# Neighborhood Change and the Valuation of Urban Amenities: Incorporating Dynamic Behavior into the Hedonic Model*

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**Abstract**

Neighborhood change is often described as the coevolution of housing prices and specific neighborhood amenities over time. Current amenity levels may provide both current consumption value and information to inform households' expectation about future consumption and housing wealth values. By developing and estimating a dynamic model of neighborhood choice, we decompose these impacts and find heterogeneous effects across a set of time-varying amenities that are often associated with gentrification. We find that households have a positive willingness to pay for higher-income neighbors, higher-education neighbors, and a higher rate of neighborhood owner occupation, but that only current neighborhood income levels signal future neighborhood improvements, all else equal. These results offer policymakers important insights on the long-term consequences of current changes to neighborhood characteristics.

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# 1 Introduction

Neighborhood change is often described as the coevolution of specific neighborhood amenities and housing prices over time. It remains an important yet controversial topic from a public-policy standpoint.[1] Prior sorting literature has documented the fact that improvements to neighborhood amenities will invariably be reflected in increased housing prices and rents. We seek to answer whether these improvements additionally signal continued neighborhood change and, if so, which amenities are the most relevant to predict future neighborhood change.

Neighborhood change, by definition, is driven by individual households' choices of where to live. These choices are an inherently dynamic decisions as neighborhood characteristics (amenities and demographics) serve dual roles by directly affecting current utility and by determining households' expectations regarding future neighborhood characteristics. In the context of neighborhood change, the tradeoff is particularity salient. Optimal policy requires understanding *why* households sort in addition to *where* households sort, thus distinguishing between current amenity value and predictive channels. Using a novel dataset that contains information on housing, individual, and neighborhood attributes in Los Angeles metropolitan area, this paper estimates the willingness to pay for neighborhood attributes including neighborhood income, education rates, and home ownership rates. We find that all three are important determinants of where households choose to live. However, the channels through which they affect sorting differ substantially.

Los Angeles presents an ideal empirical setting. It is the second largest metropolitan area in the United States with a population of almost 13 million (US Census). Additionally, it is the largest city in California, which is considered the most diverse state in the union;

[1] As noted by Ellen and Ding (2016) :"The term gentrification inevitably generates controversy and disagreement. People disagree about its definition, its causes, and, above all, its consequences. All seem to agree, however, that whatever gentrification is, it is becoming more prevalent in U.S. cities."

according to World Population Review, California ranks first for linguistic diversity, second for racial and ethnic diversity, and third for socioeconomic diversity.[2] Finally, the current policy environment increases the likelihood of neighborhood change in Los Angeles; in September of 2021, Governor Gavin Newsom signed into law California Senate Bill 9 (SB-9) and Senate Bill 10 (SB-10), which expand housing production and increase population density in the state. Prior literature (Sims (2011), Autor, Palmer, and Parthak (2014 and 2019), and Diamond, McQuade, and Qian (2018)) have shown that supply-side housing policies impact both the prices and the characteristics of the affected neighborhoods.

The starting point for our analysis is a particularly rich set of data describing both housing transactions and time-varying household and neighborhood attributes. Our data is constructed from a number of sources. The first source is data on all housing transactions in the Los Angeles metropolitan area from 1990-2008, which provides rich information on housing characteristics and sales prices. It also provides a unique house identifier and buyer and seller names, which can be used to track households through time. The second source is demographic information on buyers, which is obtained from mortgage applications. The third source is geocoded amenity data, which come primarily from GeoLytics and the Neighborhood Change Data Base. By combining these three sources of data, we can observe the decisions that households make about where to live and how often to move as well, as the household and neighborhood characteristics that potentially determine these decisions.

We develop and estimate a dynamic model of location choice in the spirit of Rosen (1974).[3] Our model expands the existing hedonic framework by allowing households to face moving costs and to be forward-looking with respect to continuous amenities of interest, these

[2]https://worldpopulationreview.com/state-rankings/most-diverse-states

[3]Rosen's property value hedonic model, which has been used extensively in the environmental and urban literature, has long been considered the "workhorse" model of amenity valuation. See, for example, applications focusing on school quality (Black (1999), Downes and Zabel (2002), Gibbons and Machin (2003)), climate (Albouy, Graf, Kellogg, and Wolff (2016)), safety (Gayer, Hamilton, and Viscusi (2000), Davis (2004), Greenstone and Gallagher (2008)), and environmental quality (Palmquist (1982), Chay and Greenstone (2005), Bento, Freedman, and Lang (2015).

amenities' implicit prices, and overall housing prices. Specifically, we allow for the fact that a change in the consumption level of an amenity affects not only current utility (through both increased price and increased amenity consumption), but also the stream of future utilities. Thus, we re-define the familiar optimality condition to require that the marginal increase in current price is equal to the marginal benefit of an increase in current amenity consumption *plus* the associated change in future utility flow.

Our dynamic estimator, which combines insights from the dynamic discrete-choice literature described (Hotz and Miller (1993) and Arcidiacono and Miller (2011)) with those in the hedonic literature (Ekeland, Heckman, and Nesheim (2004) and Bishop and Timmins (2019)), allows for both moving costs and dynamic, forward-looking behavior of the households and adds relatively little to the computational burden; we require only the additional first-stage estimation of the change in future utility flows associated with an amenity change today. We show that once this additional first stage is estimated, the parameters of the utility function may be recovered using existing methods associated with Rosen's framework. Additionally, as our estimator has a low computational burden, we are able to control for a rich set of amenities describing neighborhoods, demographic characteristics describing buyers, as well as control for both unobserved neighborhood attributes and unobserved household preference heterogeneity, within a dynamic framework.

We define neighborhood at a fine level of geography (U.S. Census Block Group) and focus our analysis on a set of amenities that are likely to have predictive power in terms of neighborhood change: median neighborhood income, mean educational attainment, and homeownership rates. We find that all three are important determinants of where households choose to live and that the mean household is willing to pay, respectively, \$384, \$289, and \$164 per year to increase these neighborhood amenities by one half of a standard deviation.

These figures give us the willingness to pay for one year of the flow utility received from the amenity in addition to the willingness to pay because of future amenity or price impacts. Using our dynamic model to decompose these two effects, we find large heterogeneity across amenities. For example, the neighborhood homeownership rate delivers little direct utility to residents in a given period. However, the homeownership rate serves as an important predictor of neighborhood change; high rates of homeownership today signal increased future utility flows in the future. In contrast, median neighborhood income delivers large flow-utilities to residents in the current period, but does not serve as a predictor of increased future utilities. Finally, neighborhood educational attainment provides large flow-utilities to residents in the current period (like income) and serves as a predictor of neighborhood change (like homeownership). However, current educational attainment predicts a negative future impact, i.e., high levels of educational attainment, conditional on other covariates, signal decreased flow utilities in future.

This paper proceeds as follows: In Section 2, we present our data. In Section 3 and Section 4, we introduce our dynamic model and we develop our estimation strategy. In Section 5, we present the results from our estimation of where and why households sort in Los Angeles. Finally, Section 6 concludes.

# 2 Data

For this analysis, we employ a rich, two-sided panel dataset describing both repeat sales of houses and repeat purchase decisions of buyers. The data cover five counties in the Los Angeles metropolitan area (Los Angeles, Orange, Riverside, San Bernadino, and Ventura) over the period 1990 to 2008. We bring together three sources of data: data on all housing transactions in the Los Angeles metropolitan area, data on the demographic information of buyers, which are obtained from mortgage applications, and data on geocoded amenities, which come primarily from US Census data. By combining these three sources of data, we are able to observe the decisions that households make about where to live and how often to move, as well as the household and neighborhood characteristics that potentially determine these decisions.

The real estate transactions data were purchased from Dataquick/CoreLogic and include dates, prices, loan amounts, and buyers', sellers', and lenders' names for all transactions over the period of our data. In addition, the data for the final observed transaction for each house include characteristics such as exact street address, square footage, year built, lot size, number of rooms, number of bathrooms, and number of units in the building. Unique property identifiers combined with the fact that the data are a complete census of all transactions, allow us to perfectly follow properties through time knowing whether or not they sold in any given year.

The process of cleaning the data involves a number of cuts. Many of these are made in order to deal with the fact that we only see housing characteristics at the time of the most recent sale, but we need to use housing characteristics from all sales as controls in our hedonic price regressions. We therefore seek to eliminate any observations that reflect major housing improvement or degradation. First, to control for land sales or re-builds, we drop all transactions where "year built" is missing or with a transaction date that is prior to "year built". Second, in order to control for property improvements (*e.g.*, an updated kitchen) or degradations (e.g., water damage) that do not present as re-builds, we drop any house that ever appreciates or depreciates in excess of 50 percentage points of the county-year mean price change. We also drop any house that moves more than 40 percentile points between consecutive sales in the county-year distribution. Additionally, we drop transactions where the price is missing, negative, or zero. After using the consumer price index to convert all transaction prices into 2000 dollars, we drop one percent of observations from each tail to minimize the effect of outliers. Finally, as we merge-in data describing amenity data using each property's geographic coordinates, we drop properties where latitude and longitude are missing.

While these transactions data provide a rich set of housing characteristics, we merge in two additional property-level characteristics: spatially-interpolated violent crime rates, measured at city centroids, from the RAND California database and spatially-interpolated pollution levels ($PM_{10}$) from the California Air Resources Board. Summary statistics for the housing panel are given in Table 1. The sample is comprised of 1,242,464 transactions.

Table 1: Property Transactions Summary Statistics

| Variable | Mean | Median | Std. Dev. | Min. | Max. | Observations |
|---|---|---|---|---|---|---|
| Sales Price (in 2000 dollars) | 263,829 | 222,791 | 156,413 | 31,110 | 1,234,245 | 1,242,464 |
| Year | 1999.29 | 1999 | 4.75 | 1990 | 2008 | 1,242,464 |
| House Age | 30.55 | 29 | 19.97 | 0 | 155 | 1,242,464 |
| Lot Square Footage | 8,239 | 6,600 | 10,734 | 0 | 130,680 | 1,242,464 |
| House Square Footage | 1,632 | 1,502 | 622 | 400 | 9,991 | 1,242,464 |
| Number of Bathrooms | 2.16 | 2 | 0.73 | 1 | 10 | 1,242,464 |
| Violent Crime Rate (per 100,000) | 379.63 | 322.65 | 262.88 | 12.82 | 3,834.10 | 1,242,464 |
| PM10 (avg. annual concentration) | 70.97 | 66.41 | 21.57 | 35.88 | 150.97 | 1,242,464 |

We merge these transactions data with two additional sources of data: demographic data describing the household buying the property and amenity data describing the characteristics of each neighborhood. We describe these in turn.

For the estimation of our model, we require the ability to see households through time. Thus, we create the panel of buyers following the algorithm developed in Bayer, McMillan, Murphy, and Timmins (2016). We restrict this sample to those households that are observed to purchase three or fewer times during the sample period to limit the impact of professions. Additionally, we use buyers' names from the transactions dataset along with the common variables of purchase date, Census tract,[4] loan value, and lender name, to merge-in data describing

[4]Census tracts are small, relatively homogeneous geographic units defined by the Census Bureau. They

household race and income from the Home Mortgage Disclosure Act dataset (available for all households taking out a mortgage), again following procedures described in Bayer, McMillan, Murphy, and Timmins. We successfully match approximately 75% of households in the transactions sample to the HMDA sample. We restrict this sample to households with annual incomes between $25,000 and $500,000 (in 2000 dollars) and to household with non-missing race. Note that these sample cuts account for less than two percent of the matched sample. Summary statistics for the household panel are given in Table 2. The sample is comprised of 855,845 households.

Table 2: Household Summary Statistics

| Variable | Mean | Median | Std. Dev. | Min. | Max. | Observations |
|---|---|---|---|---|---|---|
| Income | 86,384 | 72,452 | 53,759 | 25,000 | 500,000 | 855,845 |
| Hispanic | 0.13 | 0 | 0.34 | 0 | 1 | 855,845 |
| Black | 0.05 | 0 | 0.22 | 0 | 1 | 855,845 |
| Asian | 0.26 | 0 | 0.44 | 0 | 1 | 855,845 |
| White | 0.55 | 1 | 0.5 | 0 | 1 | 855,845 |

Finally, we merge-in data describing amenities at the neighborhood level, which we define as a block group. Block groups are the second smallest unit of geography within the US Census and the smallest unit for which the Census tabulates and publishes data on our amenities of interest. This is an ideal level of geography as it is fine enough to capture the notion of local neighborhood characteristics and provides variation within Census Tracts. This is important as it will allow us to isolate within-Tract variation in prices and amenities to identify the casual effect of amenities on prices; as Census Tracts are designed by the Census Bureau to be small, relatively homogeneous geographic units, the within-Tract variation in amenities (conditional on a rich set of housing characteristics) is plausibly exogenous.

Our amenities of interest are block group measures of the median household income,

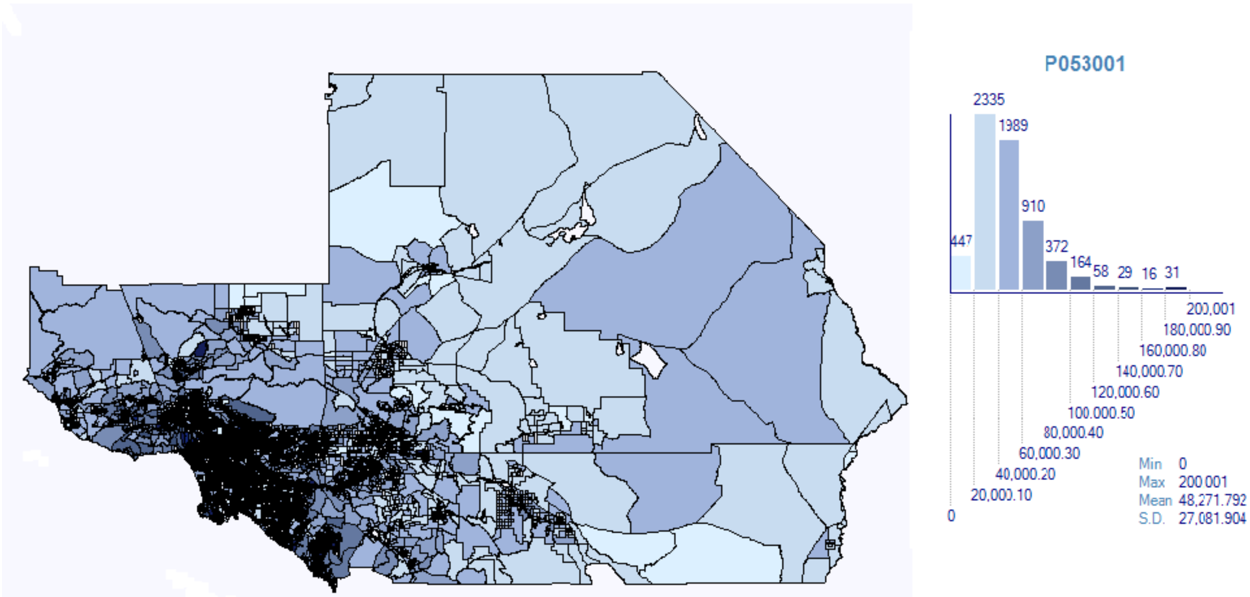
contain between 2,500 and 8,000 individuals on average and vary in geographic size according to population density. [U.S. Census Bureau] the percentage of residents holding a college degree or higher, and the rate of homeownership. The underlying data for these measures come from the publicly available summary files for the 1990 Census, the 2000 Census, and the 5-year estimates from the 2009 American Community Survey. We purchased the block-group-level measures from GeoLytics who take the underlying data and harmonize the geographic identifiers to consistently defined, year-2000 block groups. Summary statistics for these variables of interest are given in Table 3.

Table 3: Neighborhood Amenity Summary Statistics

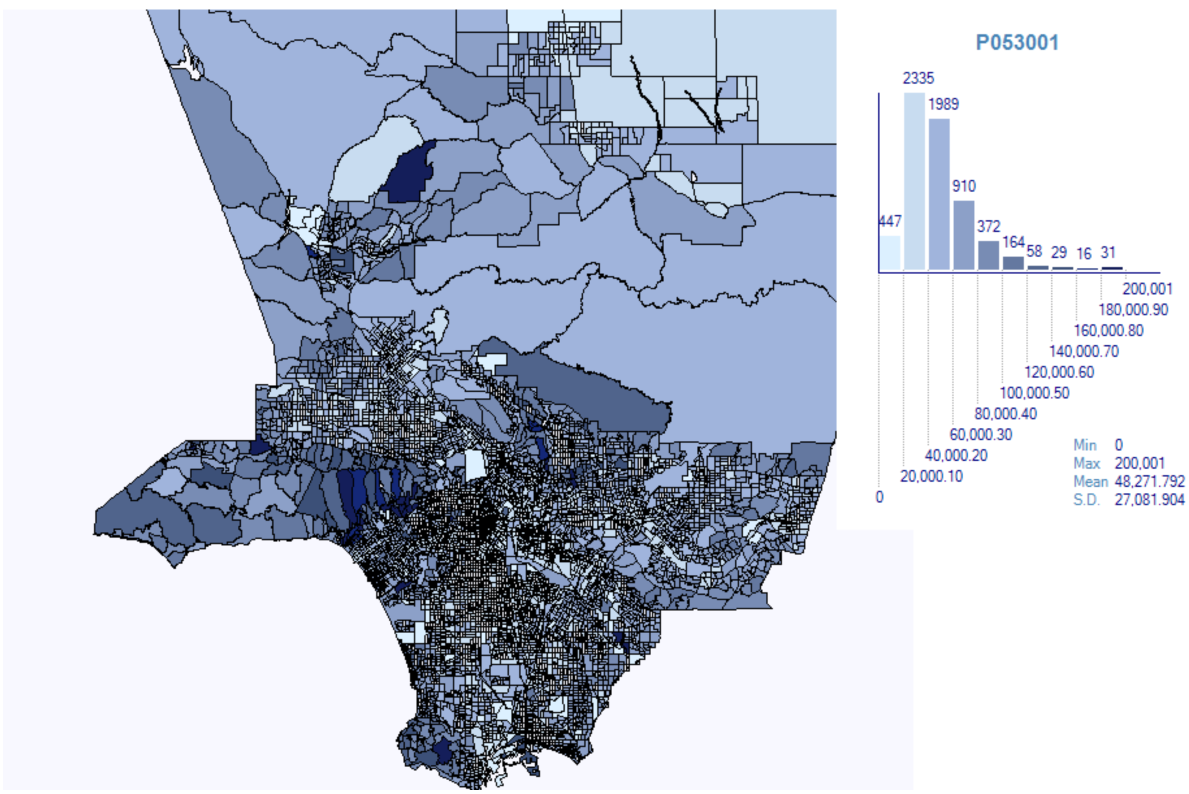
| Variable | Mean | Median | Std. Dev. | Min. | Max. | Observations |
|---|---|---|---|---|---|---|
| Median Household Income | 58,830 | 56,330 | 18,920 | 24,470 | 118,810 | 855,845 |
| Percent with College Degree | 27.33 | 24.97 | 15.42 | 3.13 | 66.20 | 855,845 |
| Rate of Homeownership | 72.14 | 78.24 | 19.56 | 17.52 | 97.46 | 855,845 |

There is considerable variation in our three amenities of interest. For example, Figure 1 illustrates spatial variation in Median Household Income across neighborhoods. Panel 1a illustrates this variation for all five counties of the L.A. metropolitan area, while Panel 1b zooms in on the more densely populated Los Angeles County. Figure 2 illustrates neighborhood-specific, temporal variation in Median Household Income. Panel (a) illustrates the distribution of changes over the nineteen-year period of 1990 to 2009, while Panel (b) shows the analogous distribution for the nine-year period of 2000 to 2009. The first notable feature is the considerable heterogeneity in block-group specific time trends. The second is that there is limited persistence in the trends; the standard deviation of the nineteen-year difference is only slightly larger than the standard deviation of the nine-year difference. Perfect persistence would be consistent with an almost doubling of this standard deviation.

Figure 1: Spatial Variation in Median Household Income (2000 dollars)

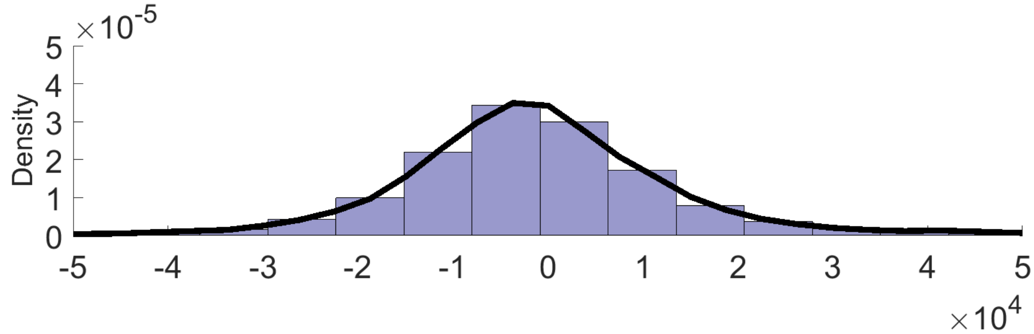



(a) Los Angeles Metropolitan Area (Five Counties)

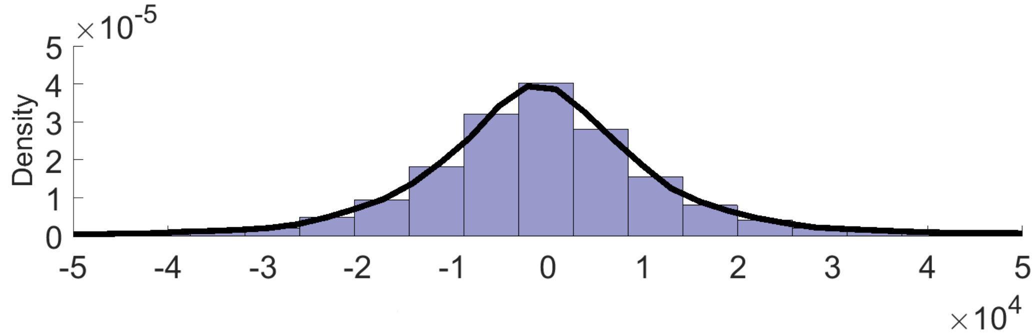



(b) Los Angeles County

Figure 2: Temporal Variation in Median Household Income (2000 dollars)




(a) Change in Neighborhood Median Household Income between 1990 and 2009




(b) Change in Neighborhood Median Household Income between 1990 and 2019

# 3 A Dynamic Model of Hedonic Demand

In the traditional model of hedonic demand associated with Rosen (1974), households make a one-time consumption-of-amenities decision to maximize current utility. For our application, we seek to decompose the current flow of utility associated with the consumption of amenities from amenities' role in the formation of expectations regarding neighborhood change. Thus, we develop a dynamic model of hedonic demand that specifies households as making a sequence of consumption-of-amenities decisions that each maximize the expected discounted stream of future utilities.

Our dynamic framework flexibly specifies households (denoted $i \in \{1, \ldots, N\}$) having heterogenous preferences over a vector of housing-related amenities, $x_{i,t}$ (which may be fixed, such as lot size, or time-varying, such as neighborhood median income). We allow each household's preferences and moving costs to differ based on a vector of observable attributes, $z_{i,t}$ (which may be fixed, such as race, or time-varying, such as income). We also allow preferences to differ across households by a vector of unobserved household- and amenity-specific preference shocks, $\eta_{i,t}$, and allow moving costs to vary by an unobserved shock to moving costs, $\epsilon_{i,t}$.[5]

Household $i$ begins each period $t$ with an endowment vector of $x$, which is determined by the household's current residence and is denoted $x^e_{i,t}$. In each period, the household then chooses how much of each amenity in the vector $x_{i,t}$ to consume. This choice is continuous over the support of $x$. If the household chooses to consume an amenity level which differs from that offered by its current residence, $x^e_{i,t}$, then the household must reoptimize by moving to a house that offers its chosen level of amenities and incurring a moving cost. When this moving cost is sufficiently high (relative to any potential gains from moving), the household will choose to stay in its current residence and consume the endowed quantity of amenities. In the standard Rosen framework, there is no endowment of amenities and no cost associated with reoptimization.

Thus, we specify the household as facing a two-part, discrete-continuous choice in each period; the household first makes the discrete decision of whether or not to reoptimize (i.e., move) and, conditional on reoptimizing, then makes the decision of how much of each amenity to consume.[6] This framework facilitates the estimation strategy described in the next section

[5]The treatment and timing of $\eta_{i,t}$ with respect to the estimation of this model is discussed in detail in Section 3.3.

[6]See Dubin and McFadden (1984) and Hanemann (1984) for seminal papers that estimate discrete-continuous demand models.

of the paper, where we adapt the insights of the dynamic, discrete-choice literature.

The first, discrete choice of whether or not to move (and incur the associated moving cost) is denoted $d^m_{i,t} = m$, where $m \in \{0, 1\}$. The second, continuous choice of how much $x$ to consume, is denoted $d^x_{i,t} = x$. This second choice is made only if $d^m_{i,t} = 1$; if the household chooses $d^m_{i,t} = 0$, it makes no further decision and consumes the endowment level of amenities (i.e., $d^x_{i,t} = x^e_{i,t}$).

We write the household's problem as choosing both $d^m_{i,t}$ and $d^x_{i,t}$ to maximize the expected discounted sum of per-period flow utilities, which is given by:

$$E\Big[\sum_{\tau=t}^{T} \beta^{\tau-t}\big(u^f_m(s_{i,\tau}, x_{i,\tau}, \epsilon_{i,\tau})\big)\Big] \tag{1}$$

where:

$$\begin{aligned} u^f_0(s_{i,t}) &= u(x^e_{i,t}, z_{i,t}, \eta_{i,t}; \alpha) - r(x^e_{i,t}; \gamma_t) \\ u^f_1(s_{i,t}, x_{i,t}, \epsilon_{i,t}) &= u(x_{i,t}, z_{i,t}, \eta_{i,t}; \alpha) - r(x_{i,t}; \gamma_t) - MC(z_{i,t}, x^e_{i,t}; \delta) - \epsilon_{i,t} \end{aligned} \tag{2}$$

The state vector, $s_{i,t}$, is comprised of all state variables (except $\epsilon_{i,t}$) that affect the household's decisions, $d^m_{i,t}$ and $d^x_{i,t}$. Thus, $s_t = [x^e_{i,t}, z_{i,t}, \eta_{i,t}, \gamma_t, \Omega_{i,t}]$ where $\Omega_{i,t}$ is household $i$'s information set at time $t$. We assume that the shock to moving costs, $\epsilon_{i,t}$, has no predictive power for $s_{i,t+1}$, conditional on $s_{i,t}$ and $d_{i,t}$ and that it is distributed $i.i.d.$ over both households and time according to the Logistic distribution with scale parameter $\sigma_\epsilon$.[7]

The choice-specific flow utilities are defined for each possible realization of the discrete choice, $d^m_{i,t} = m$ where $m \in \{0, 1\}$, and are comprised of three components. The first component of utility, $u(x_{i,t}, z_{i,t}, \eta_{i,t}; \alpha)$, is known up to the parameter vector $\alpha$ and captures the direct

[7] The first assumption is analogous to the familiar Conditional Independence assumption made in Rust (1987) and allows us to write the transition densities as $q = q_s(s_{i,t+1}|s_{i,t}, d_{i,t})q_\epsilon(\epsilon_{i,t+1})$. Equation (2) embeds the Additive Separability (in $\epsilon$) assumption of Rust (1987).

effect of amenities on utility. It is a function of consumed amenities in period $t$, $x_{i,t}$, which is given by the household's endowed amenities, $x^e_{i,t}$, in the case that the household chooses to not move. The second component is the implicit amenity rental price, $r(x_{i,t};\gamma_t)$, which is known up to the parameter vector, $\gamma_t$. This rental-equivalence price component, (entering with a negative sign) shows that a higher consumption of $x_{i,t}$ reduces consumption of other goods and services. This price, often called rental equivalent or user cost, captures the true, annual economic cost of owning a house. To build from a more primitive foundation, we treat the arguments of the rental price function, $(x_{i,t},\gamma_t)$, as states, rather than then rental-price outcomes, $r_{it}$, themselves. The third component is the moving cost, $MC(z_{i,t},x^e_{i,t};\delta)+\epsilon_{i,t}$, which is paid only if the household moves and is known up to the parameter vector, $\delta$. This moving cost component reflects the fact that the household may choose to consume any quantity of $x_{i,t}$, but will need to pay the moving cost if it chooses a level of $x_{i,t}$ is other than $x^e_{i,t}$. While the flow utility is specified here as quasilinear for simplicity, this is not necessary for either identification or for the estimation approach that we describe in the following section.[8]

If the household chooses to move in the current period $t$, it knows that it will optimally make decisions $d^m_{i,t}$ and $d^x_{i,t}$ in all future periods. This allows us to define the direct choice-specific value function associated with moving, $v_1(s_{i,t},x_{i,t})$. This function specifies the lifetime utility (excluding $\epsilon_{i,t}$) that a household will receive from choosing to move in the current period (given any subsequent choice of $x_{i,t}$). Therefore, if the household chooses to move, $x_{i,t}$ is chosen to maximize $v_1(s_{i,t},x_{i,t})+\epsilon_{i,t}$:

$$x^*_{i,t}(s_{i,t}) = argmax_x \; v_1(s_{i,t},x_{i,t})+\epsilon_{i,t}$$

The household also knows that it will behave optimally in the future if it chooses to

[8]Bishop and Timmins (2019) estimates a non-quasilinear utility function. See also Ekeland, Heckman, and Nesheim (2004) and Murray (1983) for discussions of the functional form of the utility function in hedonic models and the value of allowing for non-quasilinear preferences.

not move in the current period. However, when choosing to not move, there is no subsequent choice of $x_{i,t}$ and the household consumes its endowment level of amenities, $x^e_{i,t}$.

Given the respective amenity-choice outcomes associated with each of the discrete-choice alternatives, we may now define the indirect choice-specific value functions for not moving and moving, $v_0(s_{i,t})$ and $v_1(s_{i,t})$, respectively. The function $v_0(s_{i,t})$ recognizes that the endowment vector, $x^e_{i,t}$, is an element of $s_{i,t}$. The function $v_1(s_{i,t})$ is defined assuming that $x_{i,t}$ is chosen optimally, i.e., $v_1(s_{i,t}) = v_1(s_{i,t}, x^*_{i,t}(s_{i,t}))$, and is no longer a function of $x_{i,t}$ itself (but rather a function of the variables that determine the optimal $x^*_{i,t}(s_{i,t})$ only). These indirect choice-specific value functions are given by:

$$\begin{aligned} v_0(s_{i,t}) &= u^f_0(s_{i,t}) + \beta E[max\{v_0(s_{i,t+1}), v_1(s_{i,t+1}) - \epsilon_{i,t+1}\}|s_{i,t}, d^m_{i,t} = 0, d^x_{i,t} = x^e_{i,t}] \quad (3) \\ v_1(s_{i,t}) &= \bar{u}^f_1(s_{i,t}) + \beta E[max\{v_0(s_{i,t+1}), v_1(s_{i,t+1}) - \epsilon_{i,t+1}\}|s_{i,t}, d^m_{i,t} = 1, d^x_{i,t} = x^*_{i,t}(s_{i,t})] \end{aligned}$$

where, following convention, the value functions are defined excluding the current period's shock to moving costs. For notational convenience, we let $\bar{u}^f_1(s_{i,t})$ denote the flow utility associated with moving absent the shock to moving costs, $\epsilon_{i,t}$. We do not need to denote an equivalent term for the flow utility associated with not moving, as moving costs (including the idiosyncratic shock) are only paid if the household, in fact, moves (i.e., in this case, an analogously defined $\bar{u}^f_0(s_{i,t})$ would exactly equal $u^f_0(s_{i,t})$). Note that the function $\bar{u}^f_1(s_{i,t})$ is an indirect flow utility as it is defined assuming that $x_{i,t}$ is chosen optimally, *i.e.*, $\bar{u}^f_1(s_{i,t}) = \bar{u}^f_1(s_{i,t}, x^*_{i,t}(s_{i,t}))$.

The household will choose to move if $v_1(s_{i,t}) - \epsilon_{i,t} > v_0(s_{i,t})$. If the household chooses to move (i.e., $d^m_{i,t} = 1$), $x_{i,t}$ is chosen to maximize the associated direct value function, $v_1(s_{i,t}, x_{i,t})$, which we may now write as:

$$v_1(s_{i,t}, x_{i,t}) = \bar{u}^f_1(s_{i,t}, x_{i,t}) + \beta E[max\{v_0(s_{i,t+1}), v_1(s_{i,t+1}) - \epsilon_{i,t+1}\}|s_{i,t}, d^m_{i,t} = 1, d^x_{i,t} = x_{i,t}] \quad (4)$$

# 4 Estimation

Estimation of the traditional, hedonic model begins with the separate first-stage estimation of the implicit price of the amenity of interest, i.e., the estimation of the gradient of the housing price function relating amenity levels to housing prices/rents. In the second stage, the parameters of the utility function are recovered (treating as known the first-stage estimates of the hedonic price gradient).[9] Our estimation framework retains this simple intuition of the Rosen framework while adapting it for a dynamic context. In addition, our estimation strategy adds only an additional estimation stage that recovers the impact of this period's amenity choice on future utility streams.

This is in contrast to the substantial computational burden that often accompanies the estimation of dynamic models. The standard, well-known computational difficulty lies in the fact that $v_0(s_{i,t})$ and $v_1(s_{i,t})$ are defined recursively. In our case, we have an additional recursive structure to contend with, as the choice-specific value function associated with moving is also a function of the optimal amenity vector, $x^*_{i,t}(s_{i,t})$, while at the same time $x^*_{i,t}(s_{i,t})$ is itself the solution to a problem involving this value function. This additional complication (a doubly-recursive structure) means that our model would be computationally prohibitive to estimate using a full-solution method, for example the approach described by Rust (1987).

To estimate the model, we combine insights from the dynamic discrete-choice literature

[9]Because of the well-documented difficulties associated with estimating Rosen's second stage (Brown and Rosen (1982), Mendelsohn (1985), Bartik (1987), and Epple (1987)), much of the previous literature has forewent estimation past the first-stage and focused only on estimating the local effects of a policy change. However, more recent papers, such as Ekeland, Heckman, and Nesheim (2004), Bajari and Benkard (2005), Heckman, Matzkin, and Nesheim (2010), and Bishop and Timmins (2019), show how to estimate the willingness-to-pay function (*i.e.*, allow willingness-to-pay to vary with the amenity of interest) while avoiding the identification and endogeneity issues laid out in earlier papers. Nesheim (2015) and Chernozhukov, Galichon, Henry, and Pass (2017) discuss the identification of multi-amenity hedonic models. Kuminoff and Pope (2014) discusses the conditions of the hedonic model under which changes in the equilibrium price function identify willingness-to-pay parameters.

(Hotz and Miller (1993) and Arcidiacono and Miller (2011)) with insights from the hedonic literature (Ekeland, Heckman, and Nesheim (2004) and Bishop and Timmins (2019)), and yet retain the well-known intuition associated with the classic Rosen framework. Employing a two-step estimation approach, we show that the estimation of our dynamic model is reduced to a familiar, and computationally feasible, environment. In a first stage, we recover estimates of the hedonic price function parameters (i.e., we estimate the first stage described by Rosen) as well as estimates of the future value associated with this period's choice. In a second stage, we treat the first-stage estimates as "data" and recover the remaining structural parameters using existing hedonic methods.[10] As our estimation approach has a relatively low computational burden, we are able to additionally control for both unobserved house/neighborhood attributes, as well as very rich unobserved individual preference heterogeneity.[11]

In the Appendix, we present a simple, parameterized version of the model and illustrate its small-sample properties using Monte Carlo experiments.

## 4.1 Two-Step Estimation

We now present the details of our two-step estimation approach to the discrete-continuous framework.

Given the Logit assumption, we may rewrite the indirect choice-specific value functions in the familiar form:

$$
\begin{aligned}
v_0(s_t) &= u_0^f(s_{i,t}) + \beta\sigma_\epsilon E[log\Big(e^{v_0(s_{t+1})/\sigma_\epsilon} + e^{v_1(s_{t+1})/\sigma_\epsilon}\Big)|s_t, d_t^m = 0, d_t^x = x_t^e] \\
v_1(s_t) &= \bar{u}_1^f(s_{i,t}) + \beta\sigma_\epsilon E[log\Big(e^{v_0(s_{t+1})/\sigma_\epsilon} + e^{v_1(s_{t+1})/\sigma_\epsilon}\Big)|s_t, d_t^m = 1, d_t^x = x_t^*(s_{i,t})]
\end{aligned}
$$

[10]See Ekeland, Heckman, and Nesheim (2004), Heckman, Matzkin, and Nesheim (2005), Heckman, Matzkin, and Nesheim (2010), and Bishop and Timmins (2019).

[11]For an application of a simplified version of this framework with a single amenity, no wealth effects, and no unobserved heterogeneity, see Bishop and Murphy (2011).

From our model, the conditional probability of a household choosing to move in period $t$ is given by:

$$P_1(s_t) = \frac{e^{v_1(s_t)/\sigma_\epsilon}}{e^{v_0(s_t)/\sigma_\epsilon} + e^{v_1(s_t)/\sigma_\epsilon}} \tag{5}$$

where, to simplify the notation (here and for the remainder of the Section), we suppress the $i$ subscripts.

Taking the log of Equation (5) and rearranging terms yields: $log(e^{v_0(s_t)/\sigma_\epsilon} + e^{v_1(s_t)/\sigma_\epsilon}) = v_1(s_t)/\sigma_\epsilon - log(P_1(s_t))$, allowing us to rewrite the indirect choice-specific value functions as:

$$\begin{aligned} v_0(s_t) &= u_0^f(s_t) + \beta\sigma_\epsilon E[-log(P_1(s_{t+1})) + v_1(s_{t+1})/\sigma_\epsilon | s_t, d_t^m = 0, d_t^x = x_t^e] \\ v_1(s_t) &= \bar{u}_1^f(s_t) + \beta\sigma_\epsilon E[-log(P_1(s_{t+1})) + v_1(s_{t+1})/\sigma_\epsilon | s_t, d_t^m = 1, d_t^x = x_t^*] \end{aligned}$$

Each choice-specific value function is now written as a function of the associated flow utility, the probability of moving in the next period, and the value of moving in the next period, all conditional on the decisions made this period.

It is helpful to formalize the mechanism through which this period's decisions determine next period's state variables directly and, therefore, determine future utility. To that end, we make the following assumption:

**Assumption 1**: This period's choice of $x_t$ determines next period's endowment, $x_{t+1}^e$, but does not affect the transitions of the other state variables in $s_{t+1}$, i.e., $q(s'_{-x_{t+1}^e} | s'_{-x_t^e}, d_t^x = x) = q(s'_{-x_{t+1}^e} | s'_{-x_t^e})$.

The first implication of Assumption 1 is that the expected value associated with moving next period is only dependent on this period's discrete (move) decision, $d_t^m$, via the channel of the impact of this period's choices on expected future moving costs.

$$E[v_1(s_{t+1})|s_t, d_t^m = 1, d_t^x = x_t^*(s_t)] - E[v_1(s_{t+1})|s_t, d_t^m = 0, d_t^x = x_t^e] =$$

$$-\big(E[MC(z_{t+1}, x_{t+1}^e; \gamma^M)|s_t, d_t^m = 1, d_t^x = x_t^*(s_t)] - E[MC(z_{t+1}, , x_{t+1}^e; \gamma^M)|s_t, d_t^m = 0, d_t^x = x_t^e]\big) \tag{6}$$

The second implication of Assumption 1 is that, conditional on moving this period, the expected value associated with moving next period is only dependent on this period's continuous choice via the channel of the impact of this period's choices on expected future moving costs. This is an example of what Arcidiacono and Miller (2011) refer to as finite dependence.

$$\frac{\partial E[v_1(s_{t+1})|s_t, d_t^m = 1, d_t^x = x_t]}{\partial x_t} = -\frac{\partial E[MC(z_{t+1}, x_{t+1}^e; \gamma^M)|s_t, d_t^m = 1, d_t^x = x_t]}{\partial x_t} \tag{7}$$

Intuitively, what Assumption 1 and Equations (6) and (7) yield is that both mobility and amenity-consumption choices made in this period only affect the amenity endowment in the next period. However, if the household moves and reoptimizes their consumption in the next period, their endowment only matters through one channel: how much moving costs they will pay next period.

While Assumption 1 is not necessary for identification, it greatly simplifies the estimation of the model by eliminating the need to repeatedly solve the model via backward recursion or fixed point iteration. This is because all of the determinants of household behavior are either directly specified in the model ($u_m^f(s_t)$ and $MC(z_t, x_t^e)$) or are directly recoverable from the data ($P_1(s_{t+1})$). This can be seen in greater detail in the derivation of the likelihood function.

## 4.2 The Formation of the Likelihood

In this subsection, we discuss the formation of the likelihood of observing individual household choices. The two components of the joint likelihood are the likelihood contribution of the continuous choice and the likelihood contribution of the discrete choice. In this subsection, we outline each of these components separately. In the following subsection, we discuss under which conditions can estimation be based on only one of the two decisions.[12]

### 4.2.1 Contribution of the Continuous Choice

The likelihood contribution for the continuous choice is straightforward to form, as we need only to consider the case when $d_t^m = 1$, as the continuous choice is only made when the household chooses the discrete decision of a move. If a household chooses to move, the optimal choice of $x_t$ is given by:

$$x_t^*(s_t) = argmax_x \; v_1(s_t, x_t) - \epsilon_t = argmax_x \Big( u(x_t, z_t, \eta_t; \alpha) - r(x_t; \gamma_t) - MC(z_t, x_t^e; \delta) - \epsilon_t - \beta E[\sigma_\epsilon log(P_1(s_{t+1})) - E[v_1(s_{t+1})|s_t, d_t^m = 1, d_t^x = x_t] \Big) \quad (8)$$

Using Equation (7), the optimal choice of $x_t$ may be simplified to:

$$x_t^*(s_t) = argmax_x \Big( u(x_t, z_t, \eta_t; \alpha) - r(x_t; \gamma_t) - \beta E[\sigma_\epsilon log(P_1(s_{t+1})) - MC(z_{t+1}, x_{t+1}^e; \delta)|s_t, d_t^m = 1, d_t^x = x_t] \Big) \quad (9)$$

For notational convenience, we let $ELP_{\mathrm{m}}(s_t, x_t)$ denote the expected future log probabil-

[12] The derivation of the likelihood for the continuous choice builds upon the static hedonic likelihood developed in Heckman, Matzkin, and Nesheim (2005) and the derivation of the likelihood for the discrete choice builds upon the approach outlined in Arcidiacono and Miller (2011). One is not constrained to a likelihood-based approach; minimum Distance or GMM estimators may also be used.

ity of moving associated with this period's moving decision, $ELP_{\text{m}}(s_t, x_t) = E[log(P_1(s_{t+1}))|s_t, d_t^m = \text{m}, d_t^x = \text{x}]$. Analogously, we let $EMC_{\text{m}}(s_t, x_t)$ denote the expected future moving costs associated with this period's moving decision, $EMC_{\text{m}}(s_t, x_t) = E[MC(z_{t+1}, x_{t+1}^e))|s_t, d_t^m = \text{m}, d_t^x = \text{x}]$. This allows us to simplify Equation (9) to:

$$x_t^*(s_t) = argmax_x \Big( u(x_t, z_t, \eta_t; \alpha) - r(x_t; \gamma_t) - \beta(\sigma_\epsilon ELP_1(s_t, x_t) + EMC_1(s_t, x_t)) \Big) \tag{10}$$

with the first-order condition for finding the optimal choice of $x_t$ given by:

$$u'(x_t, z_t, \eta_t; \alpha) - r'(x_t); \gamma_t) - \beta(\sigma_\epsilon ELP_1'(s_t, x_t) + EMC_1'(s_t, x_t)) = 0 \tag{11}$$

Equation (11), forms the basis of the likelihood contribution of the continuous choice. For most specifications, it will be impossible to derive a closed form solution for the density of $x$ conditional on the covariates, $s_t$. However, if we assume that the marginal utility function $u'(x_t, z_t, \eta_t; \alpha)$ is separable in $\eta_t$, then we can solve for a closed form solution for $\eta_t$. Once we solve for $\eta_t$ (as a function of data and given parameter values) it is straightforward to use a change of variables to form the likelihood. Although this model is dynamic, if $ELP_1'(s_t, x_t)$ and $EMC_1'(s_t, x_t)$ are known, the approach to forming the likelihood outlined in Heckman, Matzkin, and Nesheim (2005) will apply.[13]

This suggests a natural two-step estimation approach. In the first stage, we estimate the transition probabilities of the time-varying amenities, the choice probability function, and the price function.[14] Using those estimates, we are able to construct estimates of $ELP_1'(s_t, x_t)$ and $EMC_1'(s_t, x_t)$, which we denote by $\widehat{ELP_1}'(s_t, x_t)$ and $\widehat{EMC_1}'(s_t, x_t)$. We then use a change of variables (from $x_t$ to $\eta_t$) to form the likelihood. A specific example of how to construct the

[13]Heckman, Matzkin, and Nesheim (2005) show how to form the likelihood in a non-parametric setting. In this case, we parameterize both the density and the utility function, however, the basic change-of-variables method of writing the likelihood still applies.

[14]For a detailed discussion of transitioning neighborhood amenities, see Lee and Lin (2018).

likelihood function is provided in the empirical specification in Section 5.1.

As $\widehat{ELP_1}'(s_t, x_t)$ and $\widehat{EMC_1}'(s_t, x_t)$ are functions of $x_t$, they will be correlated with the error, $\eta_t$, for reasons discussed in Epple (1987) and Bartik (1987). That is, as $x_t^*(s_t)$ is itself a function of $\eta_t$, $ELP_1'(s_t, x_t)$ and $EMC_1'(s_t, x_t)$ will be correlated with $\eta_t$. However, this is not a threat to identification as all we require is that $\eta_t$ is independent of the state variables $s_t$.[15,16]

An analysis of Equation (11) is helpful to understand how controlling for dynamic behavior affects the interpretation of results. As we have normalized the coefficient on rental price to one, each household's marginal willingness to pay, which is our desired object of interest, is given by $u'(x_t, z_t, \eta_t; \alpha)$. Our full dynamic model will recover this as equal to $r'(x_t; \gamma_t) - \beta(\sigma_\epsilon ELP_1'(s_t, x_t) + EMC_1'(s_t, x_t))$. The traditional Rosen-style models would typically focus only on $r'(x_t; \gamma_t)$, which is equal to $u'(x_t, z_t, \eta_t; \alpha) + \beta(\sigma_\epsilon ELP_1'(s_t, x_t) + EMC_1'(s_t, x_t))$. In other words, the traditional static approach can be interpreted as a reduced form that captures the two channels through which current neighborhood amenities affect household sorting: the determination of current utility flows (through $u(x_t, z_t, \eta_t; \alpha)$) and the determination of expectations over future utility flows (through $ELP_1'(s_t, x_t)$ and $EMC_1'(s_t, x_t)$). By directly estimating $ELP_1'(s_t, x_t)$ and $EMC_1'(s_t, x_t)$, we are able to disentangle these two effects.

### 4.2.2 Contribution of the Discrete Choice

As is standard in discrete-choice models, only *differences* in utility matter when estimating the dynamic, discrete-choice component. Using Equation (6), the difference in utilities is

[15] If marginal willingness to pay is a function of $x_t$, i.e., if $u'(x_t, z_t, \eta_t; \alpha)$ is a function of $x_t$, then the same techniques can be used to control for the fact that $x_t$ will be correlated with $\eta_t$ in the marginal willingness-to-pay function. One could alternatively estimate (11) directly and "correct" for the endogeneity of $ELP_1'(s_t, x_t)$ and $EMC_1'(s_t, x_t)$ (and $u'(x_t, z_t, \eta_t; \alpha)$) by using market dummies as instruments, as suggested in Bartik (1987).

[16] See Ekeland, Heckman, and Nesheim (2004) and Kuminoff and Pope (2014) (and the papers cited therein) for a discussion of the identification of the hedonic model.

simply given by:

$$v_1(s_t) - v_0(s_t) = \bar{u}_1^f(s_t) - u_0^f(s_t) - \beta\sigma_\epsilon\big(ELP_1(s_t, x_t^*) + ELP_0(s_t, x_t^e)\big) - \beta\big(EMC_1(s_t, x_t^*) + EMC_0(s_t, x_t^e)\big) \quad (12)$$

One remaining detail is to how to construct $\bar{u}_1^f(s_t) = \bar{u}_1^f(s_t, x_t^*(s_t))$ as it requires finding the optimal choice of $x_t$. A computationally burdensome approach would be to solve for $x_t^*(s_t)$ using a numerical search. Instead, we use a computationally simpler solution that is analogous to the estimation of conditional choice probabilities by directly estimating the reduced-form non-parametric policy function, $x_t^*(s_t)$.[17]

Once a solution for the flow utility functions, $u_0^f(s_t)$ and $\bar{u}_1^f(s_t)$, has been found, forming the likelihood for the discrete choice is straightforward. We first estimate transition probabilities for each time-varying variable and associated choice probabilities. Second, we use Equations (5) and (12) to form the likelihood. As with the continuous-decision likelihood, a specific example of how to construct the likelihood function is provided in the empirical specification in Section 5.1.

## 4.3 The Role of Unobserved Heterogeneity

We now discuss the role of unobserved preference heterogeneity in the model's estimation, i.e., we discuss the estimation routine in light of the idiosyncratic shock to preferences, $\eta_t$. If $\eta_t$ is a component of the state vector, $s_t$, then the first-stage estimation of the conditional choice probabilities, $P_1(s_t)$, should be conditioned upon the unobserved $\eta_t$. We consider two alternative solution methods under the assumption that $\eta_t$ can be written as the sum two

[17] Gayle, Hincapié, and Miller (2022) refer to this as conditional choice density estimation.

components.

$$\eta_t = \eta_t^{pre} + \eta_t^{post} \tag{13}$$

The first component, $\eta_t^{pre}$, by definition, is observed before both the move and quantity of amenity consumption decisions are made. The second component, $\eta_t^{post}$, by definition, is an i.i.d. shock that is observed after the agent decides to move but before deciding how much of the amenity to consume. The two solutions methods reflect different assumptions on the first component, $\eta_t^{pre}$.

In the first case, we assume that $\eta_t^{pre} = 0$, thereby assuming that no amenity-preference shock is observed (to the household) at the time of the household's discrete-choice decision of whether or not to move, but that the amenity-preference shock, $\eta_t^{post}$, is observed at the time of the household's continuous-choice decision of how much $x_t$ to consume.[18] The conditional choice probability, $P_1(s_t)$, is then no longer a function of $\eta_t$ and may be estimated in a completely separate first stage. This means that Maximum Likelihood estimation of the key parameters may be conducted using the continuous-choice likelihood contribution only.

In the second case, we specify that $\eta_t^{pre}$ is discrete and follows a finite mixture distribution. As $\eta_t^{pre}$ is fully observable to the household at all times, the first-stage estimation of the conditional choice probabilities, $P_1(s_t)$, needs to be conditioned upon $\eta_t^{pre}$ (but not on $\eta_t^{post}$). As $\eta_t^{pre}$ follows a finite mixture distribution, techniques similar to those developed in Arcidiacono and Miller (2011) may be applied. Note that in Arcidiacono and Miller (2011), there is a single unobserved variable while, in this case, each element of $\eta_t^{pre}$ is an unobserved state and we will have as many unobserved variables as number of amenities. As this component of unobserved heterogeneity is potentially time-varying, households would form expectations over future values of $\eta_t^{pre}$ in the same manner as they form expectations over other time-varying variables in the model.[19] A natural simplification would be to allow the

[18]The period $t$ shock to moving costs, $\epsilon_t$, is always observed at time $t$.

[19]Another possibility may be to assume $\eta_t^{pre}$ is continuous and that $\eta_t^{post} = 0$ . As the value of $\eta_t$ is

discrete component of $\eta_t^{pre}$ to be fixed through time for each household.

# 5 Empirical Specification and Results

## 5.1 Empirical Specification

We treat each county in the L.A. metropolitan area as a separate market, denoted $k$, and allow the parameters of the rent function to vary by market/county. We use data from the following five counties: L.A., Orange, Riverside, San Bernadino, and Ventura. Our primary amenities of interest, which we denote by $x$, are block group measures describing the median household income, the college degree rate, and home ownership rate, as well as a composite amenity discussed below. Thus, in our application, the number of amenities is given by $A = 4$. In addition to our amenities of interest, we include as controls the additional amenities of property age, square footage, lot size, number of rooms, violent crime exposure, $PM_{10}$ exposure, and full sets of Census-tract and year fixed effects and denote these controls $h$. The rental price function is given by:

$$log(r_{i,t}) = \gamma_{0,t,k} + \sum_{a=1}^{A-1} f(x_{a,i,t}; \gamma_{a,1,k}) + h'_{i,t}\gamma_{2,k} + e_{i,t} \quad (14)$$

$r_{i,t}$ is the household's observed housing rental equivalent in the data. In our simplest specification, we follow the literature and define the yearly rental equivalent for owners as

necessary for both steps of the two-step estimation routine, recovery of the structural parameters could be done using an iterative procedure between the first-stage conditional choice probability estimation and the second-stage utility parameter estimation. For any given value of $\eta_t$ and $s_t$, one could recover a first-stage estimate of $ELP'(s_t, x_t)$ and, conditional upon $\widehat{ELP'}(s_t, x_t)$, the likelihood associated with the second-stage regression. The residual from the second-stage regression would itself be an estimate of $\eta_t$, which could be used to re-estimate $\widehat{ELP'}(s_t, x_t)$ in the first stage and re-recover an updated $\widehat{\eta}$ in the second stage. This iterative procedure would continue until the estimates of $\eta_t$ converge and would be required at each iteration of the likelihood.

0.075∗(property value).[20] An important determinant in the literature of the mapping from property value to rental equivalent (i.e. 0.075) is expected property value appreciation. Therefore when estimating the full dynamic model, we include an adjustment to the 0.075 scaling constant to capture the fact that Equation (14) can be combined with estimated transition probabilities (Equation (17) below) to predict how amenity choices this period affect housing appreciation.[21] Finally, given estimates of the parameters of Equation (14), we can recover a catch-all amenity, $x_A = h'_{i,t}\gamma_{3,k} + e_{i,t}$ following Sieg, Smith, Banzhaf, and Walsh (2002) and Bajari and Benkard (2005).

The deterministic component of moving costs is given by:

$$MC(z_{i,t}, x^e_{i,t}) = 0.06 * Price(x^e_{i,t})\delta_1 + z'_{i,t}\delta_2. \tag{15}$$

where the vector of household attributes, $z_{i,t}$, includes a constant, race, income, and year.

The direct component of utility is given by:

$$u(x_t, \eta_t; \alpha) = \alpha_0 + x'_t(\alpha_{1,i} + \eta_t) \tag{16}$$

where $\eta \sim N(0, \Sigma_\eta)$ and $\alpha_{1,i} = z'_{i,t}\alpha_1$.

The transition probabilities of the $a^{th}$ element of the amenity vector $x$ are given by:

$$x^e_{a,t+1} = \rho_{0,a,k} + \rho_{1,a,k}x_t + \rho_{2,a,k}t + \nu_{a,t+1} \tag{17}$$

[20] For approaches that allow rental equivalent rates (also called user-cost rates) to vary by some combination of local geography and year, see Bieri, Kuminoff, and Pope (2022) and Bishop, Dowling, Kuminoff, and Murphy (2022).

[21] The adjustment is $\tilde{r}_{i,t} = (0.075 - adj(x_{i,t}))*$(property value) where $adj(x_{i,t}) = \left(\sum_{a=1}^{A-1} E[f(x_{a,i,t+1};\gamma_{a,1,k}) - f(x_{a,i,t};\gamma_{a,1,k})|s_t, x_{i,t}]\right) - \left(\sum_{a=1}^{A-1} E[f(\bar{x}_{t+1};\gamma_{a,1,k}) - f(\bar{x}_t;\gamma_{a,1,k})|s_t, \bar{x}_t]\right)$. Intuitively, as the 0.075 figure already captures average expected appreciation, we adjust this by how much the expected appreciation from a given choice of $x_{it}$ differs from expected appreciation from the average value of $x$, which is denoted by $\bar{x}_t$.

Letting $r'(x_{i,t})$, $ELP_1'(s_{i,t}, x_{i,t})$, and $EMC_1'(s_{i,t}, x_{i,t})$ equal the vector of derivatives of the rental function, the expected future log probability of moving, and expected future moving costs with respect to each element of $x$, the first-order condition for the household's optimal choice of $x$ is given by:[22]

$$\widehat{r}'(x_{i,t}) = z_i'\alpha_1 - \beta\sigma_\epsilon \widehat{ELP_1}'(s_{i,t}, x_{i,t}) - \beta\delta_1 \widehat{EMC_1}'(s_{i,t}, x_{i,t}) + \eta_{i,t} \tag{18}$$

where "hats" denote variables estimated in the first stages of estimation.[23] Using (18), to solve for $\eta_{i,t}$, we can form the likelihood of observing the continuous decision variable, $x$. The likelihood contribution of observation $i$ is given by:[24]

$$L^x(\alpha, \Sigma_\eta, \sigma_{epsilon}, \delta_1) = (2\pi)^{-J/2}|\Sigma|^{-1/2} exp\{-\frac{\eta_{i,t}'\Sigma^{-1}\eta_{i,t}}{2}\}|\frac{\partial \eta_{i,t}}{\partial x_{i,t}}| \tag{19}$$

The marginal likelihood contribution of the discrete choice to form the likelihood is based on the probability of an agent choosing to move in any period. This probability is determined by the utility difference:

$$\begin{aligned} v_1(s_t) - v_0(s_t) = & \big(x_t^*(s_t) - x_t^e\big)'\alpha_{1,i} - \big(\hat{r}(x_t^*(s_t)) - \hat{r}(x_t^e)\big) - 0.06 * Price(x_{i,t}^e)\delta_1 - z_{i,t}'\delta_2 \\ & - \beta\sigma_\epsilon\big(\widehat{ELP_1}(s_t, x_{i,t}^*) - \widehat{ELP_0}(s_t, x_t^e)\big) - \beta\delta_1\big(\widehat{EMC_1}(s_t, x_{i,t}^*) - \widehat{EMC_0}(s_t, x_t^e)\big) \end{aligned} \tag{20}$$

The probability of moving is given by:

$$P_1(s_t) = \frac{e^{(v_1(s_t) - v_0(s_t))/\sigma_\epsilon}}{1 + e^{(v_1(s_t) - v_0(s_t))/\sigma_\epsilon}} \tag{21}$$

[22] For example the $j^{th}$ element of $r'(x)$ is given by $\frac{\partial r(x)}{\partial x_j}$.

[23] Note that if we were to set $\beta = 0$, we would be faced with the standard (static) Rosen second-stage regression equation.

[24] This is the likelihood contribution of a household assuming that they moved and made a continuous choice. If they did not move, the endowment must have been consumed and the likelihood contribution is effectively one.

and the likelihood contribution of observation $i$ is given by:[25]

$$L^m(\alpha, \Sigma_\eta, \sigma_{epsilon}, \delta_1, \delta_2) = P_1(s_t)^{1(d_t^m=1)}(1 - P_1(s_t))^{1(d_t^m=0)} \tag{22}$$

Finally, to solve for both the discrete- and continuous-choice likelihoods, we must also estimate the reduced-form policy functions $\hat{P}_1(s_t)$ and $\hat{x}^*(s_t)$. We obtain $\hat{P}_1(s_t)$ by estimating the probability of reoptimizing with a flexible Logit and $\hat{x}(s_t)$ by estimating a flexible regression of $x_t$ on $s_t$. Both estimations are conducted separately for each market, $k$.

## 5.2 Results

We estimate the rental equivalent function, Equation (14), separately for each county. Our identification strategy relies on exploiting the within-tract variation in our amenities, while also controlling for important property-level determinants of price, such as square foot, number of bedrooms, and lot size. Per the Census Bureau, local agencies can include up to 9 block groups within a tract. "The guidelines specified an ideal size for a BG of 400 housing units, with a minimum of 250, and a maximum of 550 housing units. The guidelines further required that BG boundaries follow clearly visible features, such as roads, rivers, and railroads." Nationally, there are an average of 3.7 block groups per tract. In our application to the Los Angeles MSA, there are 2.4 block groups per tract.

As discussed above, the derivative of the rental-equivalent function is a measure of *where* amenities households sort. The derivative, often called the implicit price, measures how much extra it would cost a household per year to consume a marginal increase in the amenity. At a household's chosen vector of amenity consumption levels, this will be equal to

[25] An advantage of using information from the discrete choice is that it facilitates estimating the moving cost parameter, $MC$. Using only the continuous-choice *controls* for moving costs, however, it does not recover an estimate of these costs.

$u' - (\beta\sigma_\epsilon ELP_1' + \beta\delta_1 EMC_1')$. That is, the implicit price will capture the direct consumption value of the amenity through $u'$ as well as the predictive power of the amenity through $(\beta\sigma_\epsilon ELP_1' + \beta\delta_1 EMC_1')$. Amenity-specific average values of these implicit prices are presented in Column 1 of Table 4. For ease of interpretation, the third column scales these implicit prices to capture a one-half standard deviation increase in the amenities.

Table 4: Hedonic Price Function Results

| | Implicit Price of 1-Unit | Implicit Price of 0.5 s.d. |
|---|---|---|
| Median Household Income | 40.65 | 384 |
| Percent with College Degree | 37.34 | 289 |
| Rate of Homeownership | 16.92 | 164 |
| Neighborhood Amenity Index | 20,019 | 4,124 |
| Observations | 855,845 | 855,845 |

For the following preliminary estimation results, we make the assumption that $\eta_{i,t}$ is observed after the household decides whether to re-optimize, but before the household decides how much crime to consume, i.e., $\eta_t^{pre} = 0$. Taking the rental price parameters, $\gamma$, as given, we use the likelihood contribution associated with the continuous choice of how much amenities to consume to estimate the utility parameters $\alpha$ and $\Sigma_\eta$, as well as the scale of the logistic moving-cost shock, $\sigma_\epsilon$ and the financial moving cost parameter, $\delta_1$. Then, taking $\gamma$, $\alpha$, $\Sigma_\eta$, $\sigma_\epsilon$, and $\delta_1$ as given, we then use the likelihood contribution associated with the discrete moving choice to estimate the parameters of the psychological moving cost function, $\delta_2$.

The estimates of the utility parameters, $\alpha$, which reflect households' willingness-to-pay for our four amenities of interest, are presented in Table 5. We find that the average household has a positive willingness to pay for all amenities and that in all cases this willingness to pay is increasing in income. The mean household is willing to pay \$37.47 per year to increase their block group's median income by \$1,000. This translates to a willingness-to-pay of \$354

Table 5: Utility Parameter Results

| | | WTP for 1-Unit | Mean WTP for 0.5 s.d. |
|---|---|---|---|
| Median Household Income | mean($\mathbf{z}'_{i,t}\boldsymbol{\alpha}_1$) | 37.47 | 354 |
| | $\alpha_{Income}$ | 0.22 | |
| Percent with College Degree | mean($\mathbf{z}'_{i,t}\boldsymbol{\alpha}_1$) | 62.40 | 483 |
| | $\alpha_{Income}$ | 0.12 | |
| Rate of Homeownership | mean($\mathbf{z}'_{i,t}\boldsymbol{\alpha}_1$) | 7.12 | 69 |
| | $\alpha_{Income}$ | 0.08 | |
| Neighborhood Amenity Index | mean($\mathbf{z}'_{i,t}\boldsymbol{\alpha}_1$) | 24,976 | 5,145 |
| | $\alpha_{Income}$ | 143 | |
| Observations | | 855,845 | 855,845 |

per year to increase their block group's median income by one half of a standard deviation.[26] The corresponding willingness-to-pay figures for a one percentage point (one half standard deviation) increase in college graduation rates and ownership rates are $62.40 ($483) and $7.12 ($69), respectively. The effect of an additional $1,000 in income increases willingness-to-pay in all cases and can be translated into income elasticities of 0.50, 0.17, and 1.00 calculated at the mean income of $\$86,384$ (in 2000 dollars).

Moving costs are comprised of two terms, financial moving costs, which are given by $0.06 * Price(x^e_{i,t})\delta_1$ house value and psychological moving costs which are given by $z'_{i,t}\delta_2 + \epsilon_{it}$. We estimate $\hat{\delta}_1 = 0.42$ and the mean, median, and standard deviation of estimated financial moving costs to be $6,058, $5,129, and $3,313, respectively. The mean, median, and standard deviation of estimated psychological moving costs to be $52,622, $50,506, and $8,664, respectively. While these may appear high, as discussed in Kennan and Walker (2011), they reflect the dis-utility that would be experienced by a randomly chosen household in a randomly chosen time period if they were forced to move. Arguably a more relevant figure is the average psychological cost faced by a household that chose to move, $z'_{i,t}\delta_2 + E[\epsilon_{it}|d^m_{i,t} = 1]$,

[26]The standard deviation is measured as the average within-year standard deviation across block groups.

which we estimate to be $15,041.[27]

Finally, we turn to the decomposition of the determinants of households sorting behavior. The combined effect of an amenity on sorting behavior is shown in Equations (3) and (4) where it can be seen that $x_{i,t}$ affects both utility today and households' expectations about future utility. As discussed in Section 4.2, rearranging Equation (11) shows that this combined effect is equal to the implicit price of an amenity, $r'(x_{i,t})$ and is therefore relatively straightforward to estimate and is given in Table 4. In contrast, Table 5 isolates the role of an amenity in solely increasing utility today, effectively holding all else, including future expectations, constant. To illustrate this decomposition, we combine the results in Table 6. We see two interesting patterns. First, *where* households sort (as captured by the first column) can be very different from *why* households sort (as captured by the second column). Second, this distinction varies considerably by amenity. For the case of median income, behavior is driven by the impact of this period's choices on this period's flow utility. For the case of college graduation, behavior is strongly driven by the impact of this period's choices on this period's flow utility but expectations about future utility flows based of this period's college graduation rate decision suppress that sorting. Finally, while households sort on ownership rates, this is driven, in large part, by how they view this period's choice of ownership rate negatively affecting future utility flows.

# 6 Conclusion

In this paper, we develop and estimation a dynamic model of hedonic demand that we use to explore neighborhood change. Individual household's choice of where to live is an inherently

[27]The equivalent figures in Kennan and Walker (2011) are $312,000 and -$80,768. The fact that the estimated moving costs, conditional on moving, are positive in this paper versus negative in Kennan and Walker (2011), indicates that the unobservables are playing a smaller role in determining moving behavior in our framework and application.

Table 6: Decomposition of the Mean WTP for 0.5 s.d. Increase in an Amenity

| | Combined-effect $u(x_t) + \beta E[v(x_{t+1})\|x_t]$ | Preferences $u(x_t)$ |
|---|---|---|
| Median Household Income | 384 | 354 |
| Percent with College Degree | 289 | 483 |
| Rate of Homeownership | 164 | 69 |
| Neighborhood Amenity Index | 4,124 | 5,145 |
| Observations | 855,845 | 855,845 |

dynamic decision as neighborhood characteristics (amenities and demographics) serve dual roles by directly affecting current utility and by determining the household's expectations regarding future neighborhood characteristics. We estimate our model sing a novel dataset that contains information on housing, individual, and neighborhood attributes in Los Angeles metropolitan area, which is an ideal setting to consider gentrification and neighborhood change more broadly.

Our model explicitly allows for the fact that a change in the consumption of an amenity today affects not only current utility (through both increased consumption and increased price), but also the stream of future utility (through both future consumption levels and future prices). We accomplish this while still retaining all of the intuition associated with the classic Rosen model of hedonic demand; we simply redefine the familiar optimality condition to require that the marginal increase in current price is equal to the marginal benefit of the increase in amenity consumption this period *plus* the associated change in future utility flow. Finally, by presenting the household's problem as a two-part, discrete-continuous decision, we are able to take advantage of recent advances in the estimation of this class of model. We show how the added computational burden (compared with the static model) is reduced to a simple, first-stage logit probability estimation.

We consider three neighborhood amenities of interest defined at the level of the Census block group (along with a rich set of controls): median income at the neighborhood level, percentage of neighborhood residents with a college degree or higher, and the neighborhood rate of homeownership. We find that while households have a positive willingness to pay for each of the amenities, only an increase in current rates of homeownership serve as a predictor of increased flow utilities in future. While increases to current educational attainment do not serve to inform predictions about future flow utilities, increases to educational attainment, conditional on other covariates, signal decreased flow utilities in future.

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# Appendix - Monte Carlo Evidence

Here we only show Monte Carlo simulations for the most parsimonious specification of our model – one with a single amenity of interest and with no household demographic attributes. To simulate the data necessary for a Monte Carlo experiment of a richer model specification, we would need to solve the full model (just to create the dataset). Thus, we would face the (prohibitively high) computational burdens discussed in Section 3.3. It is important to note, however, that the *estimation* of a richer model would not face such computational constraints, given the estimation strategy presented in this paper.

For simplicity, let $x$ be a scalar amenity (a good) and let the number of markets, $J$, be equal to two. The direct component of utility is given by:

$$u(x_t, \eta_t; \alpha) = \alpha_0 + (\alpha_1 + \eta_t)x_t$$

where $\eta \sim N(0, \sigma_\eta)$. We assume the model timing outlined in Section **??**, with the shock to preferences, $\eta_t$, being observed after the decision to move has been made, but before the decision of how much $x$ to consume has been made.

The rental price function, of which we allow the parameters to vary by market, is given by:

$$r(x_t; \gamma) = \gamma_{0,j} + \gamma_{1,j}x_t + \gamma_{2,j}x_t^2 + e_t$$

and the deterministic component of movings costs is given by the constant, $MC$. The exogenous transition probabilities of the amenity $x$ are given by:

$$x_{t+1}^e = \rho_0 + \rho_1 x_t + \nu_{t+1}$$

where $\nu \sim N(0, \sigma_\nu)$.

Using this same basic structure, we recover the parameters using (i) the likelihood of observing the continuous choice, (ii) the likelihood of observing the discrete choice, and (iii) the joint likelihood of the two choices. We additionally present the results from two static approaches for comparison.

Table A.1: Monte Carlo Results

| | True Values | I Static Hedonic | II Static Discrete Choice | III Dynamic Hedonic | IV Dynamic Discrete Choice | V Dynamic Joint Likelihood |
|---|---|---|---|---|---|---|
| $\alpha_1$ | 4 | 3.680 | 3.772 | 4.014 | 3.996 | 3.996 |
| | | (0.003) | (0.017) | (0.067) | (0.018) | (0.012) |
| $\sigma_\epsilon$ | 1.2 | – | – | 1.278 | 1.196 | 1.206 |
| | | – | – | (0.271) | (0.034) | (0.051) |
| $\sigma_\eta$ | 0.25 | 0.230 | – | 0.252 | 0.335 | 0.250 |
| | | (0.002) | – | (0.012) | (0.374) | (0.007) |
| MC | 3 | – | 2.466 | – | 3.177 | 3.012 |
| | | – | (0.041) | – | (0.376) | (0.088) |

In Table A.1, we compare estimates of the model using both the static and the dynamic estimators. In Columns I and II, we present the results using the static hedonic approach and the static discrete-choice approach. The results of the dynamic hedonic estimator, the dynamic discrete-choice estimator, and joint-likelihood estimator are shown in Columns III, IV, and V, respectively. In each experiment, we set the number of draws to 500, the number of households to 1,000, and the number of time periods to 20. We fix and $\beta = 0.95$ and assume that it is know by the econometrician.

The Monte Carlo results show that the static models return biased estimates of the primary structural parameter of interest, $\alpha_1$. The estimates of the parameters from all three dynamic estimators show very little evidence of finite-sample bias.[28] Among the dynamic estimators, the variance of the dynamic hedonic estimator shown in Column III is somewhat higher than that of the dynamic discrete-choice estimator shown in Column IV.[29] This variance differential is caused by two factors: first, the sample size is approximately four times larger for the dynamic discrete-choice estimator, as data describing the continuous choice of $x_t$ is only available for households who, in fact, move (the unconditional probability of moving is 0.252), and, second, the estimation of the dynamic hedonic estimator involves estimating a non-parametric derivative which, while not affecting the consistency of the estimator, does add to the variance. That said, the simplicity and computational tractability of the dynamic hedonic estimator make it an attractive estimation approach.

[28] $\sigma_\eta$ is poorly identified in the dynamic discrete-choice estimator, as in this case it is only identified off functional form. Apart from, $\sigma_\eta$, the estimate with the largest finite-sample bias is $\sigma_\epsilon$. The mean of $\sigma_\epsilon$ from the dynamic hedonic approach (Column III) is 1.278. The median estimate of 1.226 is much closer to the true parameter.

[29] The exception to this is the estimated variance of $\sigma_\eta$.