Elasticities

Objectives:
Students will be able to

- Calculate the elasticity of demand.
- Calculate the value at which total revenue is maximized.
- Determine if demand is elastic, inelastic or neither for specified values of price.

We all know that the demand for a product will depend on the price of the product. We have functions will tell us the demand for a product given its price. What we need to know is how a change in price will affect the demand. One way to measure this sensitivity of demand can be determined by relative change. This change is $\dfrac{\left(\dfrac{\Delta q}{q}\right)}{\left(\dfrac{\Delta p}{p}\right)}$. Here the demand function *q* is a function of price *p*. Thus $q = f(p)$. Using limits (related to change in price) and Newton's Quotient, we get the following version of this function $\lim_{\Delta p \to 0} \dfrac{p}{q} \cdot \dfrac{\Delta q}{\Delta p} = \dfrac{p}{q} \cdot \dfrac{dq}{dp}$. It turns out that the negative of this function is called the elasticity of demand or *E*.

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

If the relative change in demand is less than the relative change in price ($E < 1$), then demand is called *inelastic*. If the relative change in demand is greater than the relative change in price ($E > 1$), then demand is said to be *elastic*. If the relative change in demand and the relative change in price are approximately equal ($E = 1$), then demand is said to have *unit elasticity*.

How does the elasticity of demand relate to revenue?

1. If the demand is inelastic, total revenue increases as price increases.
2. If the demand is elastic, total revenue decreases as price increases.
3. Total revenue is maximized at the price where demand has unit elasticity.

Example 1:
For the demand function $q = 25000 - 50p$, find *E*. For which value(s) of q, if any, is the total revenue maximized?

Solution:
Our first step is to find the elasticity of demand function *E* ($E = -\frac{p}{q} \cdot \frac{dq}{dp}$). To do this we need to replace $\frac{dq}{dp}$ with the derivative of the demand function *q* from above. We will also need to replace *q* with the demand function. The only thing in *E* that does not get substituted for to make the elasticity of demand function is *p*. The derivative is of the demand function is $\frac{dq}{dp} = -50$. Now we are ready to plug everything in

$$E = -\frac{p}{25000 - 50p} \cdot (-50)$$

$$E = -\frac{-50p}{25000 - 50p}$$

$$E = \frac{p}{500 - p}$$

We have now finished finding *E*.

The next part of the question asks us to find where total revenue is maximized. Total revenue will be maximized at a price *p* where the elasticity of demand function is equal to 1. Thus we need to set *E* equal to 1 and solve for *p*.

$$1 = \frac{p}{500 - p}$$

$$500 - p = p$$

$$500 = 2p$$

$$250 = p$$

This means that total revenue will be maximized at a price of 250. This finishes answering the question.

Example 2:
Find the elasticity of demand (*E*) for the demand function $q = 400 - 0.2p^2$ at the value of *p* = \$40. Is the demand elastic, inelastic, or neither at *p* = \$40?

Solution:
Again for this problem our first step is to find the elasticity of demand function. We will again need the derivative of the demand function that is given.

$$\frac{dq}{dp} = -0.4p$$

Now we will plug into the elasticity of demand function.

$$E = -\frac{p}{400 - 0.2p^2} \cdot (-0.4p)$$
$$E = -\frac{-0.4p^2}{400 - 0.2p^2}$$
$$E = \frac{0.4p^2}{400 - 0.2p^2}$$

We could simplify the function further, but it is not necessary for our purposes.

The next step is to plug in 40 for p to find out the value of the elasticity of demand function at the price $40.

$$E = \frac{0.4(40)^2}{400 - 0.2(40)^2}$$
$$E = \frac{640}{80}$$
$$E = 8$$

We are now asked if demand is elastic, inelastic or neither at a price of $40. If we look back at the definitions of elastic and inelastic, we see that since $E > 1$, then demand is elastic at the price of $40.