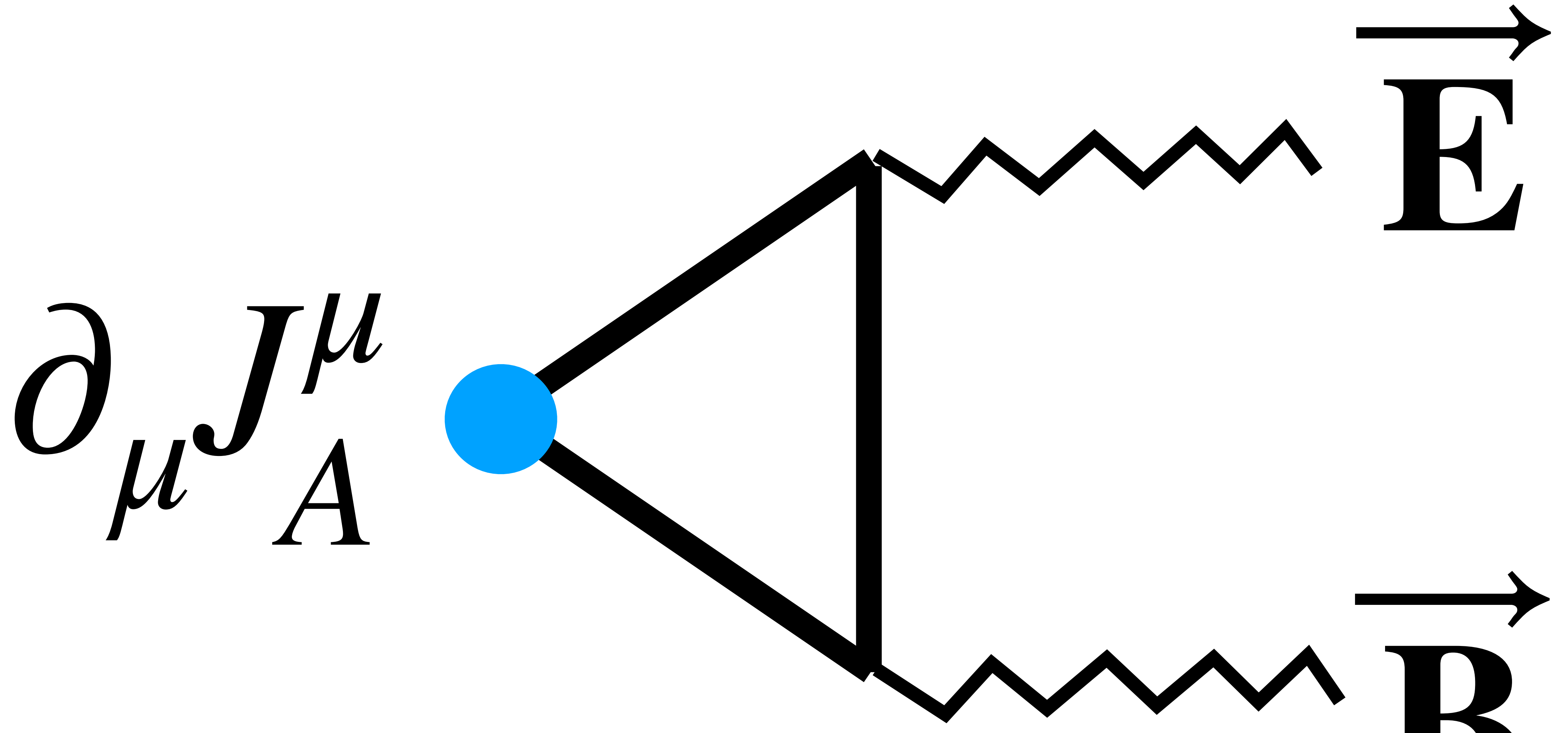

**Axial charge=(Blue)-(Red)**

Axial charge $n_A$ is the net helicity density

$$n_A = N(q_L) + N(\bar{q}_L) - (N(q_R) + N(\bar{q}_R))$$

It is P-odd and CP-odd (C-even)

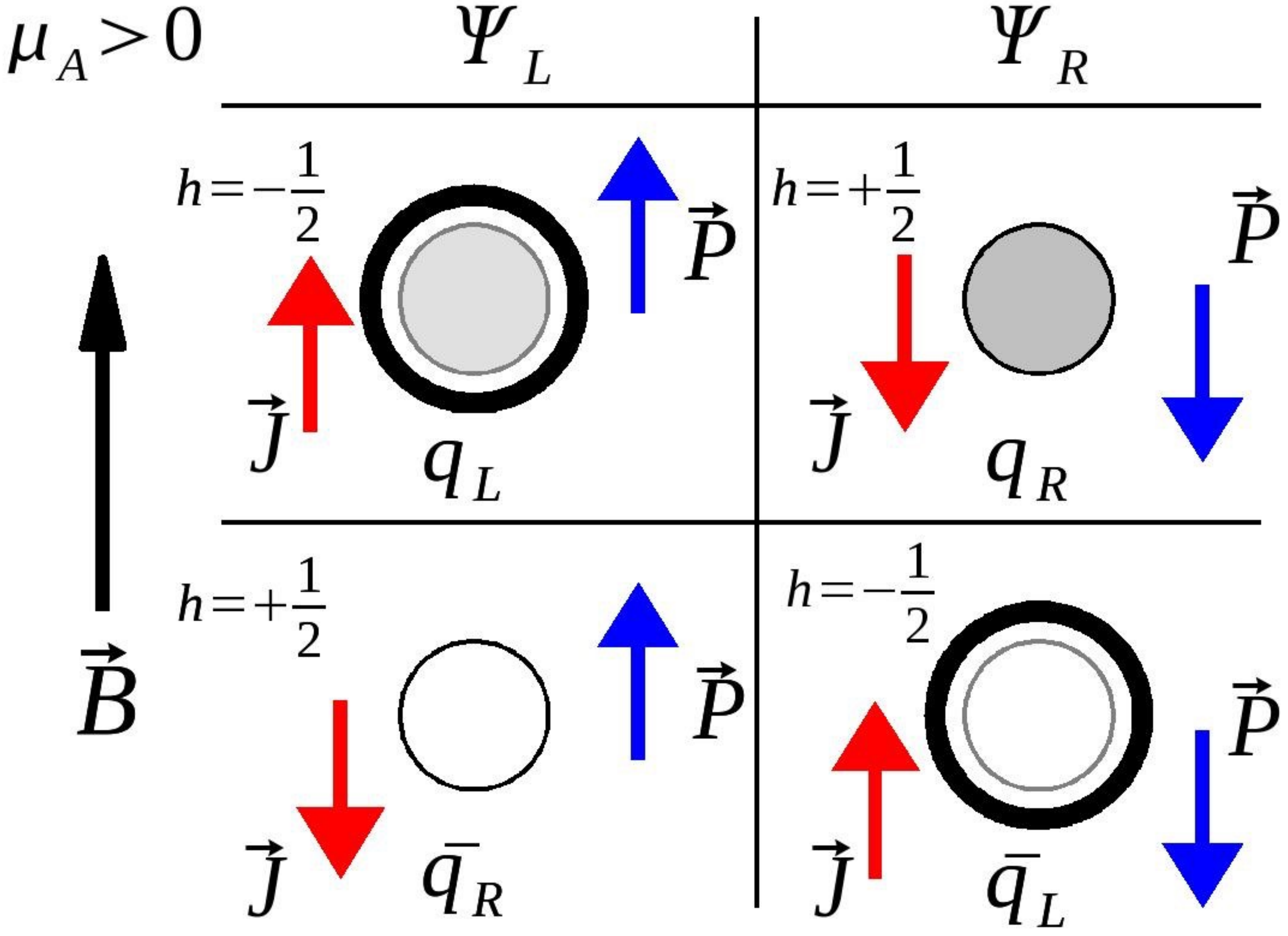
Chiral Anomaly : $\frac{dn_A}{dt} = \frac{e^2}{2\pi^2}\vec{\mathbf{E}}\cdot\vec{\mathbf{B}}$ (P-,CP-odd)




Triangle Diagram

# Chiral Magnetic Effect

(Fukushima-Kharzeev-Warringa, Vilenkin, Son-Zhitnitsky)




**Spin alignment in magnetic field leads to momentum alignment to induce a net charge current**

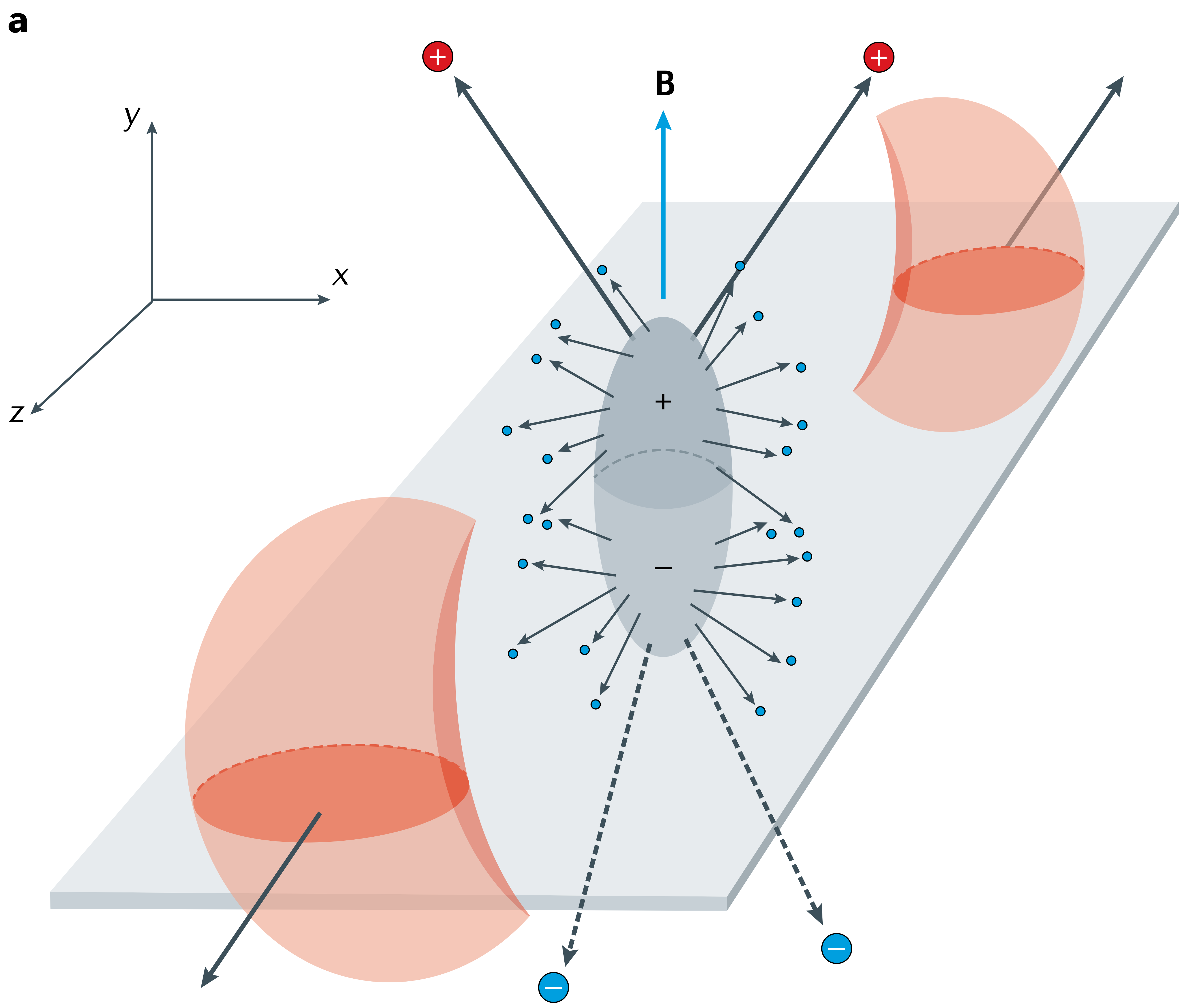
$$\vec{\mathbf{J}} = \frac{e^2}{2\pi^2}\mu_A \vec{\mathbf{B}} \quad \text{and} \quad \vec{\mathbf{J}_A} = \frac{e^2}{2\pi^2}\mu \vec{\mathbf{B}}$$

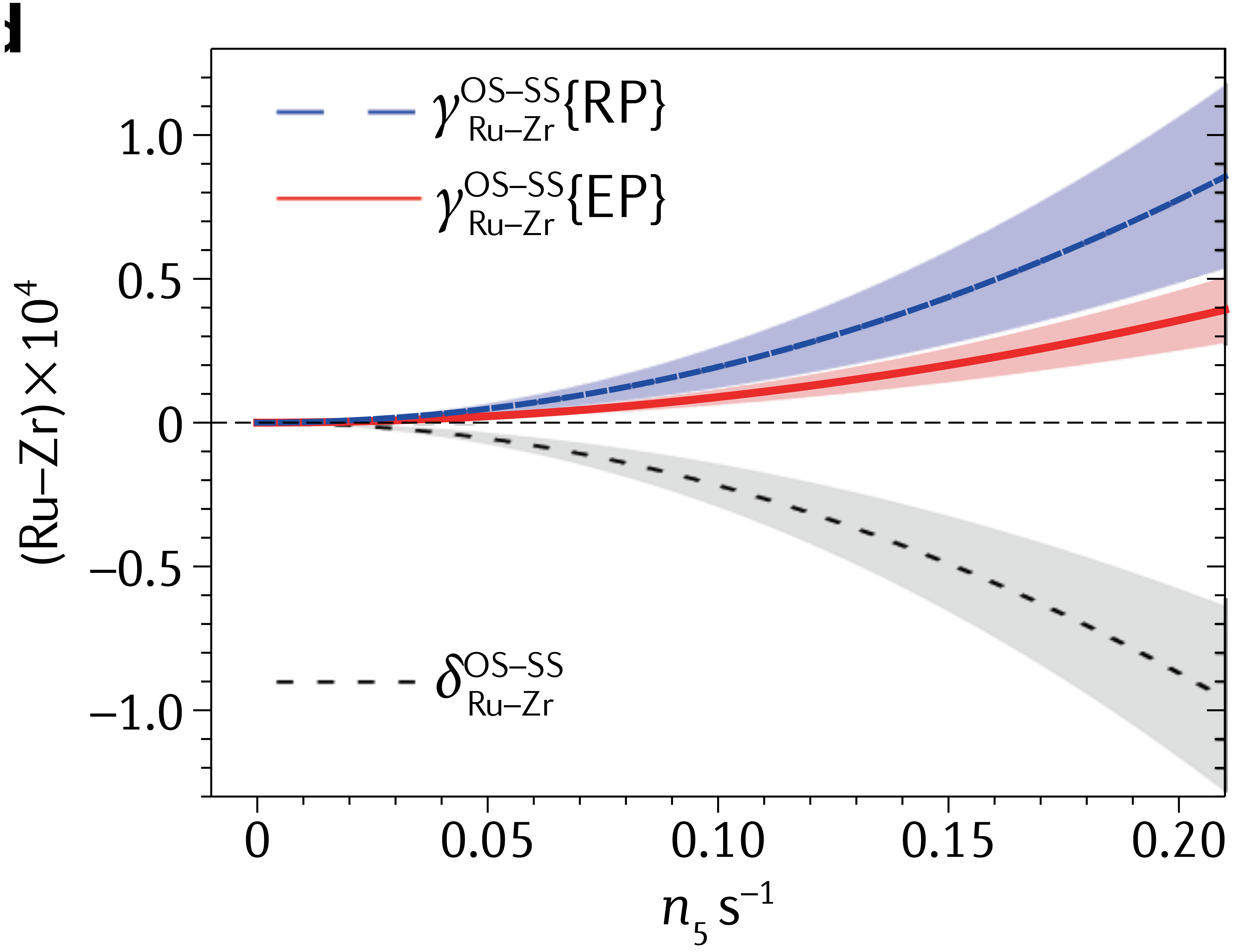
$$\text{Power } P = \vec{\mathbf{E}} \cdot \vec{\mathbf{J}} = \frac{dn_A}{dt}\mu_A = \frac{e^2}{2\pi^2}\vec{\mathbf{E}} \cdot \vec{\mathbf{B}}\mu_A$$

(Nielsen-Ninomiya)

$$\rightarrow \vec{\mathbf{J}} = \frac{e^2}{2\pi^2}\mu_A \vec{\mathbf{B}}$$

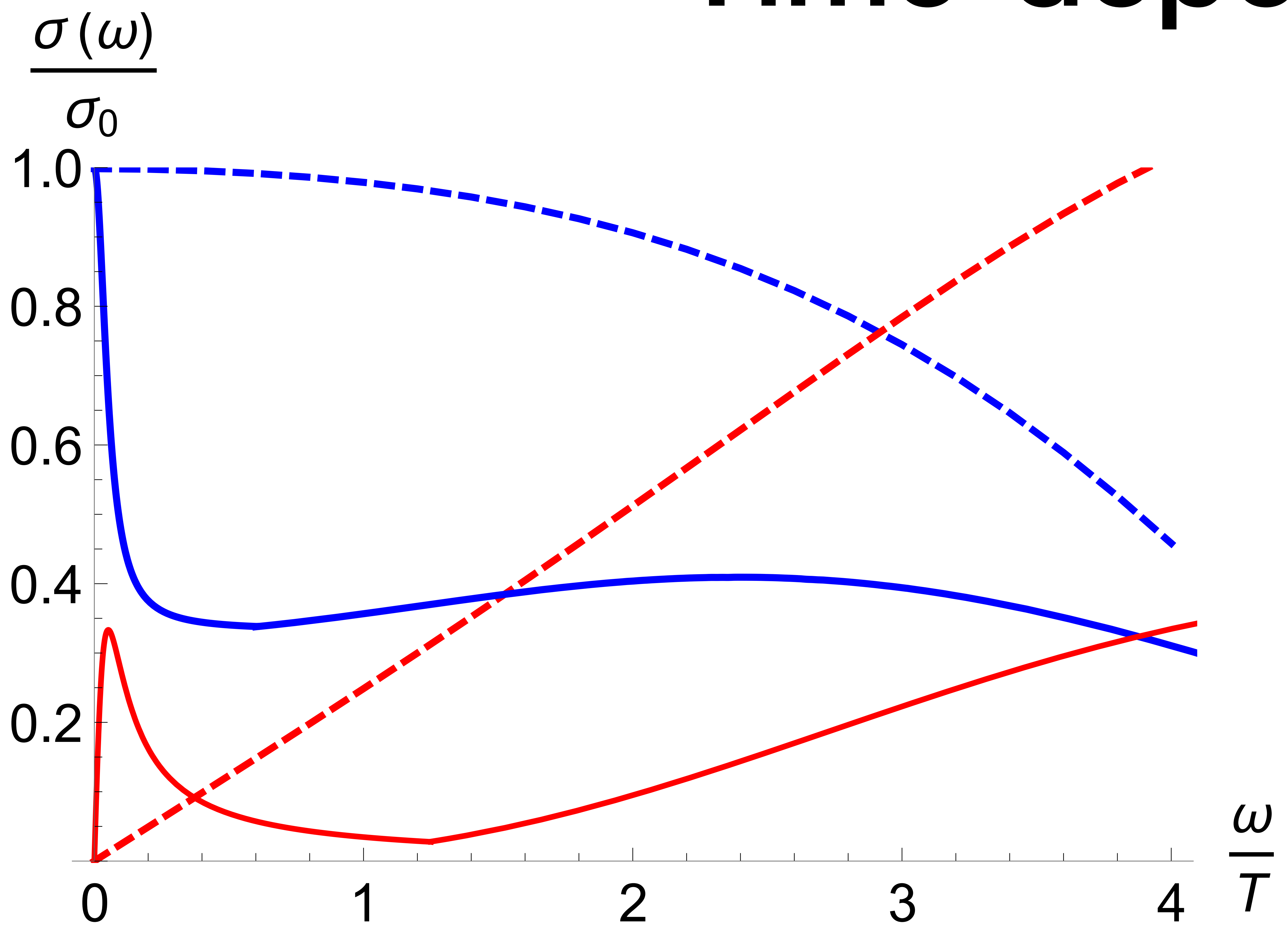
# Iso-baric Ru-Zr at RHIC




Taken from Kharzeev-Liao, Nature Reviews Physics,
Volume 3, 55–63 (2021)

# Time-dependent CME




**Solid: Perturbative QCD, Dashed: AdS/CFT**
**Blue: Real part, Red: Imaginary part**

**From PRD 95, 051901 (2017) by Kharzeev-Stephanov-Yee**

$$\vec{\mathbf{J}}(\omega) = \sigma(\omega)\vec{\mathbf{B}}(\omega)\,, \quad \sigma_0 = \frac{e^2}{2\pi^2}\mu_A$$

$$\sigma(\omega) \sim \sigma_0 - i\xi_5\omega$$

$$\xi_5 = -\frac{0.5}{\alpha_s^2\log(1/\alpha_s)}\frac{\sigma_0}{T}$$ in pQCD

(Jimenez Alba-Yee)

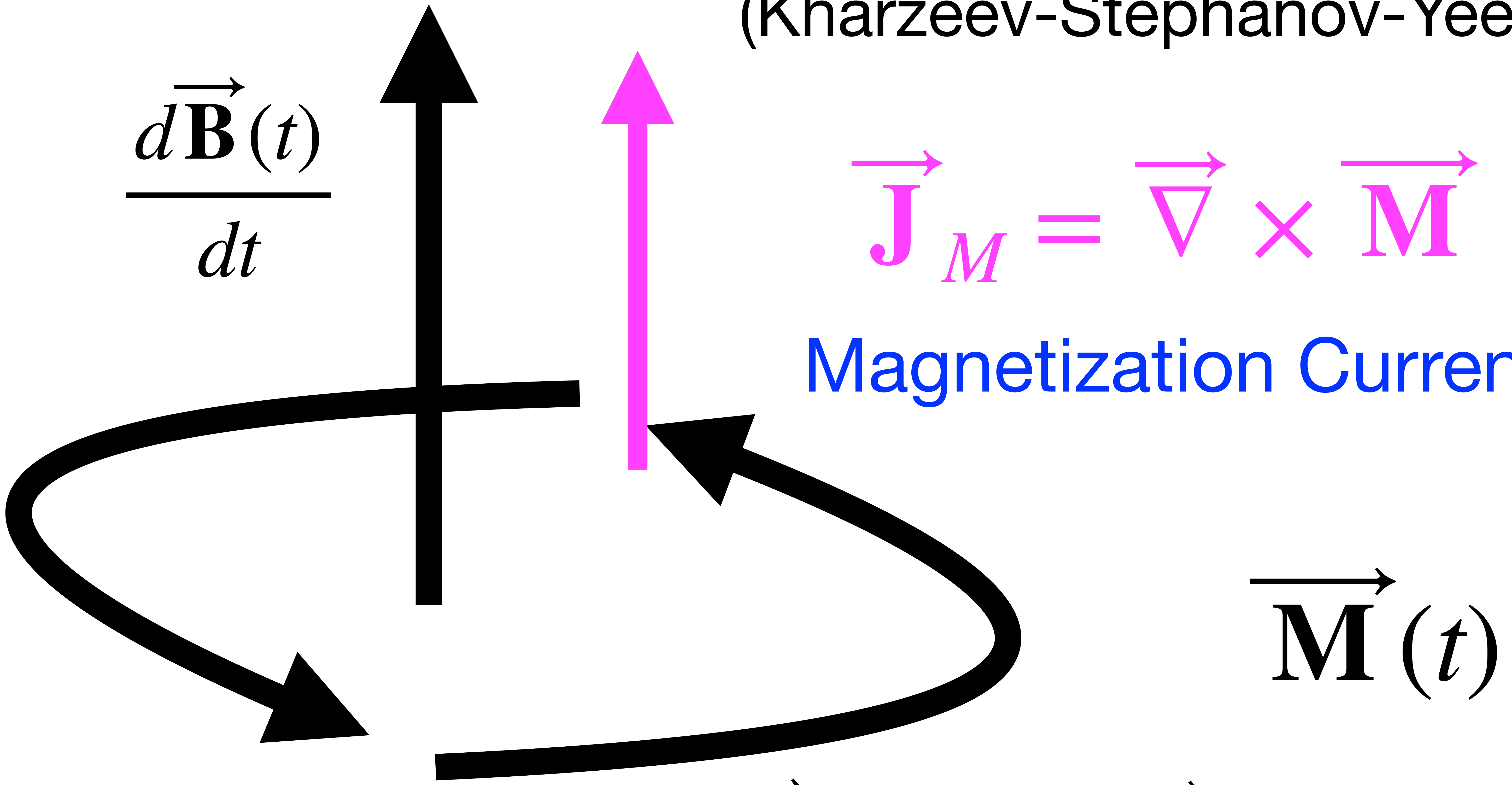
$\sigma(0) = \sigma_0$ is fixed by chiral anomaly

But, $\sigma(\omega > 0)$ is from more general effects from P-odd helicity

# Anatomy of Chiral Magnetic Effect

(Kharzeev-Stephanov-Yee)

| | $\omega \ll \tau_R^{-1}$ | $\omega \gg \tau_R^{-1}$ |
|---|---|---|
| $\boldsymbol{J}^{\mathrm{EQ}}$ | $\frac{g}{6}$ | $0$ |
| $\boldsymbol{J}^{\mathrm{KM}}$ | $1-\frac{g}{6}$ | $1-\frac{g}{6}$ |
| $\boldsymbol{J}^{\mathrm{M}}$ | $0$ | $-\frac{g}{6}$ |
| $\boldsymbol{J}^{\mathrm{total}}$ | $1$ | $1-\frac{g}{3}$ |




Faraday's Effect

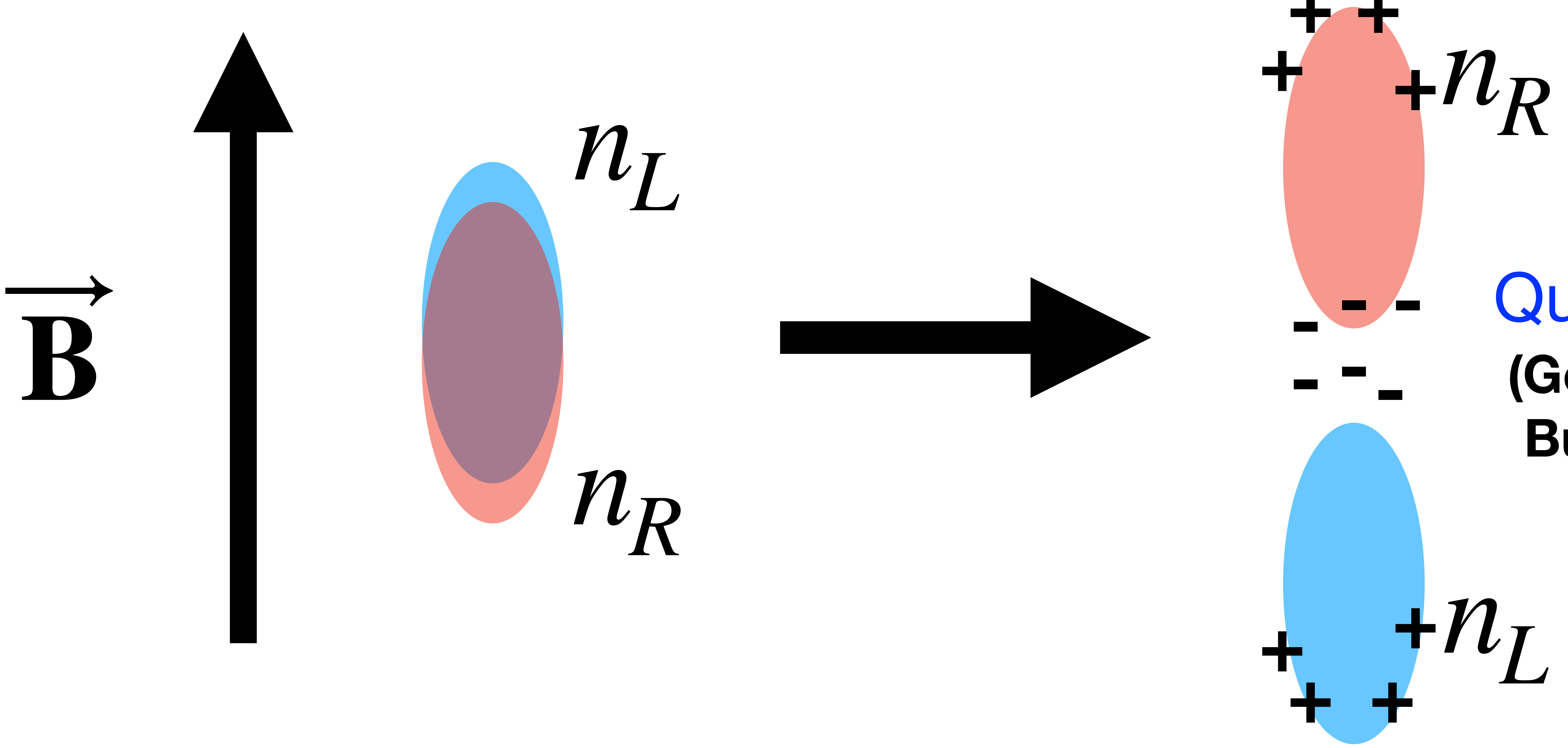
$$\vec{\mathbf{E}}(t) \sim \frac{d\vec{\mathbf{p}}}{dt} \sim \frac{d\vec{\mathbf{S}}}{dt} \sim \frac{d\vec{\mathbf{M}}}{dt}$$

Helicity g-factor

# Chiral Magnetic Wave

(Kharzeev-Yee, Burnier-Kharzeev-Liao-Yee)

$$\partial_t n + \vec{\nabla} \cdot \vec{\mathbf{J}} = \partial_t n \pm \frac{1}{4\pi^2\chi} \vec{\mathbf{B}} \cdot \vec{\nabla} n = (\partial_t + \vec{\mathbf{v}}_\chi \cdot \vec{\nabla}) n = 0$$




Quadrupole of charges

**(Gorbar-Miransky-Shovkovy, Burnier-Kharzeev-Liao-Yee)**

Hydrodynamic wave of chiral charges with velocity $\vec{\mathbf{v}}_\chi = \pm \frac{1}{4\pi^2\chi} \vec{\mathbf{B}}$

# Anomalous Transport

(Son-Surowka, Landsteiner, ...)

$$\vec{\mathbf{J}} = \sigma_B \vec{\mathbf{B}} + \sigma_V \vec{\omega} \qquad \sigma_B = \frac{\mu}{4\pi^2}, \quad \sigma_V = \frac{\mu^2}{8\pi^2}$$

Chiral Vortical Effect

$$\vec{\mathbf{P}} = \sigma_B^{\epsilon} \vec{\mathbf{B}} + \sigma_V^{\epsilon} \vec{\omega} \qquad \sigma_B^{\epsilon} = \frac{\mu^2}{8\pi^2}, \quad \sigma_V^{\epsilon} = \frac{\mu^3}{6\pi^2}$$

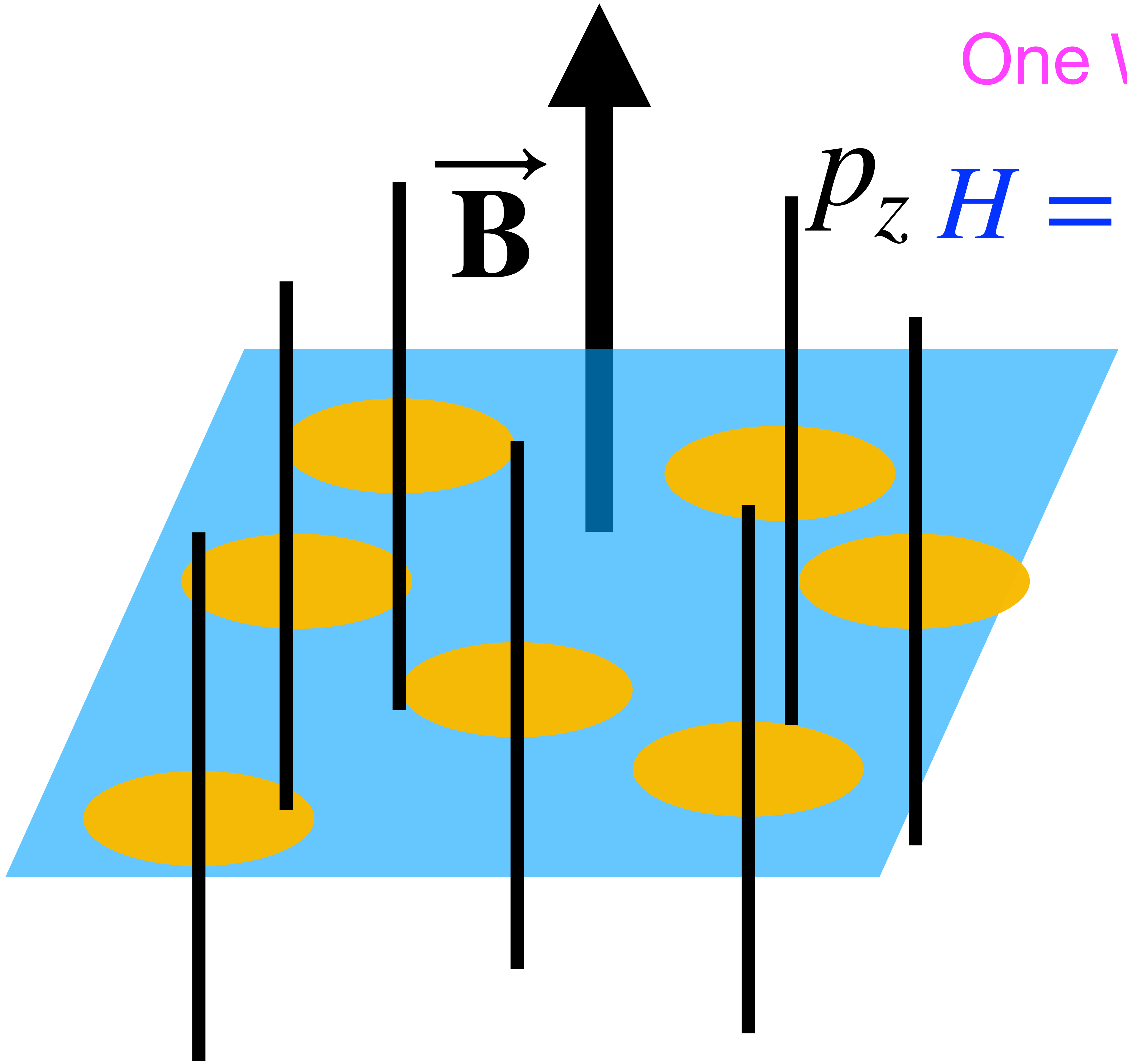
Time Reversal (T) relates $\sigma_V(k,\omega) = \sigma_B^{\epsilon}(k,\omega)$

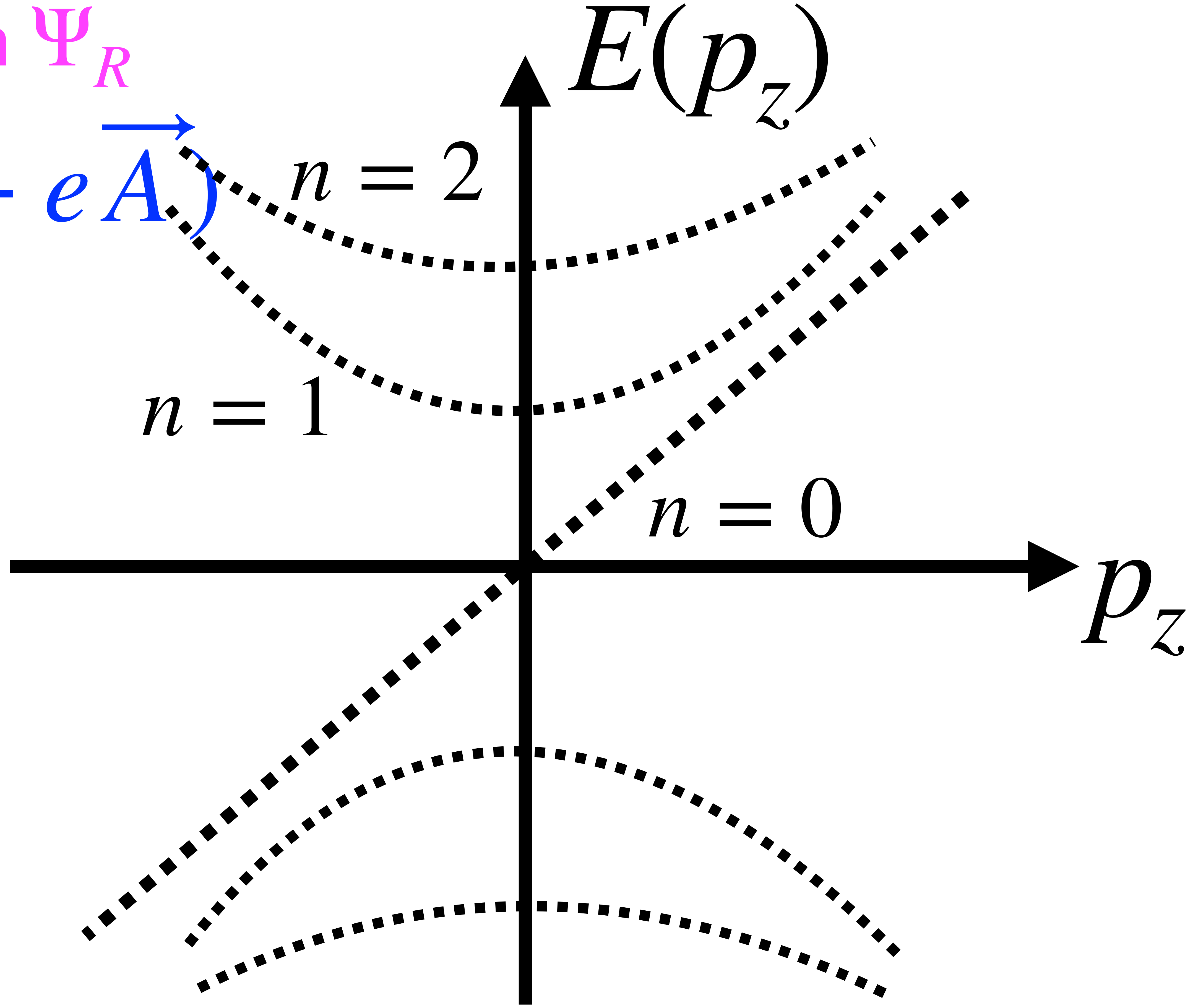
(Shiyong Li-Yee)

# Gribov's Picture of Anomaly

One Weyl Fermion $\Psi_R$

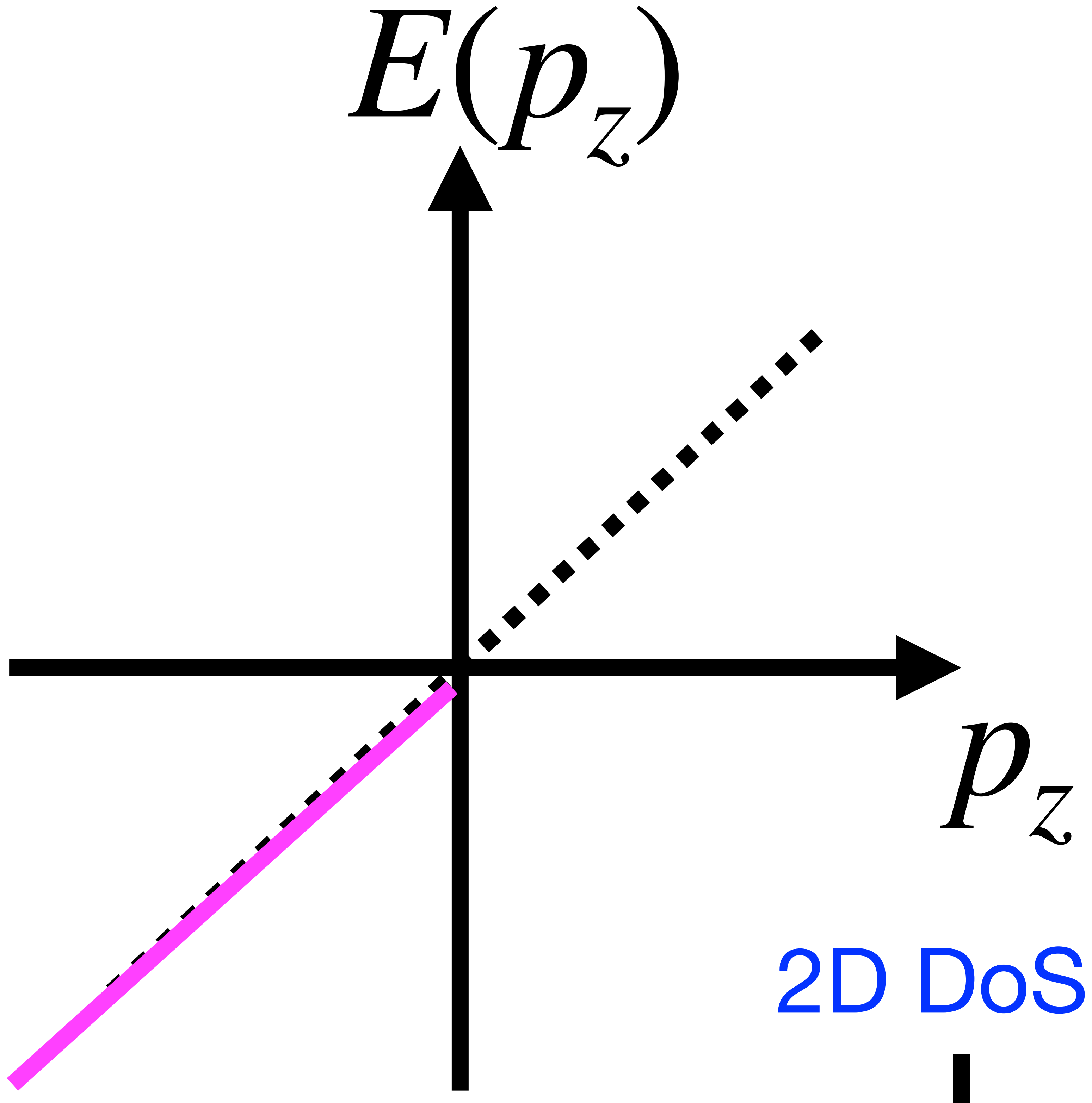
$$H = \vec{\sigma} \cdot (\vec{p} - e\vec{A})$$



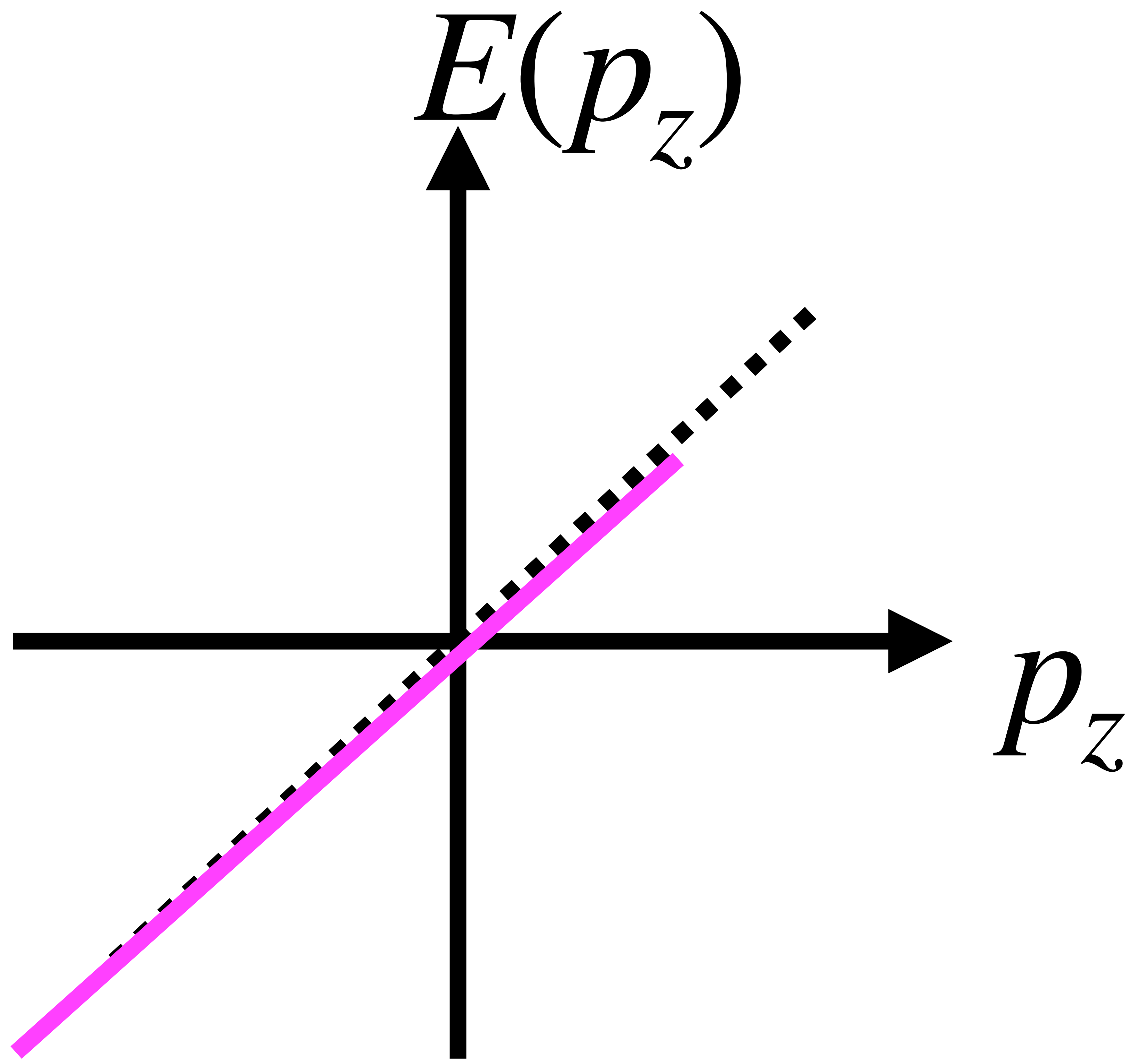




Landau Levels with 2D density of states $\dfrac{eB}{2\pi}$ Band-Crossing Chiral Zero Mode

# Spectral Flow for Anomaly




$$\frac{dp_z}{dt} = eE$$

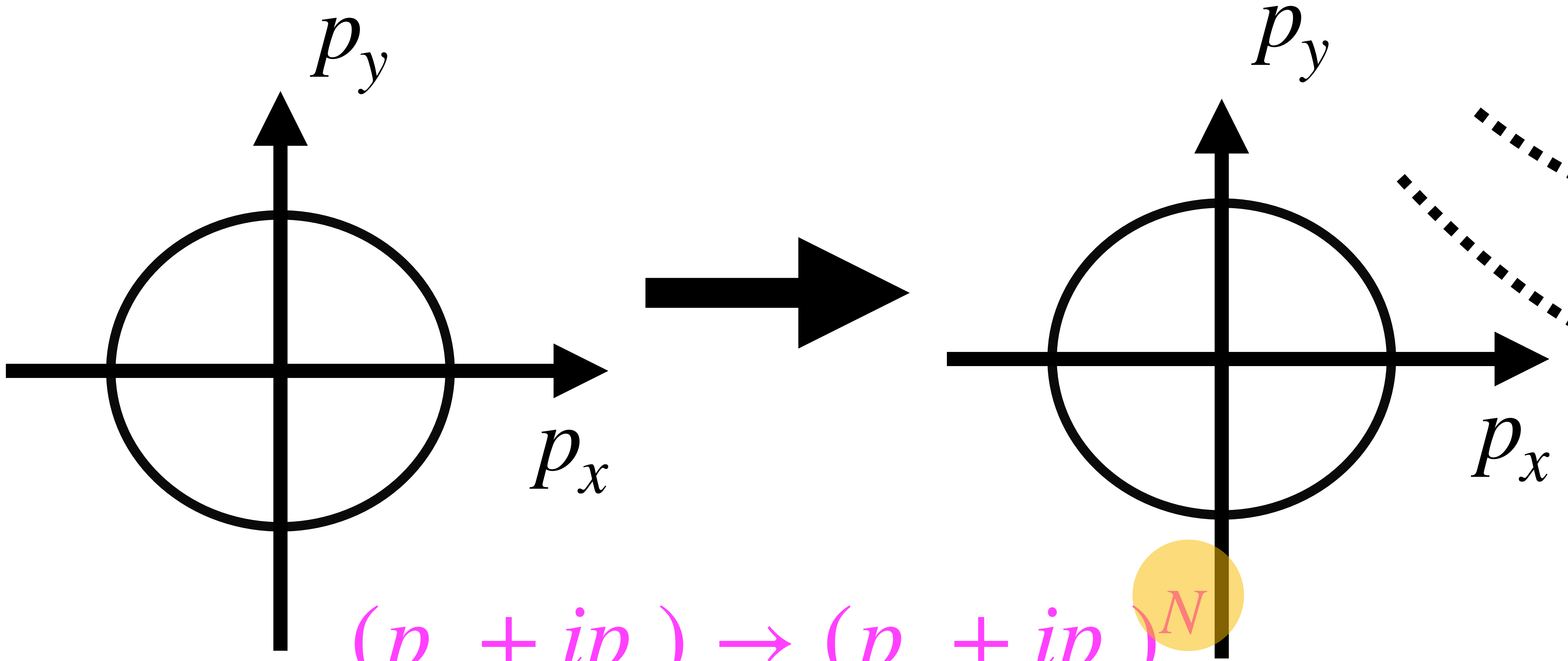



2D DoS 1D DoS

$$\frac{dn_A}{dt} = \left(\frac{eB}{2\pi}\right)\frac{1}{2\pi}\frac{dp_z}{dt} = \frac{e^2}{4\pi^2}\vec{\mathbf{E}}\cdot\vec{\mathbf{B}}$$

# Dirac/Weyl Semi-Metals

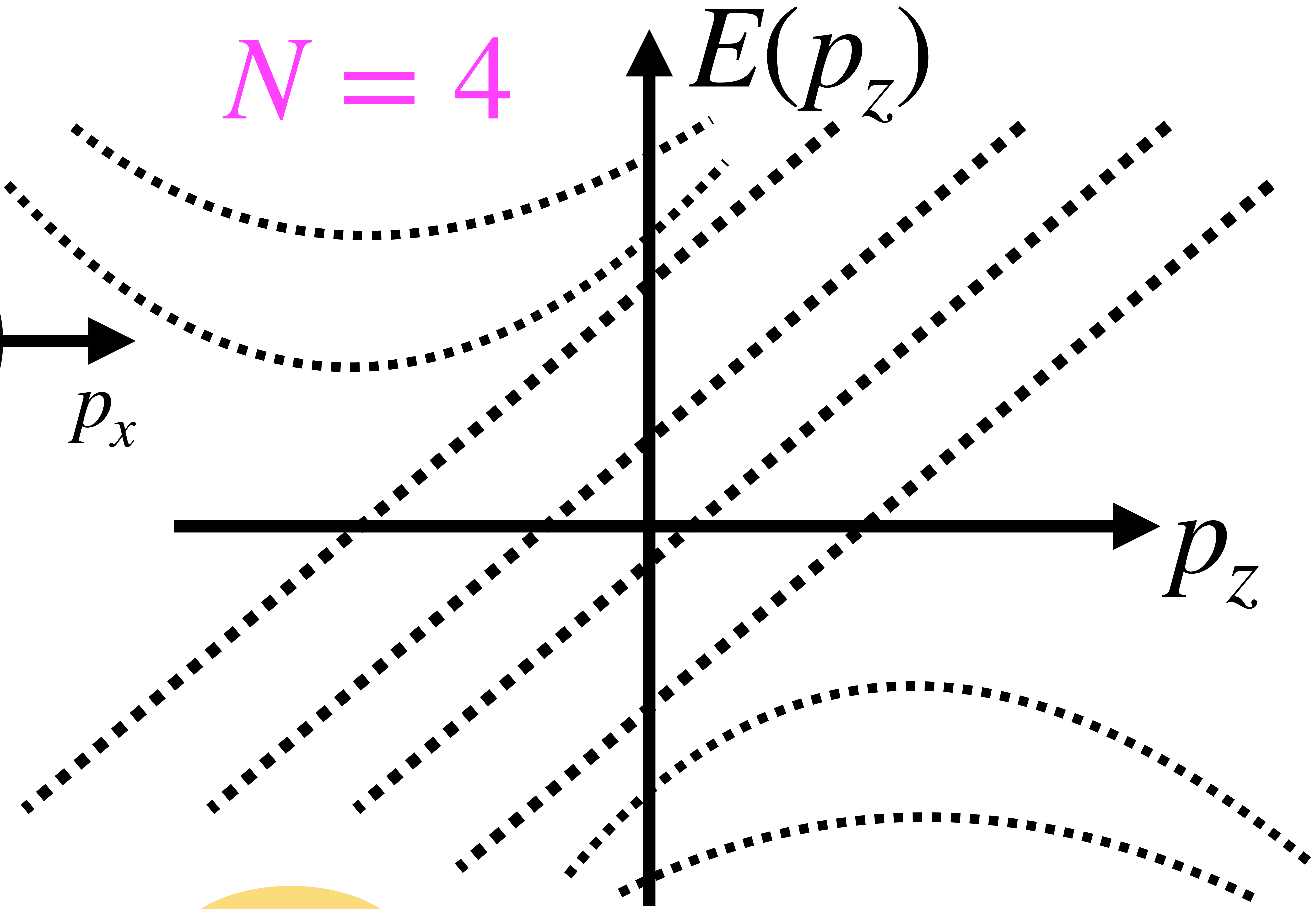
$$H = \sigma^{+}(D_x - iD_y)^N + \sigma^{-}(D_x + iD_y)^N + \sigma_z D_z \,, \qquad \vec{D} = \vec{p} - e\vec{A}$$




$(p_x + ip_y) \rightarrow (p_x + ip_y)^N$

(N-Winding Map)

$N = 4$




Chiral Anomaly : $\dfrac{dn}{dt} = N\dfrac{e^2}{4\pi^2}\vec{\mathbf{E}} \cdot \vec{\mathbf{B}}$

N=4 Chiral Zero Modes

# Topology of Generalized Spinors

(Piljin Yi-Yee)

A generalized spinor $H = P_1(\overrightarrow{\mathbf{k}})\sigma_x + P_2(\overrightarrow{\mathbf{k}})\sigma_y + P_3(\overrightarrow{\mathbf{k}})\sigma_z$

with arbitrary functions $P_1(\overrightarrow{\mathbf{k}}), P_2(\overrightarrow{\mathbf{k}}), P_3(\overrightarrow{\mathbf{k}})$ in momentum $\overrightarrow{\mathbf{k}}$

In both Heat-Kernel method (Fujikawa's method) and Diagrammatic computation, it was proven that

Chiral Anomaly : $\partial_\mu j^\mu = N_P \frac{1}{4\pi^2} \overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{B}}$

where $N_p = \frac{1}{\pi^{\frac{3}{2}}} \int d^3\overrightarrow{\mathbf{k}} \ \det\left(\frac{\partial P(k)}{\partial k}\right) e^{-P^2(\overrightarrow{\mathbf{k}})}$

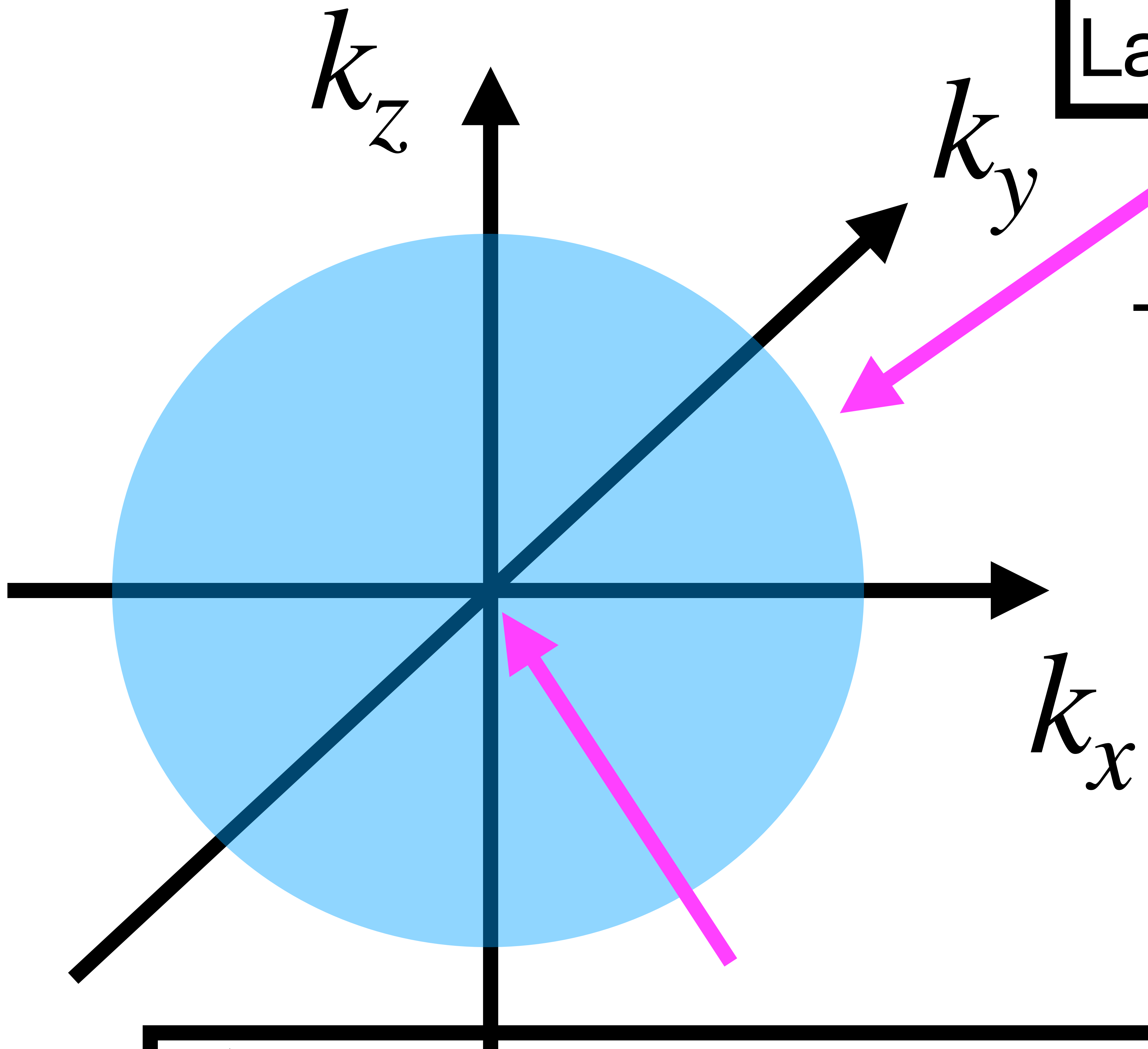
: Winding Number of the Map $\overrightarrow{\mathbf{k}} \to \overrightarrow{\mathbf{P}}(\overrightarrow{\mathbf{k}})$

# Topology of Generalized Spinors

Winding Number $N_p$ = Berry's Monopole at $|\vec{\mathbf{k}}| \to \infty$

Large $\vec{\mathbf{k}}$ where Chiral Kinetic Theory is valid

(Son-Yamamoto, Stephanov-Yin, Chen-Pu-Wang-Wang)




The spinor at $\vec{\mathbf{k}}$ is $|\psi(\vec{\mathbf{P}}(\vec{\mathbf{k}}))\rangle$ where

$$(\vec{\mathbf{P}} \cdot \vec{\sigma})|\psi(\vec{\mathbf{P}})\rangle = |\vec{\mathbf{P}}||\psi(\vec{\mathbf{P}})\rangle$$

The Berry's monopole of $|\psi(\vec{\mathbf{P}}(\vec{\mathbf{k}}))\rangle$ in $\vec{\mathbf{k}}$-space is $N_p$ times of unit monopole in $\vec{\mathbf{P}}$-space

$$A_{k^i} = \left(\frac{\partial P^j}{\partial k^i}\right) A_{P^j}$$

$\vec{\mathbf{k}} = 0$ where Anomaly happens

# UV-IR Connection

Index Theorem : $\text{Index}(D\cdot\sigma) = \frac{N_P}{8\pi^2}\int d^4x\ F\wedge F$

Anomaly : Defined in Infrared (IR)

Local Topological Density: Defined Locally in Space (UV)

Chiral Anomaly : $\partial_\mu j^\mu = N_p \frac{1}{4\pi^2}\vec{\mathbf{E}}\cdot\vec{\mathbf{B}}$

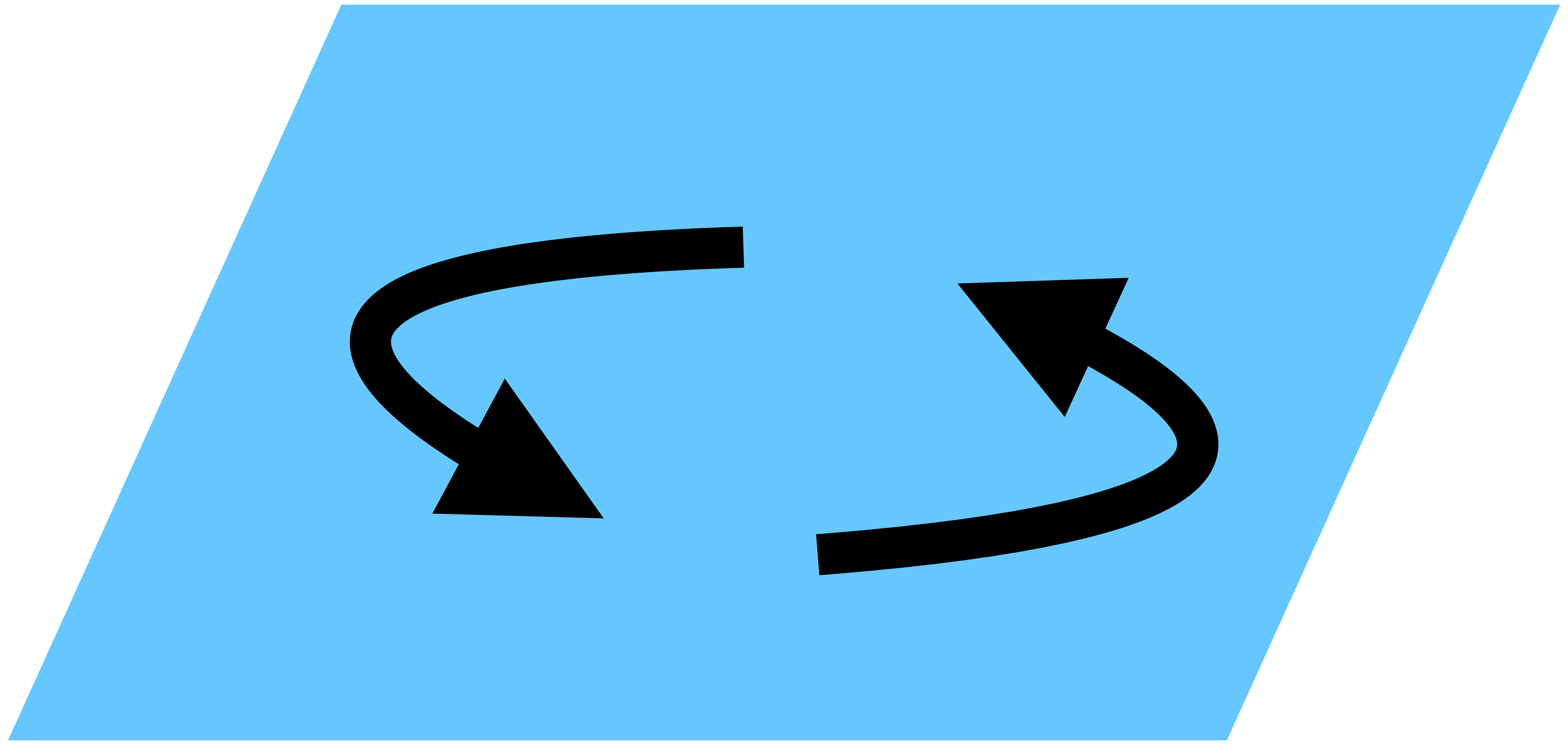
Anomaly : Defined in Infrared $\vec{\mathbf{k}} = 0$ (IR)

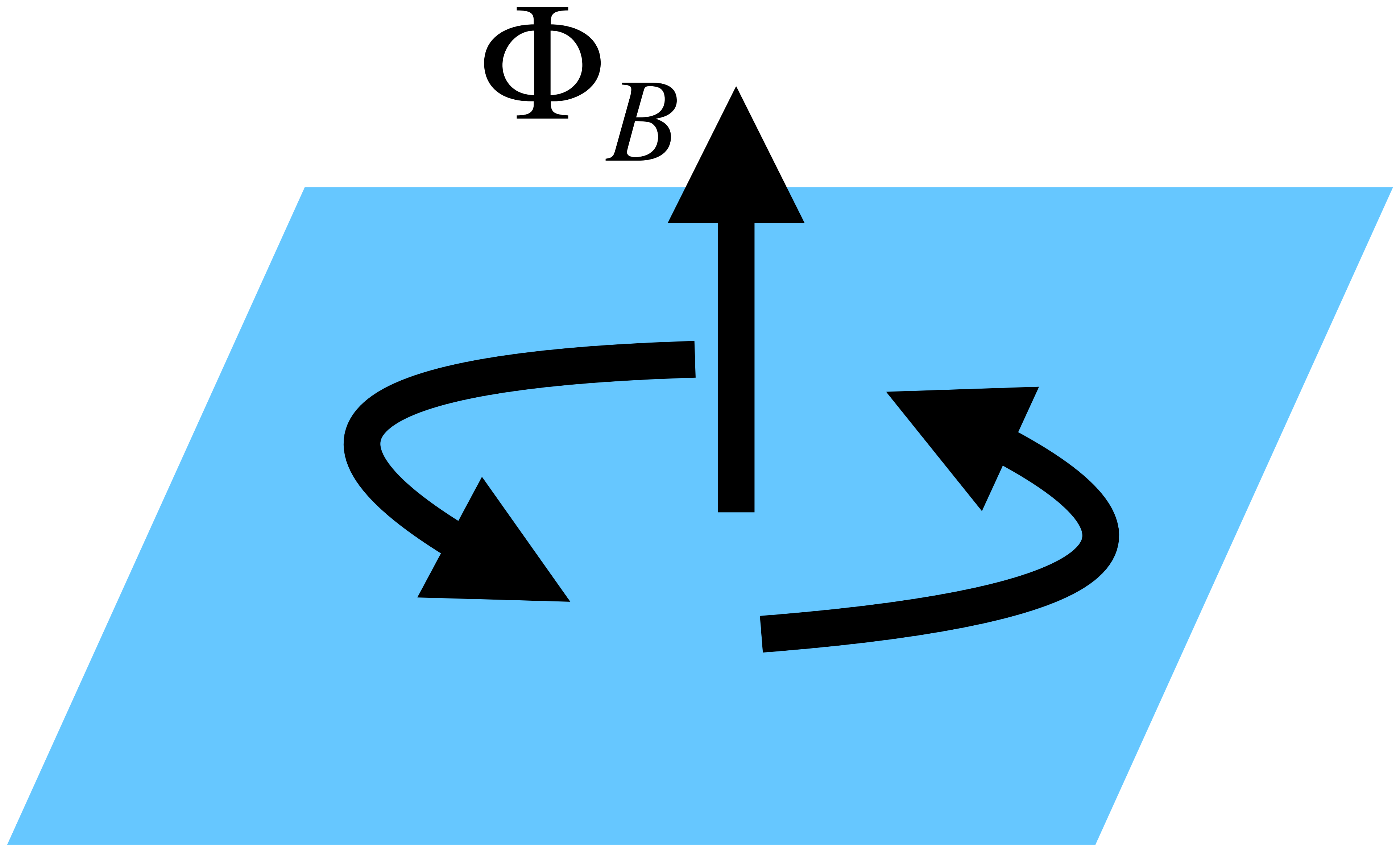
Answers the question by Fujikawa and Mueller-Venugopalan

Berry's Monopole defined in $\vec{\mathbf{k}} = \infty$ (UV)

# Spin of Magnetic Vortices

(Fukushima-Hidaka-Yee)






Superfluid Vortices

$$L_z = \hbar N \quad , \quad N = \text{Particle Number}$$

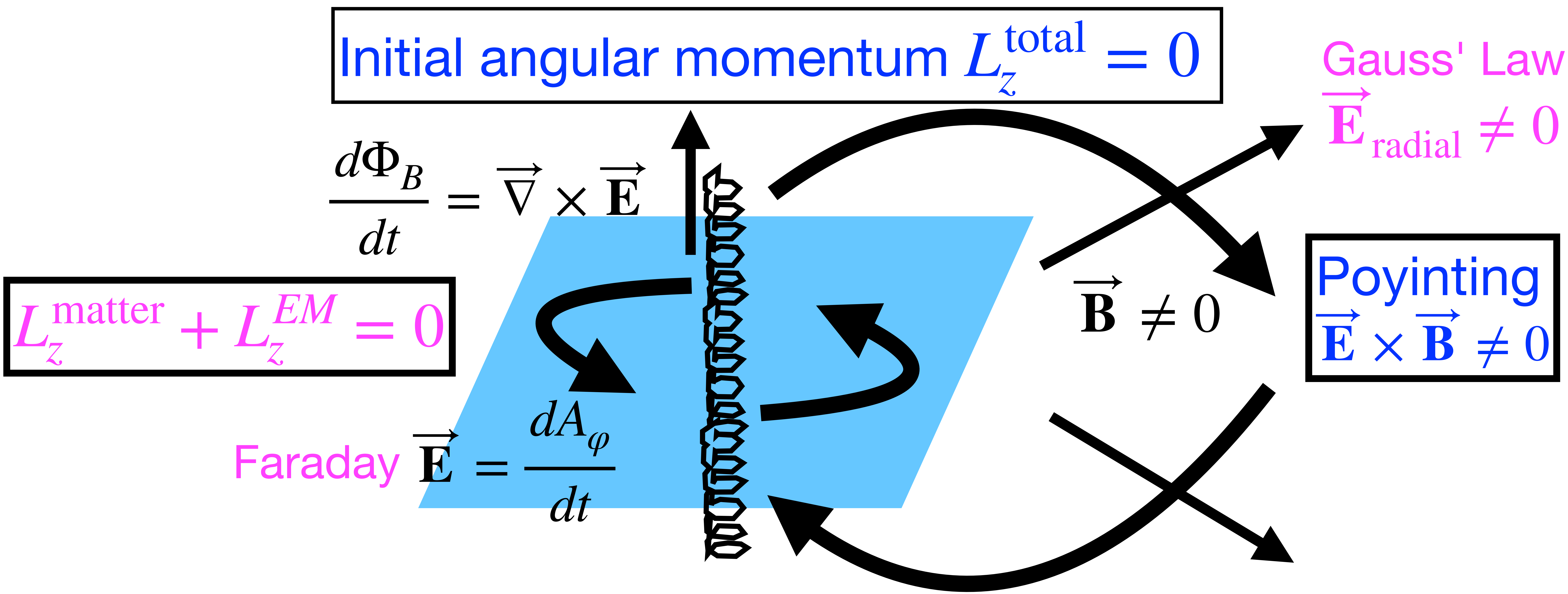
**Magnetic Vortices**

$\Pi_\varphi = p_\varphi - qA_\varphi$ : Gauge Invariant

$L_z = r \times \Pi_\varphi$ : Not a multiple of $\hbar$

Special feature of
Representation of 2D
Rotation Group SO(2)=U(1)

# Feynman's Angular Momentum Paradox

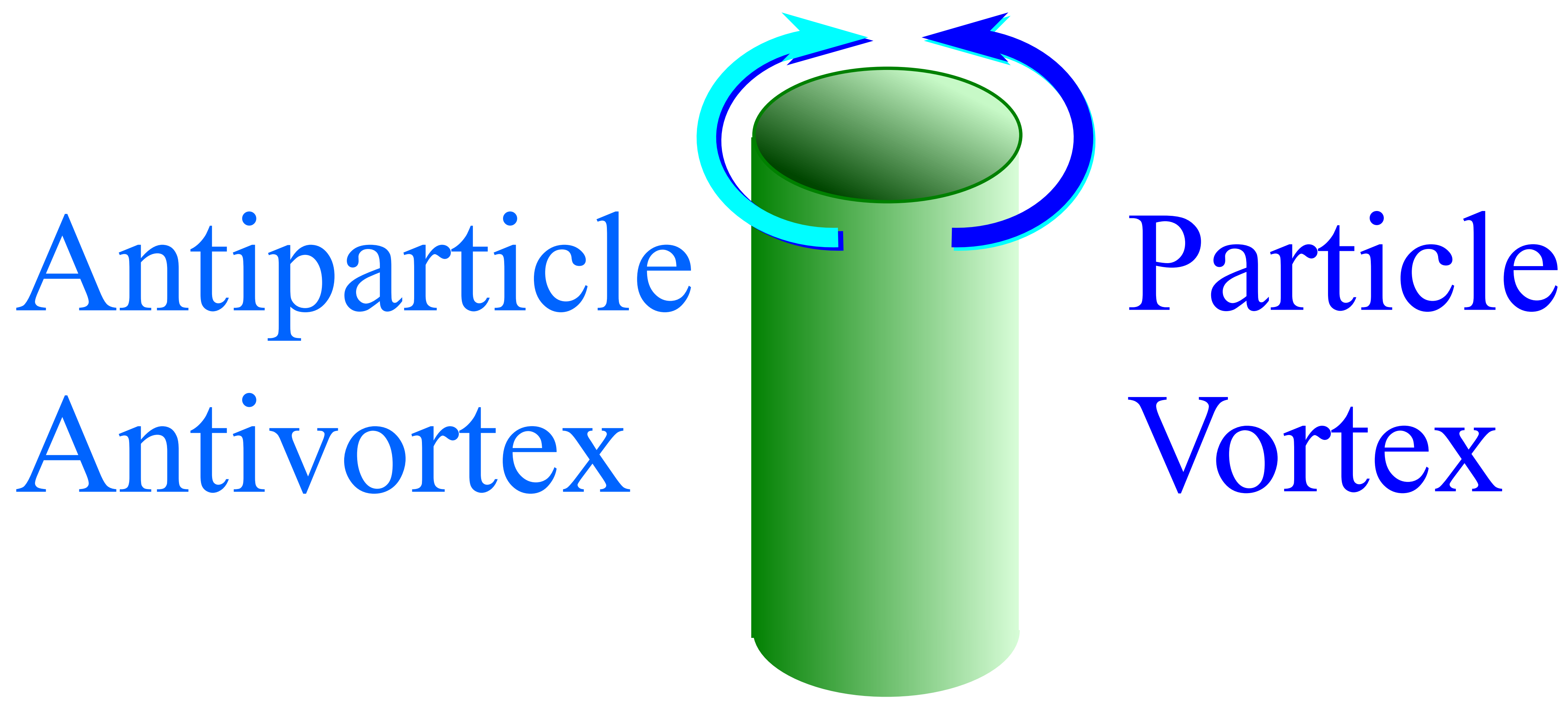



$L_z^{\text{matter}} = r \times \Pi_\varphi = r \times (P_\varphi - qA_\varphi) = -q(r \times A_\varphi)$ : Not a multiple of $\hbar$

$L_z^{\text{EM}} = r \times (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \neq 0$ : Angular momentum of EM fields

# Relativistic Magnetic Vortices

## Neutral Nielsen-Olesen Vortex




**Angular momentum from particle vortex and anti-particle anti-vortex cancel**

Angular momentum is zero for a neutral vortex, where $D_0\Phi = 0$, $\overrightarrow{P} \propto i((D_0\Phi)^*(\overrightarrow{\mathbf{D}}\Phi) - \mathrm{h.c.}) = 0$

Neutral vortex is a composite of particle vortex and anti-particle anti-vortex $\Phi \sim e^{i\varphi} \sim a + b^\dagger$

## Particle-Vortex Duality in (2+1)D

$$|D\Phi|^2 + V(|\Phi|^2) + \frac{1}{4}F_{\mu\nu}^2 \quad \leftrightarrow \quad (\partial\phi)^2 + W(|\phi|^2)$$

Magnetic Vortex — Scalar Particle

# Example : Non-Abelian CFL Vortex

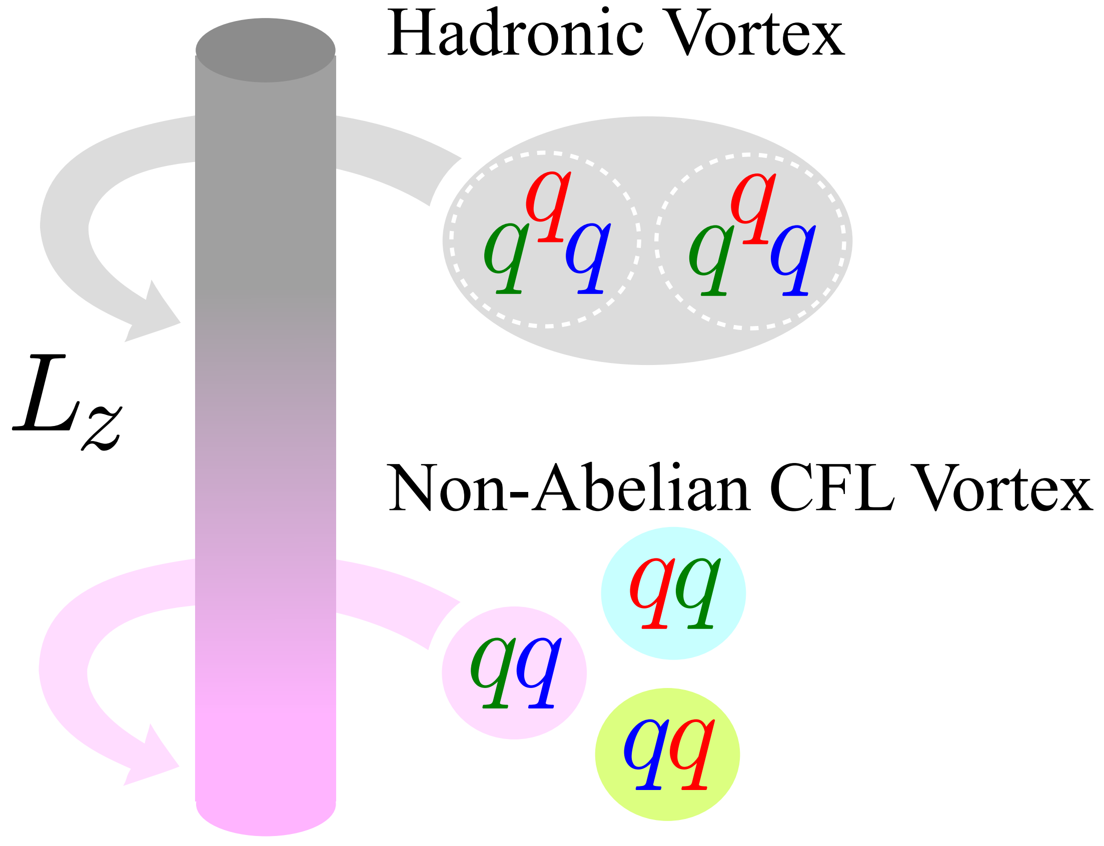
(Balachandran-Digal-Matsuura, Eto-Hirono-Nitta-Yasui, Alford et al)

For a charged vortex, there is a non-trivial cancellation $L_z^{\text{matter}} + L_z^{\text{EM}} = 0$

Non-Abelian Vortex in Color-Flavor-Locking (CFL) phase of dense 3-flavor quark matter $\langle q_\alpha^i q_\beta^j \rangle_{L,R} = \epsilon_{\alpha\beta\gamma}\epsilon^{ij\gamma}\Delta_{L,R}$

$U(1)_B \times SU(3)_L \times SU(3)_R \times SU(3)_c \to SU(3)_V$ : CFL is similar to Hidden Local Symmetry breaking for massive $\rho$-Mesons and massless Pions

A composite of Baryon Superfluid Vortex + Color Magnetic Vortex



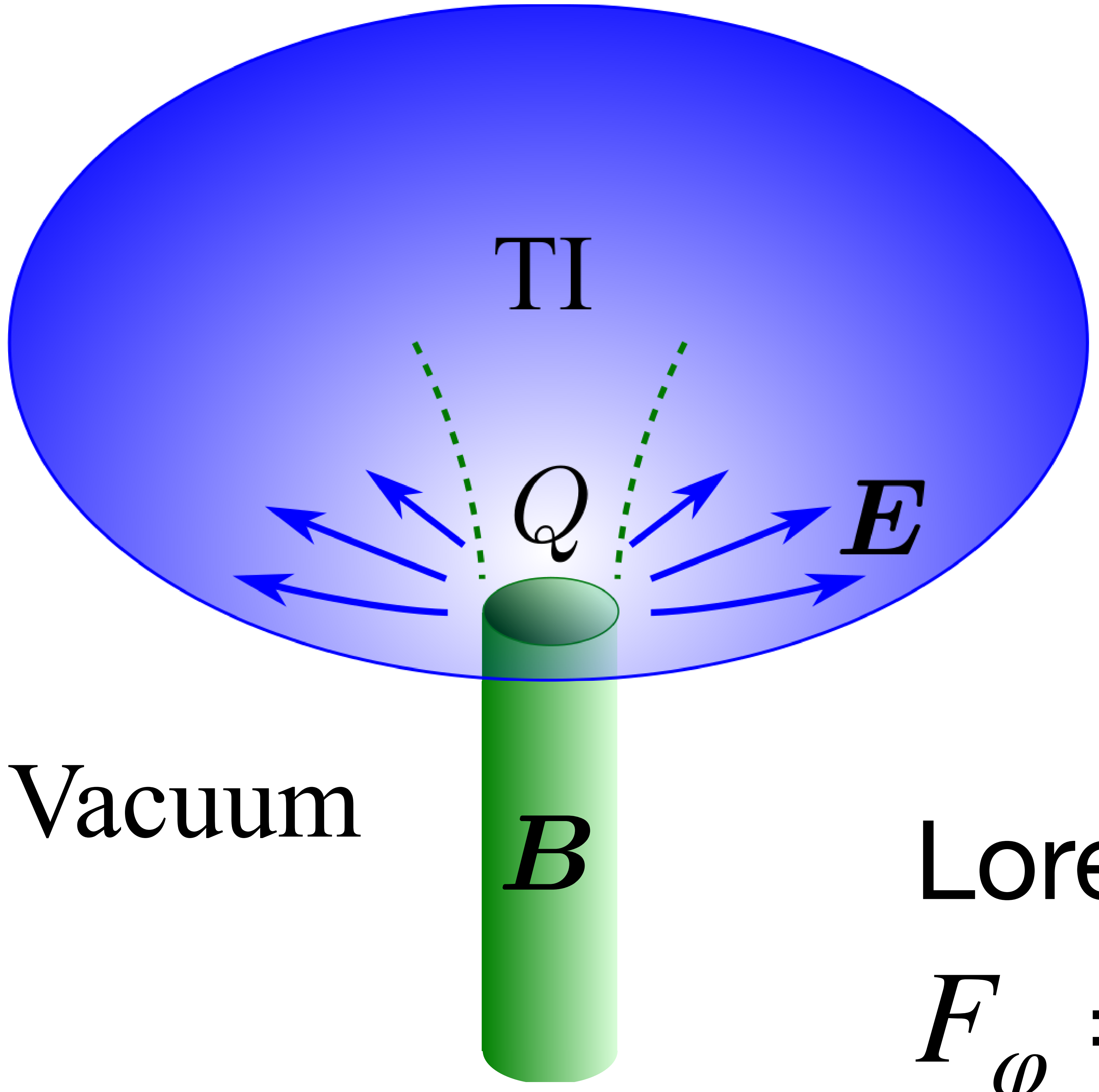

$$\Phi = e^{i\frac{\nu}{3}\varphi}\begin{pmatrix} e^{i\frac{2\nu}{3}\varphi} & 0 & 0 \\ 0 & e^{-i\frac{\nu}{3}\varphi} & 0 \\ 0 & 0 & e^{-i\frac{\nu}{3}\varphi} \end{pmatrix}\begin{pmatrix} f(r) & 0 & 0 \\ 0 & b(r) & 0 \\ 0 & 0 & b(r) \end{pmatrix}$$

$$L_z^{\text{matter}} + L_z^{\text{color fields}} = \frac{\nu}{2}N_B$$

It matches to Hadronic Phase

# Vortices in Topological Insulator Surface

(Nogueira-Nussinov-Brink)




QHE of level $\frac{1}{2}$ : $j^\mu = -\frac{e^2}{8\pi\hbar}\epsilon^{\mu\nu\alpha}F_{\nu\alpha}$

In components :

$$Q = \frac{e^2}{4\pi\hbar}B_z\,, \quad j_x = \frac{e^2}{4\pi\hbar}E_y\,, \quad j_y = -\frac{e^2}{4\pi\hbar}E_x$$

Lorentz force is zero: Zero Torque on TI Matter

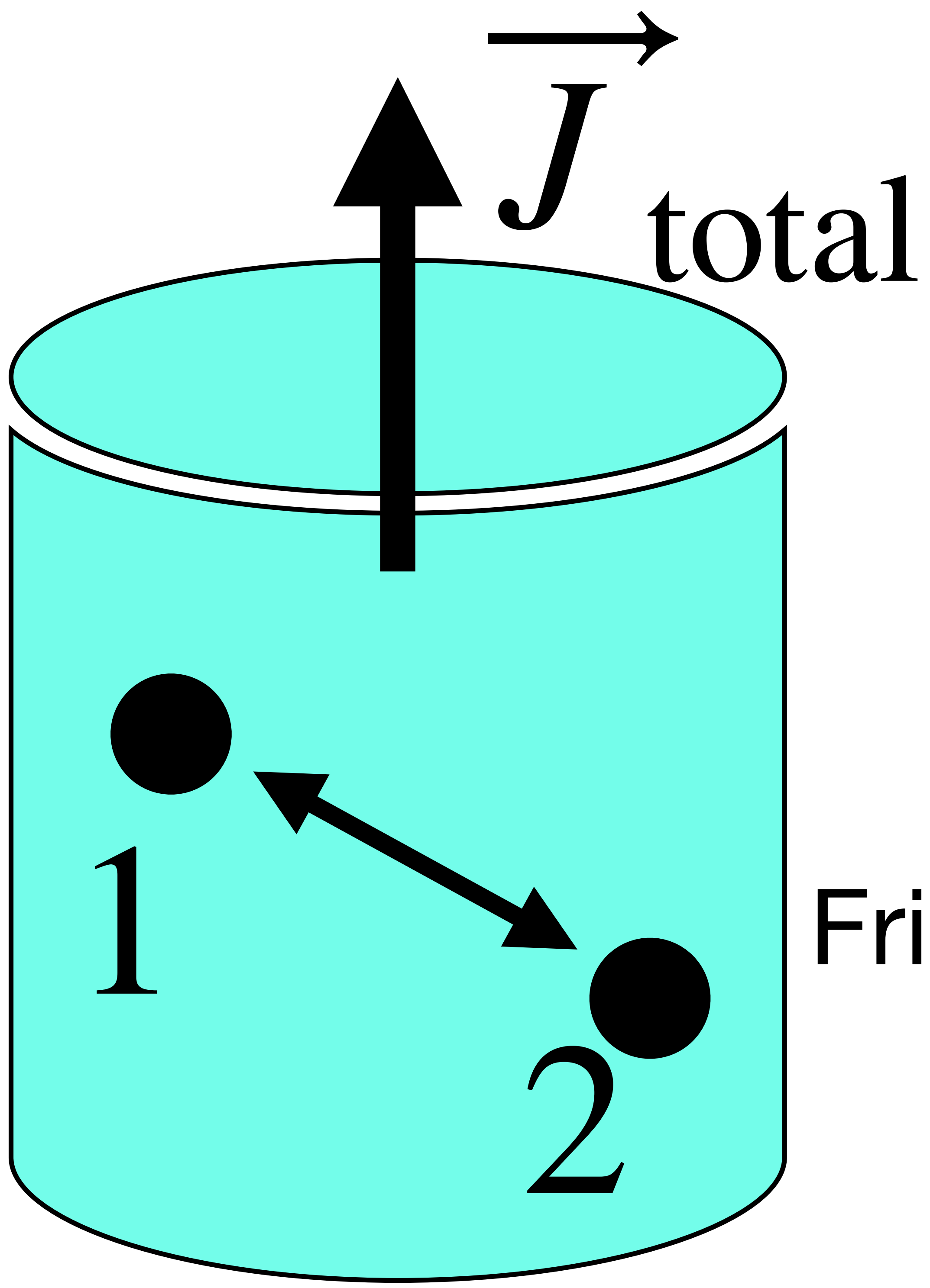
$$F_\varphi = (Q\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}})_\varphi = QE_\varphi - j_r B_z = 0$$

The EM part $L_z^{\mathrm{EM}} = \int d^3r\ r \times (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) = -\frac{e^2}{16\pi^2\hbar}\Phi_B^2 = -\frac{\nu^2}{16}\hbar$ **Fractional Angular Momentum**

No TI matter part $L_z^{\mathrm{matter}} = 0$ $\quad \frac{2e}{\hbar}\Phi_0 = 2\pi\nu$ for Superconductor Vortex

# Spin Hydrodynamics and Pseudo-Gauge Transformations

(Shiyong Li-Stephanov-Yee)




Frictionless Wall

$$\Delta E_1 = -\Delta E_2$$
$$\Delta N_1 = -\Delta N_2$$
$$\Delta \mathbf{J}_1 = -\Delta \mathbf{J}_2$$

**Global Equilibrium with Angular Momentum Conservation**

$$dS = \beta dE - \alpha dN - \beta \vec{\boldsymbol{\Omega}} \cdot d\vec{\mathbf{J}}$$

**Spin Potential**

$\vec{\mathbf{J}} = \vec{\mathbf{x}} \times \vec{\mathbf{p}} + \vec{\mathbf{S}}$ : Angular Momentum

$$dS = \beta(dE - \vec{\mathbf{v}} \cdot d\vec{\mathbf{p}}) - \alpha dN - \beta \vec{\boldsymbol{\Omega}} \cdot d\vec{\mathbf{S}}$$

$\vec{\mathbf{v}} = \vec{\boldsymbol{\Omega}} \times \vec{\mathbf{x}}$ , $\vec{\boldsymbol{\Omega}}$ = Fluid Vorticity

**Spin correction to 1st Law of Thermodynamics**

(Hattori-Hongo-Huang-Matsuo-Taya)

# Canonical EM Tensor and Spin

Total Angular Momentum Tensor

$$J^{\mu\nu\alpha} = x^{\nu}T^{\mu\alpha} - x^{\alpha}T^{\mu\nu} + S^{\mu\nu\alpha}$$

Conservation of Angular Momentum $\partial_{\mu}J^{\mu\nu\alpha} = 0$ gives

$$\boxed{T^{\mu\nu} - T^{\nu\mu} = -\partial_{\alpha}S^{\alpha\mu\nu}}$$

Canonical Energy-Momentum Tensor

It describes angular momentum exchange between spin and orbital angular momenta

# Spin Hydrodynamics

(Becattini-Tinti, Florkowski-Friman-Jaiswal-Speranza, Hattori-Hongo-Huang-Matsuo-Taya, Fukushima-Pu, Li-Stephanov-Yee, Gallegos-Gursoy-Yarom)

Canonical EM Tensor

$$T_C^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + pg^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu - \frac{1}{2}\partial_\alpha S^{\alpha\mu\nu}$$

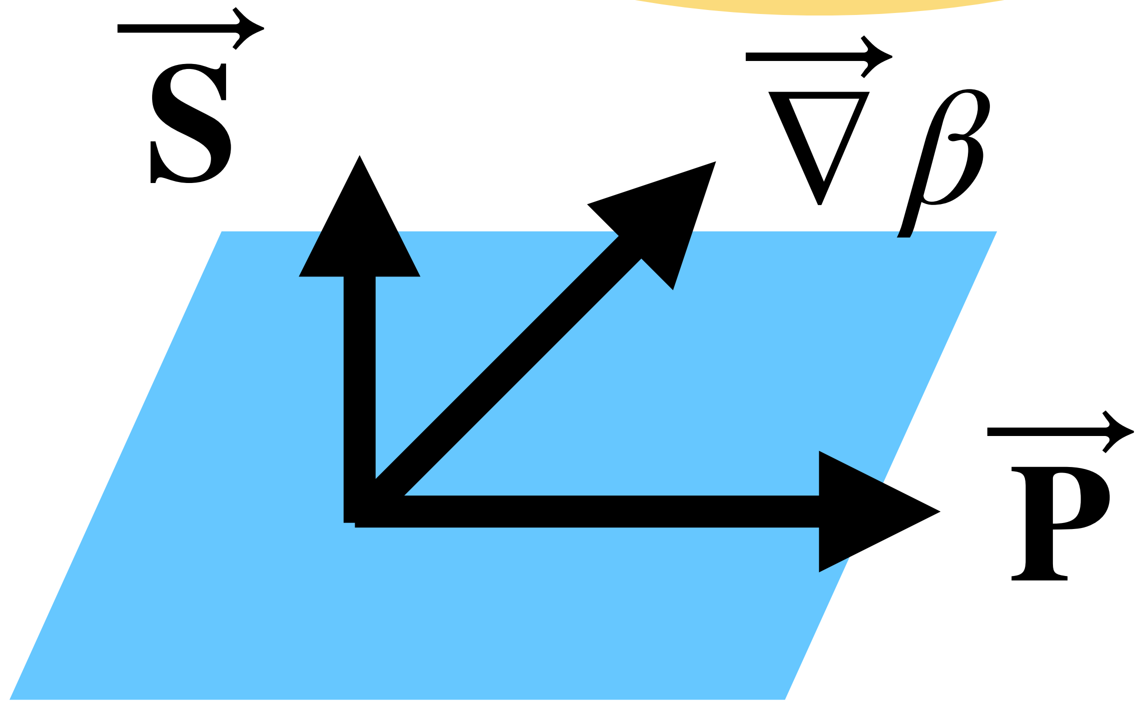
Anti-Symmetric Part

$$j^\mu = nu^\mu \, , \, s^\mu = su^\mu + \Delta s^\mu$$

The 2nd Law of Thermodynamics

$$\partial_\mu s^\mu = \partial_\mu(\Delta s^\mu - \frac{1}{2}\beta_\nu \partial_\alpha S^{\alpha\mu\nu} - \beta q^\mu) + (-\beta a_\mu + \partial_\mu \beta)(q^\mu - \frac{T}{2}S^{\mu\nu}\partial_\nu \beta) = 0$$

Thermal Hall Effect : $q^\mu = \frac{T}{2}S^{\mu\nu}\partial_\nu \beta$




Ideal Limit of Spin Hydrodynamics

N.B. A more complete list of terms can be found in Gallegos-Gursoy-Yarom '21

# Pseudo-Gauge Transformations

(Becattini-Tinti, Florkowski-Kumar-Ryblewski, Speranza-Weickgenannt)

Conservation Laws are not modified by

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\alpha(\Sigma^{\alpha\mu\nu} - \Sigma^{\mu\alpha\nu} - \Sigma^{\nu\alpha\mu}), \quad \tilde{S}^{\mu\nu\alpha} = S^{\mu\nu\alpha} - \Sigma^{\mu\nu\alpha}$$

$$\tilde{j}^\mu = j^\mu - \partial_\nu\left(\frac{a}{2}\omega^{\mu\nu}\right), \quad \tilde{s}^\mu = s^\mu - \partial_\nu\left(\frac{b}{2}\omega^{\mu\nu}\right)$$

Magnetization current

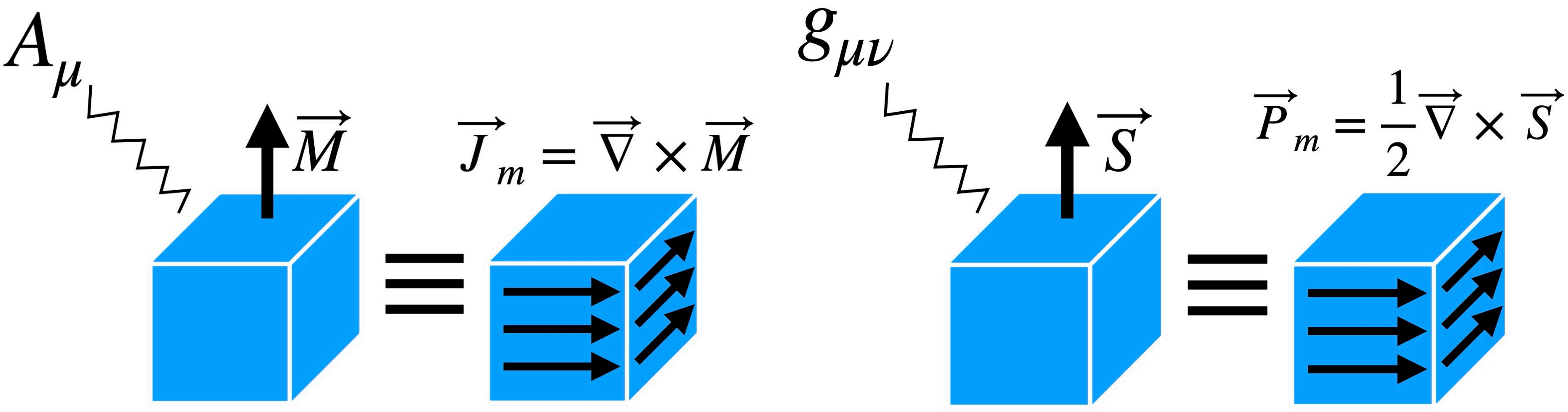
$\vec{\mathbf{j}} = \vec{\nabla} \times \vec{\mathbf{M}}, \quad \vec{\mathbf{M}} = \frac{a}{2}\vec{\omega}$

: Barnett's Effect

A special choice $\Sigma^{\mu\nu\alpha} = S^{\mu\nu\alpha}$ makes $\tilde{T}^{\mu\nu}$ symmetric Belinfante form and $\tilde{S}^{\mu\nu\alpha} = 0$, i.e. no spin tensor in theory

We get to the conventional relativistic hydrodynamics with symmetric EM tensor with no spin degrees of freedom

# Physics of Pseudo-Gauge Transformations




**Magnetization Current** ``Spinetization" Momentum

# Non-Dissipative Second Order Transports

As a result of pseudo-gauge transformation to get to symmetric Belinfante EM tensor, we obtain several Non-Dissipative Second Order Transport coefficients related to Fluid Vorticity : $\partial_\mu \tilde{s}^\mu = 0$

$$\tau^{\mu\nu} = \frac{\chi}{2}(\sigma^\mu{}_\alpha + \omega^\mu{}_\alpha)\omega^{\alpha\nu} + (\mu \leftrightarrow \nu) + 2a_0 \Delta^{\mu\nu}\omega_\nu\omega^\nu$$

$$\tau^\mu = -\frac{Tn}{2w}\Delta^\mu_\lambda \partial_\nu(\beta\chi\omega^{\lambda\nu}) - \frac{1}{2}\Delta^\mu_\lambda \partial_\nu(a\omega^{\lambda\nu})$$

$$\Delta s^\mu = -\frac{Ts}{2w}\Delta^\mu_\lambda \partial_\nu(\beta\chi\omega^{\lambda\nu}) - \frac{1}{2}\Delta^\mu_\lambda \partial_\nu(b\omega^{\lambda\nu}) + \frac{n\chi}{2w}\omega^{\mu\nu}\partial_\nu\alpha$$

It is shown that the thermodynamics can also be made conventional

$$\tilde{S}(\tilde{\varepsilon}, \tilde{n}, \omega^\mu) = s(\tilde{\varepsilon} + \Delta\varepsilon, \tilde{n} + \Delta n, \omega_\mu) - \Delta s$$

$$d\tilde{s} = \tilde{\beta} d\tilde{\varepsilon} - \tilde{\alpha} d\tilde{n}\,, \quad \tilde{s} = \tilde{\beta}(\tilde{\varepsilon} + \tilde{p}) - \tilde{\alpha}\tilde{n}$$

# Why do we have these equivalent descriptions?

**In fact, there are infinitely many equivalent descriptions by performing partial pseudo gauge transformations with $\Sigma^{\mu\nu\alpha} = tS^{\mu\nu\alpha}, \quad 0 < t < 1$. The $t = 1$ is the special point where the energy-momentum tensor becomes the symmetric Belinfante form**

$$\partial_\mu F^{\mu\nu} = J^\nu_{\text{total}} = J^\nu_{\text{hydro}} - \partial_\alpha \left( \frac{a}{2} \omega^{\nu\alpha} \right)$$

Hydrodynamics, a priori, does not distinguish between $J^\mu_{\text{total}}$ and $j^\mu_{\text{hydro}}$

$$G^{\mu\nu} = 16\pi G_N T^{\mu\nu}_B = 16\pi G_N (T^{\mu\nu}_{\text{hydro}} + \frac{1}{2}\partial_\alpha (S^{\alpha\mu\nu} - S^{\mu\alpha\nu} - S^{\nu\alpha\mu}))$$

# Equivalent Hydrodynamics

Given a microscopic theory with several constituents carrying spins, the macroscopic hydrodynamics may choose the canonical EM tensor for some degrees of freedom and the Belinfante EM tensor for other degrees of freedom.

All these discrete choices are equivalent hydrodynamic descriptions of the same microscopic theory

# Universality of Hydrodynamics comes with Generosity

**The origin of pseudo gauge transformation is the microscopic equivalence between spin and “spinetization" momentum, that hydrodynamics can not resolve macroscopically. To accommodate such microscopic equivalences for any system, that hydrodynamics can not probe in macroscopic scales, what hydrodynamics can do is to allow much more generous equivalences with continuous parameters of pseudo gauge transformation, to be able to accommodate any system**

# Thank you !