**Online Appendix D**
for '"Building Credit Histories"
by Natalia Kovrijnykh, Igor Livshits, and Ariel Zetlin-Jones

# D Proofs and Equilibrium Constructions

## D.1 Sequence Problems

In order to facilitate characterization of equilibria, we define the sequence of problems faced by each agent in the order implied by backward induction. In the middle of stage 2, after lenders have made their stage-2 offers, the borrower has observed two sets of offers, $O_1$ and $O_2$, and her own credit history $h_2^P = (x_1, q_1, j_1)$. Let $h_2^B = (O_1, h_2^P, O_2)$ denote this information set of the borrower. The borrower's stage-2 action is to choose an offer from $O_2$ (or possibly reject all offers). She does so based in part on her posterior beliefs about her own quality state induced by the history (and her understanding of lenders' strategies). We denote $\theta_2^B(e|h_2^B)$ the probability the borrower assigns in stage 2 to receiving endowment $e$ in the second period. Note that this probability is a convolution of the posterior belief of the borrower regarding her underlying quality $s$ and the probability distribution over outcomes implied by this quality. Of course, the borrower forms her posterior about her underlying quality based on public and private histories, as well as her understanding of lenders' equilibrium strategies—on the equilibrium path, it is obtained using Bayes' rule. The borrower's stage-2 action maximizes her expected payoff under $\theta_2^B$ and so solves

$$V_2\left(h_2^B\right) = \max_{(x_2,q_2,j)\in O_2 \bigcup\{(0,0,0)\}} u(q_1x_1 + q_2x_2) + \beta \sum_e \theta_2^B(e|h_2^B)u(\max\{e - x_1 - x_2, (1-\varphi)e\}).$$

At the beginning of stage 2, everyone has observed the public credit history of the borrower $h_2^P = (x_1, q_1, j_1)$. Additionally, each lender $k$ knows his private signal about the borrower's state, $\sigma_k$, and his offer to the borrower in the first stage, $(x_1^k, q_1^k)$. Thus, the private history of the lender $k$ is $h_2^k = (h_2^P, \sigma_k, (x_1^k, q_1^k))$. When choosing his second-stage offer, the $k$th lender forms expectations of other lenders' offers. Similar to the borrower, the lender forms his posterior belief $\mu_2(\sigma_{-k})$ regarding the other class' signal based in part on his understanding of equilibrium strategies. Equilibrium strategies imply a mapping from the vector of realized signals and the observed public history into an offer set $O_2$, which will be faced by the borrower. For any $(x, q)$ offered by the $k$th lender, denote by $\xi_2^k$ the probability of that offer being accepted (as perceived by the $k$th lender given the equilibrium strategies of the borrower and the other lenders).[48] Then, the optimal offer made by lender $k$ solves the following maximization problem:

$$W_2^k\left(h_2^k\right) = \max_{(x,q)} \sum_{\sigma_{-k}} \mu_2(\sigma_{-k}|h_2^k)\, \xi_2^k(x,q) \times \left[-qx - q_1 x_1 \mathbb{1}_{j_1=k} + \bar{q}\left(x + x_1 \mathbb{1}_{j_1=k}\right) \sum_e \theta_2^L(e|h_2^k, j_2 = k) \mathbb{1}_{[\varphi e \geqslant x_1 + x]}\right],$$

where $\theta_2^L(e|\cdot)$ is the lender's posterior probability that the borrower will receive endowment $e$ conditional on the lender's information at the beginning of stage 2 and the fact that her offer was accepted by the borrower.

In stage 1, the borrower chooses among offers in the set $O_1$ (and the option of rejecting all offers) to maximize

$$V_1(O_1) = \max_{(x,q,k)\in O_1 \bigcup \{(0,0,0)\}} \mathbb{E}V_2(O_1, (x,q,k), O_2(x,q,k)).$$

Note that the borrower understands that her choice of $(x, q)$ influences not only her payoffs in $V_2$ directly but also the set of offers she will receive in stage 2, $O_2$.

Similarly, lenders in stage 1 understand that the offer they make, if accepted, may influence the posteriors of other lenders in the second stage.[49] Having observed their signal, they make an offer that maximizes their expected profits:

$$W_1^k(\sigma_k) = \max_{(x,q)} \sum_{\sigma_{-k}} \mu_1(\sigma_{-k}|\sigma_k) \left[\xi_1^k(x,q)\, W_2^k\left((x,q,k), \sigma_k, (x,q)\right) + \left(1 - \xi_1^k(x,q)\right) W_2^k\left((x_{-k}, q_{-k}, -k), \sigma_k, (x,q)\right)\right],$$

where $\xi_1^k$ and $\theta_1^L$ are defined similar to their stage-2 counterparts. Note that, if accepted, the lender's offer influences her payoffs not only directly but also by affecting the offer set $O_2$ in the subsequent stage.

[48]To be more precise, $\xi_2^k = \xi_2^k\left((x,q,k)|(x_1^k, q_1^k, k), O_1^{-k}(\sigma_k, \sigma_{-k}), h_2^p, O_2^{-k}(\sigma_k, \sigma_{-k}, h_2^p)\right)$, where $\sigma_k$ is the signal observed by the k-th lender, $\sigma_{-k}$ is the signal observed by lenders of the other class, and $O_i^{-k}$ is the offer set excluding the offer made by the k-th lender in stage $i = 1, 2$.

[49]In our setting, an individual lender's deviation does not change the borrower's posterior, since the borrower is facing many lenders. However, it may affect other lenders' posterior, since lenders do not observe the offer set $O_1$, only the borrower's choice from that set.

## D.2 Preliminaries

The following expressions will be useful for our equilibrium construction throughout the rest of this appendix. We have

$$\Pr(g|A) = \frac{\alpha(1+\rho)}{\alpha(1+\rho)+(1-\alpha)(1-\rho)},\ \Pr(b|B) = \frac{(1-\alpha)(1+\rho)}{\alpha(1-\rho)+(1-\alpha)(1+\rho)},$$
$$\Pr(g|B) = \frac{\alpha(1-\rho)}{\alpha(1-\rho)+(1-\alpha)(1+\rho)},\ \Pr(b|A) = \frac{(1-\alpha)(1-\rho)}{\alpha(1+\rho)+(1-\alpha)(1-\rho)},$$
$$\Pr(g|AA) = \frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2},\ \Pr(b|BB) = \frac{(1-\alpha)(1+\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2},$$
$$\Pr(g|AB) = \alpha,\ \Pr(b|AB) = 1-\alpha,$$
$$\Pr(g|BB) = \frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2},\ \Pr(b|AA) = \frac{(1-\alpha)(1-\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2}.$$

In addition,

$$\Pr(AA|A) = \frac{1}{2}\frac{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2}{\alpha(1+\rho)+(1-\alpha)(1-\rho)}, \tag{10}$$
$$\Pr(AB|A) = \frac{1}{2}\frac{(1+\rho)(1-\rho)}{\alpha(1+\rho)+(1-\alpha)(1-\rho)}. \tag{11}$$

Moreover,

$$\Pr(\text{repay } \varphi e_h|AA) = \delta \Pr(g|AA), \quad \Pr(\text{repay } \varphi e_m|AA) = 1-\delta \Pr(b|AA),$$
$$\Pr(\text{repay } \varphi e_h|AB) = \delta \Pr(g|AB), \quad \Pr(\text{repay } \varphi e_m|AB) = 1-\delta \Pr(b|AB).$$

Recall that we restrict stage-1 offers to $\varphi e_1$ for $e_1 \in E$, and given a history $\varphi e_1$, we restrict stage-2 offers to $\varphi(e_2 - e_1)$ for $e_2 \in E$ and $e_2 > e_1$.

## D.3 Symmetric-Information Outcomes

To establish a benchmark for our analysis, consider a variant of our model environment in which all lenders' signals are public information. The multi-stage nature of period I is irrelevant in this setting, as there is no need to aggregate any information. We can thus simply restrict attention to equilibria where all borrowing occurs in the last stage of the period, which avoids any concerns of debt dilution. All loans are then competitively priced, and we can simply think of the borrowers as choosing their preferred loan size, given actuarially fair interest rates appropriate for the specific type of the borrower.

All of the equilibrium examples in the paper share one key feature of the symmetric-information benchmark. Namely, the equilibrium outcome in the limiting case as $\rho$ approaches 1 features full separation in loan sizes between the three borrower types. I.e., for $\rho$ arbitrarily close to 1, $BB$-borrowers take on a small loan, $AB$-borrowers choose a medium loan, and $AA$-borrowers get a large loan in the equilibrium of the symmetric-information environment. The restrictions on the parameter values that yield this outcome, which we will sometimes refer to as $lmh$, are as follows.

**Assumption 1** *Assume that parameter values satisfy the following conditions:*

*(i)* $(1-\delta(1-\alpha))\,e_m > e_\ell$*, or, equivalently,* $q_m^{AB} e_m > \bar{q} e_\ell$*;*

*(ii)* $(1-\delta(1-\alpha))\,e_m > \delta\alpha e_h$*, or, equivalently,* $q_m^{AB} e_m > q_h^{AB} e_h$*;*

*(iii)* $\delta e_h > e_m$*;*

*(iv)* $e_\ell > (1-\delta) e_m$*.*

We explain both formally and intuitively why these conditions imply the $lmh$ outcome in the proof of the following proposition.

**Proposition 1** *If parameter values satisfy Assumption 1, then the symmetric-information equilibrium outcome is:*

*(i) for $\rho$ arbitrarily close to 1, $BB$-borrowers get $(\varphi e_\ell, \bar{q})$, $AB$-borrowers get $(\varphi e_m, q_m^{AB})$, and $AA$-borrowers get $(\varphi e_h, q_h^{AA})$;*

*(ii) for $\rho = 0$, all borrowers receive a medium-size loan.*

**Proof.** First, note that the symmetric structure of the signals is such that the posterior regarding the underlying state of $AB$-borrowers is the same as the uninformed prior and thus does not depend on the precision of the signal. Hence, parts (i) and (ii) of Assumption 1 guarantee that $AB$-borrowers choose the medium-size loan under actuarially fair loan pricing. But these same conditions then guarantee that all borrowers choose medium-size loans when signals are completely uninformative.

Assumption 1 (iii) guarantees that, when signals are perfectly informative, $AA$-borrowers take on a large loan if all prices are actuarially fair. Assumption 1 (iv) guarantees that $BB$-borrowers in this situation choose the small loan.

Note that this set of conditions also ensures that $AA$-borrowers do not choose the small loan. To see this, note that the condition for $AA$ to prefer a medium loan to a small one is

$[1-\delta\Pr(b|AA)]e_m > e_\ell$. Note that $\Pr(b|AA) < 1-\alpha$ whenever $\rho > 0$. Hence Assumption 1 (i) ensures that AA-borrowers prefer a medium loan to a small one. □

**Corollary 1** *If parameter values satisfy Assumption 1, then* BB*-borrower prefers a medium-size loan to a large loan if both loans are priced actuarially fairly. I.e.,* $q_m^{BB} e_m \geqslant q_h^{BB} e_h$.

**Proof.** By Assumption 1 (ii), $e_m/e_h \geqslant q_h^{AB}/q_m^{AB}$. In order to establish our claim, we need to show that $q_h^{AB}/q_m^{AB} \geqslant q_h^{BB}/q_m^{BB}$, which we do by establishing $q_m^{BB}/q_m^{AB} \geqslant q_h^{BB}/q_h^{AB}$. The actuarially fair prices are

$$
\begin{aligned}
q_m^{AB} &= \bar{q}\left(1-\delta(1-\alpha)\right), \\
q_h^{AB} &= \bar{q}\delta\alpha, \\
q_m^{BB} &= \bar{q}\left[1-\delta\frac{(1-\alpha)(1+\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2}\right], \\
q_h^{BB} &= \bar{q}\delta\frac{\alpha(1-\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2}.
\end{aligned}
$$

Plugging these in, we have

$$
\begin{aligned}
\frac{q_m^{BB}}{q_m^{AB}} - \frac{q_h^{BB}}{q_h^{AB}} &= \frac{1}{1-\delta(1-\alpha)}\left[1-\delta\frac{(1-\alpha)(1+\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2}\right] - \frac{(1-\rho)^2}{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2} \\
&= \frac{(1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2-\delta(1-\alpha)(1+\rho)^2-(1-\delta(1-\alpha))(1-\rho)^2}{(1-\delta(1-\alpha))((1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2)} \\
&= \frac{(1-\delta)(1-\alpha)(1+\rho)^2-(1-\delta)(1-\alpha)(1-\rho)^2}{(1-\delta(1-\alpha))((1-\alpha)(1+\rho)^2+\alpha(1-\rho)^2)} \geqslant 0.
\end{aligned}
$$

□

**Proposition 2** *Suppose parameter values satisfy Assumption 1. Then*

*(i) there exists* $\rho^{BB} \in (0,1)$ *such that* BB*-borrowers take on* $(\varphi e_\ell, \bar{q})$ *in the symmetric-information equilibrium whenever* $\rho > \rho^{BB}$*, and they choose* $(\varphi e_m, q_m^{BB})$ *whenever* $\rho < \rho^{BB}$*;*

*(ii) there exists* $\rho^{AA} \in (0,1)$ *such that* AA*-borrowers take on* $(\varphi e_h, q_h^{AA})$ *in the symmetric-information equilibrium whenever* $\rho > \rho^{AA}$*, and they choose* $(\varphi e_m, q_m^{AA})$ *whenever* $\rho < \rho^{AA}$.

**Proof.** Since borrowers are impatient, they simply maximize the size of the loan advance they receive in period I. The medium-size loan yields $\varphi e_m q_m^\omega$ to a type-$\omega$ borrower, where $q_m^\omega = \bar{q}\left(1-\delta\Pr(b|\omega)\right)$.

(i) Note that $\Pr(b|BB)$ is increasing in $\rho$, and thus $q_m^{BB}$ is monotonically decreasing in $\rho$. On the other hand, the advance on the safe loan $(\varphi e_\ell, \bar{q})$ is not affected by $\rho$. Since the medium-size loan is preferred by BB-borrowers when $\rho = 0$, and the small loan is preferred when $\rho = 1$ (as was established in Proposition 1), there must be an interior $\rho^{BB}$, as described in the statement of this proposition.

(ii) The advance on the large loan is given by $\varphi e_h q_h^\omega$, where $q_h^\omega = \bar{q}\delta\Pr(g|\omega)$. Just like in the case above, it is straightforward to show that $\varphi e_h q_h^{AA} - \varphi e_m q_m^{AA}$ is strictly increasing in $\rho$. And since the advance to an AA-borrower from a large loan is greater than that from a medium-size loan when $\rho = 1$, and since the opposite is true when $\rho = 0$ (both premises are guaranteed by Proposition 1), there must exist an interior $\rho^{AA}$ described in the statement of this proposition. Making this argument more explicit, the large loan yields a (weakly) larger loan advance whenever

$$
\begin{aligned}
0 \leqslant q_h^{AA} e_h - q_m^{AA} e_m &= \bar{q}\delta \Pr(g|AA) e_h - \bar{q}[\Pr(g|AA) + (1-\delta)(1-\Pr(g|AA)] e_m \\
&= \bar{q}\frac{\delta\alpha(1+\rho)^2 e_h - \alpha(1+\rho)^2 e_m - (1-\delta)(1-\alpha)(1-\rho)^2 e_m}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2} \\
&= \bar{q}\frac{\alpha(\delta e_h - e_m)y - (1-\delta)(1-\alpha)e_m}{\alpha y + 1 - \alpha},
\end{aligned} \tag{12}
$$

where $y = (1+\rho)^2/(1-\rho)^2$ is strictly increasing in $\rho$. The derivative of the right-hand side of (12) with respect to $y$ is

$$
\bar{q}\alpha(1-\alpha)\frac{(\delta e_h - e_m) + (1-\delta)e_m}{(\alpha y + 1 - \alpha)^2} = \bar{q}\alpha(1-\alpha)\delta\frac{e_h - e_m}{(\alpha y + 1 - \alpha)^2} > 0.
$$

Thus $q_h^{AA} e_h - q_m^{AA} e_m$ is strictly increasing in $\rho$, implying that there is a unique root between 0 and 1. We denote this root by $\rho_{AA}$. □

## D.4 Equilibrium Outcome 1: $\mathtt{lmh}$ without Cross-Subsidization

We construct the equilibrium as follows.

### D.4.1 On-Path Actions

Stage 1:

- B-lenders make no offers;
- A-lenders offer $(\varphi e_\ell, q_h^{AA})$;

– Only borrowers with two such offers ($AA$-borrowers) accept one.

Stage 2:

– $A$-lenders whose offer was not accepted, but observe an accepted offer from the *opposite* class, learn that the borrower is $AA$ and offer $\left(\varphi\left(e_h - e_\ell\right), q_h^{AA}\right)$. Such an offer is accepted by $AA$-borrowers.

– $A$-lenders whose offer was not accepted, but who observe an accepted offer from *their* class, offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$. (Note that on the equilibrium path, this offer is not accepted by any borrowers. However, we specify this offer in order to ensure that $AB$-borrowers do not mimic $AA$-borrowers.)

– $A$-lenders who observe no accepted offer learn that the borrower is $AB$, and offer $\left(\varphi e_m, q_m^{AB}\right)$. This offer is accepted by the $AB$-borrowers.

– $B$-lenders, who *never* observe an accepted offer in stage 1 (on-path), offer $(\varphi e_\ell, \overline{q})$. This offer is accepted by the $BB$-borrowers.

Before proceeding, note a couple of things about this equilibrium. First, this equilibrium is symmetric: lenders' offers are a function of their signal and public information only, so we forego class identifiers. Second, on path, this equilibrium features full information for the borrower after stage 1. By observing the number of offers that she receives in stage 1, a borrower is certain whether she is $AA$ (offers from all lenders), $AB$ (offers from only one class of lenders), or $BB$ (no offers).

### D.4.2 Equilibrium Payoffs

The payoffs to borrowers in equilibrium are as follows:

- $AA$-borrowers: $\varphi e_\ell q_h^{AA} + \varphi\left(e_h - e_\ell\right) q_h^{AA} = \varphi e_h q_h^{AA}$;
- $AB$-borrowers: $\varphi e_m q_m^{AB}$;
- $BB$-borrowers: $\varphi e_\ell \overline{q}$.

### D.4.3 Equilibrium Conditions

Before we proceed with construction of beliefs and (off-path) strategies, we state necessary conditions on the model parameters so that our constructed equilibrium candidate is indeed an equilibrium. We later show that these conditions together with Assumption 1 are sufficient to ensure that relevant incentive constraints are satisfied.

**Condition 1** *Suppose that the model parameters satisfy*

*(i)*

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2} \leqslant 1-\delta(1-\alpha),$$

*or, equivalently,* $q_h^{AA} \leqslant q_m^{AB}$*; this ensures that the* AB*-borrowers do not accept a small loan in stage 1.*

*(ii)*

$$[1-\delta(1-\alpha)](e_m - e_\ell) \geqslant \delta\alpha(e_h - e_\ell),$$

*or, equivalently,* $q_m^{AB}(e_m - e_\ell) \geqslant q_h^{AB}(e_h - e_\ell)$*; this ensures that upon acceptance of a small loan in stage 1, prices are such that an* AB*-borrower is better off being topped up to a medium loan rather than a large one. (Note that this condition implies that* $q_m^{AB} e_m > q_h^{AB} e_h$*, which is Assumption 1(ii).)*

*(iii)*

$$\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2}(e_h - e_\ell) \geqslant \left[1-\delta\frac{(1-\alpha)(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2}\right](e_m - e_\ell),$$

*or, equivalently,* $q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AA}(e_m - e_\ell)$*; this ensures that upon acceptance of a small loan in stage 1, prices are such that an* AA*-borrower is better off being topped up to a large loan rather than a medium one.*

*(iv)*

$$e_\ell \geqslant \left(1-\delta\frac{(1-\alpha)(1+\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2}\right) e_m,$$

*or, equivalently,* $\bar{q} e_\ell \geqslant q_m^{BB} e_m$*. Note that by Assumption 1 (ii) and Corollary 1 we also have* $q_m^{BB} e_m \geqslant q_h^{BB} e_h$*. Combining, we have* $\bar{q} e_\ell \geqslant \max\{q_m^{BB} e_m, q_h^{BB} e_h\}$*. Thus, the imposed condition ensures that* BB*-borrowers prefer to take on a small loan at the risk-free price, rather than a medium or large loan at the actuarially-fair price reflecting their risk.*

*(v)*

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2} e_h \geqslant [1-\delta(1-\alpha)]\, e_m,$$

*or, equivalently,* $q_h^{AA} e_h \geqslant q_m^{AB} e_m$*; this ensures that* AA*-borrowers are better off accepting their stage-1 offer.*

#### D.4.4 Beliefs

We classify out-of-equilibrium histories and beliefs based on the size of the stage-1 loan.

1. *Small Loans:* Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$.

    - Beliefs of $A$-lenders when the loan came from *the opposite* class are

    $$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

    - Beliefs of $A$-lenders when the loan came from *their* class are

    $$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

    - Beliefs of $B$-lenders when the loan came from *the opposite* class are

    $$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < \max\{q_h^{AA}, \hat{q}_\ell\}, \\ 1 & q \geqslant \max\{q_h^{AA}, \hat{q}_\ell\}, \end{cases}$$

    where $\hat{q}_\ell e_\ell + q_m^{BB}\left(e_m - e_\ell\right) = \bar{q} e_\ell$;

    - Beliefs of $B$-lenders when the loan came from *their* class are

    $$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}. \end{cases}$$

2. *Medium Loans:* Suppose the borrower has accepted a loan $(\varphi e_m, q)$.

    - Beliefs of $A$-lenders when the loan came from the opposite class are

    $$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

    - Beliefs of $A$-lenders when the loan came from their class are

    $$\Pr\left(\sigma^- = A\right) = \begin{cases} 0 & q < q_h^{AA}, \\ 1 & q \geqslant q_h^{AA}; \end{cases}$$

- Beliefs of B-lenders when the loan came from the opposite class are

$$\Pr\left(\sigma^{-}=A\right)=\begin{cases}0 & q<\max\{\hat{q}_{m2},q_h^{AA}\},\\ 1 & q\geqslant\max\{\hat{q}_{m2},q_h^{AA}\};\end{cases}$$

  where $\hat{q}_{m2}e_m+q_h^{BB}(e_h-e_m)=\bar{q}e_\ell$.

- Beliefs of B-lenders when the loan came from their class are

$$\Pr\left(\sigma^{-}=A\right)=\begin{cases}0 & q<q_h^{AA},\\ 1 & q\geqslant q_h^{AA}.\end{cases}$$

3. *Large Loans:* Suppose the borrower has accepted a loan $(\varphi e_h, q)$. Then lenders' beliefs in this scenario going forward are irrelevant.

4. *No loans*: All lenders believe $\Pr(\sigma^{-}=A)=0$.

### D.4.5 Strategies

**Borrowers' (off-path) Strategies in Stage 1** Strategies of borrowers upon observing offer(s) in the first stage:[50]

- $AA$-borrowers: Suppose a borrower observes at least $2N-1$ offers of $\left(\varphi e_\ell, q_h^{AA}\right)$.
  - Small loan: if one lender offers $(\varphi e_\ell, q)$ with $q\neq q_h^{AA}$, the borrower accepts that offer if and only if $q>q_h^{AA}$.
  - Medium loan: if one lender offers $(\varphi e_m, q)$, the borrower accepts if and only if $q>q_h^{AA}$.
  - Large loan: if one lender offers $(\varphi e_h, q)$, the borrower accepts if and only if $q>q_h^{AA}$.
- $AB$-borrowers with N offers: Suppose that the borrower receives $N-1$ offers of $\left(\varphi e_\ell, q_h^{AA}\right)$; that is, no B-lenders make offers, but one A-lender offers something off path.
  - Small loan: if one lender offers $(\varphi e_\ell, q)$ with $q\neq q_h^{AA}$, the borrower accepts if $q>q_m^{AB}$.

[50] The list below is restricted to histories with a single deviation.

- Medium loan: if one lender offers $(\varphi e_m, q)$, the borrower accepts if $q > \hat{q}_{m1}$ where

$$\hat{q}_{m1} e_m + q_h^{AB}(e_h - e_m) = q_m^{AB} e_m.$$

- Large loan: if one lender offers $(\varphi e_h, q)$, the borrower accepts if $q e_h \geqslant q_m^{AB} e_m$.

- AB-borrowers with $N+1$ offers: Suppose that the borrower receives $N$ offers of $\left(\varphi e_\ell, q_h^{AA}\right)$ and one additional offer; that is, one B-lender made an offer.
  - Small loan: if the deviating B-lender offers $(\varphi e_\ell, q)$, the borrower accepts if and only if $q \geqslant q_h^{AA}$.
  - Medium loan: if the deviating B-lender offers $(\varphi e_m, q)$, the borrower accepts if and only if $q \geqslant \min\{\hat{q}_{m1}, q_h^{AA}\}$.
  - Large loan: if the deviating B-lender offers $(\varphi e_h, q)$, the borrower accepts if $q e_h \geqslant q_m^{AB} e_m$.
- BB-borrowers: Suppose that a borrower observes just one offer.
  - Small loan: if the deviating B-lender offers $(\varphi e_\ell, q)$, the borrower accepts if and only if $q \geqslant \hat{q}_\ell$.
  - Medium loan: if the one lender offers $(\varphi e_m, q)$, the borrower accepts if $q \geqslant \hat{q}_{m2}$.
  - Large loan: if the one lender offers $(\varphi e_h, q)$, the borrower accepts if $q e_h \geqslant \bar{q} e_\ell$.

**Lenders' Strategies in Stage 2** We next describe lenders' strategies for any credit history in stage 2 (i.e. any information set of lenders in stage 2).

1. *Small loan* $(\varphi e_\ell, q)$ from stage 1
   - If the first-stage loan came from the other class of lenders, then A-lenders
     - offer $\left(\varphi\left(e_h - e_\ell\right), q_h^{AA}\right)$ if $q \geqslant q_h^{AA}$,
     - offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$ if $q < q_h^{AA}$.
   - If the first-stage loan came from their class of lenders, then A-lenders offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$.
   - If the first-stage loan came from the other class of lenders, then B-lenders
     - offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{AB}\right)$ if $q \geqslant \max\{q_h^{AA}, \hat{q}_\ell\}$,
     - offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{BB}\right)$ if $q < \max\{q_h^{AA}, \hat{q}_\ell\}$.

- If the first-stage loan came from their class of lenders, then B-lenders offer $\left(\varphi\left(e_m - e_\ell\right), q_m^{BB}\right)$.

2. *Medium loan* $(\varphi e_m, q)$ from stage 1

   - If the first-stage loan came from the other class of lenders, then A-lenders
     - offer $\left(\varphi\left(e_h - e_m\right), q_h^{AA}\right)$ if $q \geqslant q_h^{AA}$,
     - offer $\left(\varphi\left(e_h - e_m\right), q_h^{AB}\right)$ if $q < q_h^{AA}$.
   - If the first-stage loan came from their class of lenders, then A-lenders offer $\left(\varphi\left(e_h - e_m\right), q_h^{AB}\right)$.
   - If the first-stage loan came from the other class of lenders, then B-lenders
     - offer $\left(\varphi\left(e_h - e_m\right), q_h^{AB}\right)$ if $q \geqslant q_h^{AA}$,
     - offer $\left(\varphi\left(e_h - e_m\right), q_h^{BB}\right)$ if $q < q_h^{AA}$.
   - If the first-stage loan came from their class of lenders, then B-lenders offer $\left(\varphi\left(e_h - e_m\right), q_h^{BB}\right)$.

3. *Large loan* from stage 1

   - Lenders make no offers in stage 2 if they see a large loan from stage 1.

4. *No loan* in stage 1

   - A-lenders offer $\left(\varphi e_m, q_m^{AB}\right)$.
   - B-lenders offer $(\varphi e_\ell, \bar{q})$.

#### D.4.6 Incentives

We now verify that given Assumption 1 and Condition 1, the strategies and beliefs described above constitute an equilibrium.

**Borrowers' Stage 1 Deviations.** Consider first possible deviations by borrowers in stage 1.

1. An AA-borrower could reject both stage 1 offers. Accepting is optimal as long as

$$q_h^{AA} e_h \geqslant q_m^{AB} e_m, \tag{13}$$

   which is ensured by Condition 1 (v).

2. An $AB$-borrower could accept a stage 1 offer. Rejecting is optimal as long as

$$q_m^{AB} e_m \geqslant q_h^{AA} e_\ell + q_m^{AB}(e_m - e_\ell), \tag{14}$$

or $q_m^{AB} \geqslant q_h^{AA}$, which is ensured by Condition 1 (i).

**Lenders' Stage 1 Deviations.** Since we have already specified the borrowers' and lenders' strategies following stage-1 deviation offers, all that remains is to verify that it is not optimal for lenders to deviate in stage 1.

- $A$-lenders: do they want to offer anything other than $(\varphi e_\ell, q_h^{AA})$?

  1. *Small loans*

     – An offer $(\varphi e_\ell, q)$ with $q < q_h^{AA}$ will not be accepted by anyone.

     – An offer $(\varphi e_\ell, q)$ with $q_h^{AA} < q < q_m^{AB}$ will be accepted only by AA borrowers, who will then be topped up to a large loan in stage 2, making this loan an expected loser.

     – An offer $(\varphi e_\ell, q)$ with $q > q_m^{AB}$ is accepted by AA and AB types, and thus is of course an expected loser. The AA types will be topped up to a large loan, and the AB types to a medium.

  2. *Medium loans*

     – An offer $(\varphi e_\ell, q)$ with $q < \min\{q_h^{AA}, \hat{q}_{m1}\}$ is not accepted by anyone. An $AA$ borrower would obtain payoff $q\varphi e_m + q_h^{AB}\varphi(e_h - e_m)$ which is smaller than her equilibrium payoff. An $AB$ borrower would obtain $q\varphi e_m + q_h^{AB}(e_h - e_m)$ which, given the definition of $\hat{q}_{m1}$, is smaller than her equilibrium payoff as well.

     – An offer with $\min\{\hat{q}_{m1}, q_h^{AA}\} < q < \max\{\hat{q}_{m1}, q_h^{AA}\}$ is accepted by only AB borrowers (if $\hat{q}_{m1} < q_h^{AA}$) or only by AA borrowers (if $q_h^{AA} < \hat{q}_{m1}$). In either case, the accepted offer yields negative expected profits for the lender. If only $AB$ borrowers accept, the lender expects to earn $-q + q_h^{AB}$ (per dollar of face value). From the definition of $\hat{q}_{m1}$, $(\hat{q}_{m1} - q_h^{AB})e_m = q_m^{AB}e_m - q_h^{AB}e_h \geqslant 0$ where the inequality follows from Condition 1 (ii). Hence, $q \geqslant q_h^{AB}$ so the offer yields negative expected profits. If only $AA$ borrowers accept, the lender expects to earn $-q + q_h^{AA}$ (per dollar of face value), which necessarily earns negative expected profits.

– An offer with $q \geqslant \max\left\{\hat{q}_{m1}, q_h^{AA}\right\}$ attracts both $AA$ and $AB$ borrowers and so necessarily loses money on both $AA$ and $AB$ borrowers using the previous argument.

3. *Large loans*

   – An offer with $(\varphi e_h, q)$ is accepted by $AA$ borrowers if and only if $q \geqslant q_h^{AA}$ in which case the offer loses money from $AA$ borrowers.

   – An offer with $(\varphi e_h, q)$ is accepted by $AB$ borrowers if and only if $q e_h \geqslant q_m^{AB} e_m$. Condition 1 (ii) implies $q_m^{AB} e_m \geqslant q_h^{AB} e_h$ so that $q \geqslant q_h^{AB}$. As a result, the offer loses money from $AB$ borrowers.

   – Since any offer $(\varphi e_h, q)$ that is accepted loses money on all types that accept it, $A$ lenders cannot profit by offering large loans in stage 1.

- B-lenders: do they want to offer anything in stage 1?

1. *Small loans*

   – A loan with $q < \min\left\{\hat{q}_\ell, q_h^{AA}\right\}$ is not accepted by anyone.

   – An offer with $\min\{q_h^{AA}, \hat{q}_\ell\} < q < \max\{q_h^{AA}, \hat{q}_\ell\}$ is accepted by only $BB$ borrowers (if $\hat{q}_\ell < q_h^{AA}$) or only by $AB$ borrowers (if $q_h^{AA} < \hat{q}_\ell$). In either case the accepted offer yields negative profits. If only $BB$ borrowers accept, the lender expects to earn $-q + q_m^{BB}$ (per dollar of face value). Using Condition 1 (iv), the definition of $\hat{q}_\ell$ implies $\hat{q}_\ell \geqslant q_m^{BB}$, so the offer is unprofitable. If only $AB$ borrowers accept, the lender expects to earn $-q + q_h^{AB}$ but $q \geqslant q_h^{AA} \geqslant q_h^{AB}$ so the offer is unprofitable.

   – An offer with $q \geqslant \max\{q_h^{AA}, \hat{q}_\ell\}$ is accepted by both $AB$ and $BB$ borrowers and so necessarily loses money on both $AA$ and $AB$ borrowers using the previous argument.

2. *Medium loans*

   – A loan with $q < \hat{q}_{m2}$ and $q < \min\left\{\hat{q}_{m1}, q_h^{AA}\right\}$ is not accepted by any borrowers.

   – A loan with $q \geqslant \hat{q}_{m2}$ is accepted by $BB$ borrowers and necessarily loses money on $BB$ borrowers. Expected profits (per dollar face value) is $-q + q_h^{BB}$. From the definition of $\hat{q}_{m2}$, $(\hat{q}_{m2} - q_h^{BB})e_m = \bar{q}e_\ell - q_h^{BB}e_h$. Condition 1 (iv) implies $\bar{q}e_\ell \geqslant q_h^{BB}e_h$ and hence this loan loses money.

   – A loan with $q \geqslant \min\left\{\hat{q}_{m1}, q_h^{AA}\right\}$ is accepted by $AB$ borrowers and necessarily loses money on $AB$ borrowers. Expected profits (per dollar face value) is $-q +$

$q_h^{AB}$. The definition of $\hat{q}_{m1}$ and Condition 1 (ii) immediately implies $\hat{q}_{m1} > q_h^{AB}$. Since $q_h^{AA} \geqslant q_h^{AB}$, it follows that $q_h^{AB} \leqslant \min\{\hat{q}_{m1}, q_h^{AA}\}$ so the offer necessarily loses money.

3. Large loans

   – An offer with $(\varphi e_h, q)$ is accepted by AB borrowers if and only if $qe_h \geqslant q_m^{AB}$. Condition 1 (ii) implies $q_m^{AB} e_m \geqslant q_h^{AB} e_h$ so that $q \geqslant q_h^{AB}$. As a result, the offer loses money from AB borrowers.

   – An offer with $(\varphi e_h, q)$ is accepted by BB borrowers if and only if $qe_h \geqslant \bar{q}e_\ell$. By Condition 1 (iv), $\bar{q}e_\ell \geqslant q_h^{BB} e_h$ so that $q \geqslant q_h^{BB}$ and hence this offer loses money.

   – Since any offer $(\varphi e_h, q)$ that is accepted loses money on all types that accept it, B lenders cannot profit by offering large loans in stage 1.

This completes our characterization of this equilibrium.

## D.5 Equilibrium Outcome 2: $\mathtt{lmh}$ with Cross-Subsidization

We construct an equilibrium with terminal loans $\varphi e_\ell$, $\varphi e_m$, $\varphi e_h$ for BB-, AB-, and AA-borrowers, respectively, in which both AA- and AB-borrowers accept loans in the first stage. We then establish a set of sufficient conditions for it to be an equilibrium. We construct the equilibrium as follows.

### D.5.1 On-Path Actions

Stage 1:

– G-class A-lenders offer $\varphi e_\ell$ at $q^A = \Pr(AA|A) q_h^{AA} + \Pr(AB|A) q_m^{AB}$;

– F-class A-lenders offer $\varphi e_\ell$ at $q_m^{AB}$;

– B-lenders offer nothing;

– AA-borrowers accept a loan from a G-class lender;

– AB-borrowers accept a loan from an A-lender.

Stage 2:

– F-class A-lenders who observe stage-1 loan of $(\varphi e_\ell, q^A)$ from a G-class lender offer $\varphi(e_h - e_\ell)$ at $q_h^{AA}$;

– Accepted-class $A$-lenders and $B$-lenders who see stage-1 loan of $(\varphi e_\ell, q^A)$ or $(\varphi e_\ell, q_m^{AB})$ offer $\varphi(e_m - e_\ell)$ at $q_m^{AB}$;

– $B$-lender who sees no loan offers $\varphi e_\ell$ at $q = \bar{q}$.

#### D.5.2 Equilibrium Payoffs

The payoffs to borrowers in equilibrium are as follows:

- $AA$-borrowers: $\varphi e_\ell q^A + \varphi(e_h - e_\ell) q_h^{AA}$;
- $AB$-borrowers with $A$ signals from $G$-class lenders: $\varphi e_\ell q^A + (e_m - e_\ell) q_m^{AB}$;
- $AB$-borrowers with $A$ signals from $F$-class lenders: $\varphi e_m q_m^{AB}$;
- $BB$-borrowers: $\varphi e_\ell \overline{q}$.

#### D.5.3 Equilibrium Conditions

Before we proceed with construction of beliefs and (off-path) strategies, we state necessary conditions on the model parameters so that our constructed equilibrium candidate is indeed an equilibrium. We later show that these conditions together with Assumption 1 are sufficient to ensure that relevant incentive constraints are satisfied.

**Condition 2** *Suppose that the model parameters satisfy*

*(i)*

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2} > 1 - \delta(1-\alpha),$$

*or, equivalently,* $q_h^{AA} > q_m^{AB}$. *Note that this condition is the reverse of Condition 1 (i) and is a sufficient condition for Condition 1 (v).*

*(ii)*

$$[1 - \delta(1-\alpha)](e_m - e_\ell) \geqslant \delta\alpha(e_h - e_\ell),$$

*or, equivalently,* $q_m^{AB}(e_m - e_\ell) \geqslant q_h^{AB}(e_h - e_\ell)$. *Note that this is the same condition as Condition 1 (ii).*

*(iii)*

$$\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2}(e_h - e_\ell) \geqslant \left[1 - \delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2 + (1-\alpha)(1+\rho)^2}\right](e_m - e_\ell),$$

*or, equivalently,* $q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AA}(e_m - e_\ell)$*. Note that this is the same condition as Condition 1 (iii).*

*(iv)*

$$e_\ell \geqslant \left(1 - \delta + \delta \frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2 + (1-\alpha)(1+\rho)^2}\right) e_m,$$

*or, equivalently,* $\bar{q} e_\ell \geqslant q_m^{BB} e_m$*. Note that by Assumption 1 (ii) and Corollary 1 we also have* $q_m^{BB} e_m \geqslant q_h^{BB} e_h$*. Combining, we have* $\bar{q} e_\ell \geqslant \max\left\{q_m^{BB} e_m, q_h^{BB} e_h\right\}$*. Note that this is the same condition as Condition 1 (iv).*

### D.5.4 Beliefs

We classify out-of-equilibrium histories and beliefs based on the size of the stage-1 loan and the class of the lenders who made the loan.

1. *Small Loans*

   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from a G-class lender.
     - Beliefs of G-class lenders when $\sigma^G = A$ are

       $$\Pr(\sigma^F = A) = \begin{cases} \Pr(AA|A) & \text{if } q \geqslant q^A, \\ 0 & \text{if } q < q^A; \end{cases}$$

     - Beliefs of G-class lenders when $\sigma^G = B$ are

       $$\Pr(\sigma^F = A) = \begin{cases} \Pr(AB|A) & \text{if } q > q_m^{AB}, \\ 0 & \text{if } q \leqslant q_m^{AB}; \end{cases}$$

     - Beliefs of F-class lenders are

       $$\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant q^A, \\ 0 & \text{if } q < q^A. \end{cases}$$

   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from an F-class lender.
     - Beliefs of G-class lenders are

       $$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^{AB}, \\ 0 & \text{if } q < q_m^{AB}; \end{cases}$$

– Beliefs of F-class lenders when $\sigma^F = A$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AA|A) & \text{if } q > q^A, \\ 0 & \text{if } q \leqslant q^A; \end{cases}$$

– Beliefs of F-class lenders when $\sigma^F = B$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AB|B) & \text{if } q \geqslant q^A, \\ 0 & \text{if } q < q^A. \end{cases}$$

2. *Medium Loans*

Define

$$\begin{aligned}
\tilde{q}_{m1} :& \quad \tilde{q}_{m1} e_m + q_h^{AA}(e_h - e_m) = q^A e_\ell + q_h^{AA}(e_h - e_\ell), & (15)\\
\tilde{q}_{m2} :& \quad \tilde{q}_{m2} e_m + q_h^{AA}(e_h - e_m) = q_m^{AB} e_m, & (16)\\
\tilde{q}_{m3} :& \quad \tilde{q}_{m3} e_m + q_h^{AB}(e_h - e_m) = q^A e_\ell + q_m^{AB}(e_m - e_\ell), & (17)\\
\tilde{q}_{m4} :& \quad \tilde{q}_{m4} e_m + q_h^{AB}(e_h - e_m) = q_m^{AB} e_m, & (18)\\
\tilde{q}_{m5} :& \quad \tilde{q}_{m5} e_m + q_h^{AA}(e_h - e_m) = q^A e_\ell + q_m^{AB}(e_m - e_\ell). & (19)
\end{aligned}$$

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from a G-class lender.
  – Beliefs of G-class lenders when $\sigma^G = A$ are

$$\Pr(\sigma^F = A) = \begin{cases} \Pr(AA|A) & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

  – Beliefs of G-class lenders when $\sigma^G = B$ are

$$\Pr(\sigma^F = A) = \begin{cases} \Pr(AB|B) & \text{if } q > \tilde{q}_{m2}, \\ 0 & \text{if } q \leqslant \tilde{q}_{m2}; \end{cases}$$

  – Beliefs of F-class lenders when $\sigma^F = A$ are

$$\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

– Beliefs of F-class lenders when $\sigma^F = B$ are

$$\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m3}, \\ 0 & \text{if } q < \tilde{q}_{m3}. \end{cases}$$

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from an F-class lender.
  – Beliefs of G-class lenders when $\sigma^G = A$ are

$$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

  – Beliefs of G-class lenders when $\sigma^G = B$ are

$$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant \tilde{q}_{m4}, \\ 0 & \text{if } q < \tilde{q}_{m4}; \end{cases}$$

  – Beliefs of F-class lenders when $\sigma^F = A$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AA|A) & \text{if } q \geqslant \tilde{q}_{m1}, \\ 0 & \text{if } q < \tilde{q}_{m1}; \end{cases}$$

  – Beliefs of F-class lenders when $\sigma^F = B$ are

$$\Pr(\sigma^G = A) = \begin{cases} \Pr(AB|B) & \text{if } q \geqslant \tilde{q}_{m5}, \\ 0 & \text{if } q < \tilde{q}_{m5}. \end{cases}$$

3. *Large Loans*

- Suppose the borrower has accepted a loan $(\varphi e_h, q)$. Then lenders' beliefs in this scenario going forward are irrelevant.

4. *No Loans*

$\Pr(\sigma^- = A) = 0.$

### D.5.5 Strategies

We now describe strategies beginning with borrowers' strategies in Stage 1 given any sets of offers arising from a single lender's deviation.

**Borrowers' (off-path) Strategies in Stage 1**

Define

$$\tilde{q}_{h1}: \quad \tilde{q}_{h1} e_h = q^A e_\ell + q_h^{AA}(e_h - e_\ell), \tag{20}$$

$$\tilde{q}_{h2}: \quad \tilde{q}_{h2} e_h = q_m^{AB} e_m, \tag{21}$$

$$\tilde{q}_{h3}: \quad \tilde{q}_{h3} e_h = q^A e_\ell + q_m^{AB}(e_m - e_\ell). \tag{22}$$

1. *AA-Borrowers*

   Suppose a borrower observes at least $(N-1)$ offers $(\varphi e_\ell, q^A)$ from G-class lenders and at least $(N-1)$ offers $(\varphi e_\ell, q_m^{AB})$ from F-class lenders.

   - Suppose one lender offers $(\varphi e_\ell, q)$ where $q$ is not prescribed by the equilibrium. The borrower's strategy is to accept the deviation offer if and only if $q > q^A$.
   - Suppose one lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q \geqslant \tilde{q}_{m1}$.
   - Suppose one lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q \geqslant \tilde{q}_{h1}$.

2. *AB-Borrowers with A signals from G-class lenders*

   Suppose a borrower observes $(N-1)$ offers $(\varphi e_\ell, q^A)$ from G-class lenders and no offers from F-class lenders.

   - Suppose one G-class lender offers $(\varphi e_\ell, q)$ where $q \neq q^A$. The borrower's strategy is to accept the deviation offer if and only if $q > q^A$.
   - Suppose one G-class lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{m3}$.
   - Suppose one G-class lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h3}$.

   Suppose a borrower observes N offers $(\varphi e_\ell, q^A)$ from G-class lenders and one offer from an F-class lender.

   - If the F-class lender's offer is $(\varphi e_\ell, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \max\{q_m^{AB}, \tilde{q}_{l1}\}$ where

$$\tilde{q}_{\ell 1} e_\ell + q_h^{AA}(e_h - e_\ell) = q^A e_\ell + q_m^{AB}(e_m - e_\ell).$$

- If the F-class lender's offer is $(\varphi e_m, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \max\{\tilde{q}_{m1}, \tilde{q}_{m5}\}$.[51]
- If the F-class lender's offer is $(\varphi e_h, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h3}$.

3. *AB-Borrowers with A signals from F-class lenders*

Suppose a borrower observes one offer from a G-class lender and N offers $(\varphi e_\ell, q_m^{AB})$ from F-class lenders.

- If the G-class lender offers $(\varphi e_\ell, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_m^{AB}$.
- If the G-class lender offers $(\varphi e_m, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \min\{\tilde{q}_{m4}, \max\{\tilde{q}_{m2}, \tilde{q}_{m1}\}\}$.
- If the G-class lender offers $(\varphi e_h, q)$, the borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h2}$.

Suppose a borrower observes no offers from G-class lenders and $(N-1)$ offers $(\varphi e_\ell, q_m^{AB})$ from F-class lenders.

- Suppose one F-class lender offers $(\varphi e_\ell, q)$ where $q \neq q_m^{AB}$. The borrower's strategy is to accept the deviation offer if and only if $q > q_m^{AB}$.
- Suppose one F-class lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \min\{\tilde{q}_{m1}, \tilde{q}_{m4}\}$.[52]
- Suppose one F-class lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{h2}$.

4. *BB-Borrowers*

Suppose a borrower observes at most one offer.

- Suppose one lender offers $(\varphi e_\ell, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \tilde{q}_{l2}$, where

$$\tilde{q}_{\ell 2} e_\ell + q_m^{BB}(e_m - e_\ell) = \bar{q} e_\ell.$$

[51] Note, $\tilde{q}_{m5}$ is the price the borrower would accept if he can obtain an AA-priced loan from the G-class lenders. However, a price $\tilde{q}_{m1}$ is needed to ensure G-class lenders believe F-class lenders have an A signal.

[52] $\tilde{q}_{m1}$ is the price needed to "fool" F-class lenders about the G-class lenders' signal. Note that $\tilde{q}_{m1} \geqslant \tilde{q}_{m2}$ so that if the borrower has "fooled" F-class lenders such a loan is worth accepting. $\tilde{q}_{m4}$ is the price that justifies a BA borrower accepting the loan without "fooling" F-class lenders. In either case, the BA borrower would want to accept this loan.

- Suppose one lender offers $(\varphi e_m, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > \min\{\tilde{q}_{m6}, \max\{\tilde{q}_{m4}, \tilde{q}_{m7}\}\}$, where

$$\tilde{q}_{m6} e_m + q_h^{BB}(e_h - e_m) = \bar{q} e_\ell,$$
$$\tilde{q}_{m7} e_m + q_h^{AB}(e_h - e_m) = \bar{q} e_\ell.$$

- Suppose one lender offers $(\varphi e_h, q)$. The borrower's strategy is to accept the deviation offer if and only if $q > e_\ell / e_h$.

**Lenders' Strategies in Stage 2** We next describe lenders' strategies for any credit history in stage 2 (i.e. any information set of lenders in stage 2).

1. *Small Loans*

   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from a G-class lender.
     - When G-class lenders have signal $\sigma^G = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - When G-class lenders have signal $\sigma^G = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
     - Suppose F-class lenders have signal $\sigma^F = A$. If $q \geqslant q^A$, they offer $(\varphi(e_h - e_\ell), q_h^{AA})$ and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - Suppose F-class lenders have signal $\sigma^F = B$. If $q \geqslant q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$, and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from an F-class lender.
     - Suppose G-class lenders have signal $\sigma^G = A$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_h - e_\ell), q_h^{AA})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - Suppose G-class lenders have signal $\sigma^G = B$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
     - When F-class lenders have signal $\sigma^F = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
     - When F-class lenders have signal $\sigma^F = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.

2. *Medium Loans*

   - Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from a G-class lender.
     - When G-class lenders have signal $\sigma^G = A$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.
     - When G-class lenders have signal $\sigma^G = B$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.
     - Suppose F-class lenders have signal $\sigma^F = A$. If $q \geqslant \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AA})$ and if $q < \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.

- Suppose F-class lenders have signal $\sigma^F = B$. If $q \geqslant \tilde{q}_{m3}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$, and if $q < \tilde{q}_{m3}$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from an F-class lender.
  - Suppose G-class lenders have signal $\sigma^G = A$. If $q \geqslant \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AA})$ and if $q < \tilde{q}_{m1}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.
  - Suppose G-class lenders have signal $\sigma^G = B$. If $q \geqslant \tilde{q}_{m4}$, they offer $(\varphi(e_h - e_m), q_h^{AB})$ and if $q < \tilde{q}_{m4}$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.
  - When F-class lenders have signal $\sigma^F = A$, they offer $(\varphi(e_h - e_m), q_h^{AB})$.
  - When F-class lenders have signal $\sigma^F = B$, they offer $(\varphi(e_h - e_m), q_h^{BB})$.

3. *Large Loans*

   If the borrower accepted a loan $(\varphi e_h, q)$ from any lender, all lenders offer $(0, 0)$.

4. *No Loans*

   If a lender has signal $\sigma = A$, they offer $(\varphi e_m, q_m^{AB})$. If a lender has signal $\sigma = B$, they offer $(\varphi e_\ell, \bar{q})$.

**Borrowers' Strategies in Stage 2** For any history and any set of loan offers in Stage 2, the borrower accepts the loan with the highest $qx$.

**Features of pricing thresholds** Under our above assumptions, it is useful to note a few relationships between the various thresholds characterizing the off-equilibrium-path beliefs.

**Lemma 1** *Suppose that Assumption 1 and Condition 2 are satisfied. Then the thresholds constructed in (15)-(22) satisfy the following conditions:*

*(i)* $\tilde{q}_{m1} > \tilde{q}_{m3} > \tilde{q}_{m4} > \tilde{q}_{m2}$,

*(ii)* $\tilde{q}_{m3} > \tilde{q}_{m5} > \tilde{q}_{m2}$,

*(iii)* $\tilde{q}_{m1} > \Pr(AA|A) q_h^{AA} + \Pr(AB|A) q_h^{AB}$,

*(iv)* $\tilde{q}_{h1} \geqslant \tilde{q}_{h3}$.

**Proof:**

(i) Inequality $\tilde{q}_{m4} > \tilde{q}_{m2}$ follows from $q_h^{AA} > q_h^{AB}$. Inequality $\tilde{q}_{m3} > \tilde{q}_{m4}$ follows from Condition 2 (i) and the definition of $q^A = \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}$. To show that $\tilde{q}_{m1} > \tilde{q}_{m3}$, rewrite (15) and (17) as

$$\tilde{q}_{m1}e_m = q^A e_\ell + q_h^{AA}(e_m - e_\ell),$$
$$\tilde{q}_{m3}e_m = q^A e_\ell + q_m^{AB}(e_m - e_\ell) - q_h^{AB}(e_h - e_m).$$

Using Condition 2 (i), we have $\tilde{q}_{m1} > \tilde{q}_{m3}$.

(ii) Inequality $\tilde{q}_{m3} > \tilde{q}_{m5}$ follows from Condition 2 (i). Inequality $\tilde{q}_{m5} > \tilde{q}_{m2}$ follows from Condition 2 (i) and the definition of $q^A$.

(iii) Using (15),

$$\tilde{q}_{m1}e_m + q_h^{AA}(e_h - e_m) = q^A e_\ell + q_h^{AA}(e_h - e_\ell),$$
$$\tilde{q}_{m1}e_m = q^A e_\ell + q_h^{AA}(e_m - e_\ell),$$
$$\tilde{q}_{m1}e_m = \left[\Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}\right] e_\ell + q_h^{AA}(e_m - e_\ell).$$

Moreover, $\Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB} < q_h^{AA}$ by Condition 2 (i). Hence, $\tilde{q}_{m1} > \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_h^{AB}$.

(iv) Follows from Condition 2 (ii). □

### D.5.6 Incentives

We now verify that given Assumption 1 and Condition 2, the strategies and beliefs described above constitute an equilibrium.

**Borrowers' Stage-1 Deviations** Consider first possible deviations by borrowers in stage 1. We will show that part (i) of Condition 2 together with Assumption 1 preclude them.

1. An AA-borrower could reject the stage-1 loan. Accepting is optimal as long as

$$q^A e_\ell + q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AB} e_m. \tag{23}$$

Since $q^A = \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}$, $\Pr(AA|A) > \Pr(AB|A)$, and $q_h^{AA} > q_m^{AB}$ by

Condition 2 (i), we have

$$q^A e_\ell + q_h^{AA}(e_h - e_\ell) > q_m^{AB} e_\ell + q_m^{AB}(e_h - e_\ell) = q_m^{AB} e_h > q_m^{AB} e_m.$$

Thus (23) holds.

2. An $AB$-borrower with a stage-1 offer from F-class lenders could reject it and either obtain $(\varphi e_m, q_m^{AB})$ in stage 2 or obtain $(\varphi e_\ell, \bar{q})$ in stage 2. Accepting is optimal as long as

$$q_m^{AB} e_m \geqslant \bar{q} e_\ell. \tag{24}$$

Note, since $q_m^{AB} = (1 - \delta(1-\alpha))\bar{q}$, when $(1 - \delta(1-\alpha))e_m > e_\ell$ as in Assumption 1 (i), this condition is satisfied.

3. An $AB$-borrower with a stage-1 offer from G-class lenders could reject it. Accepting is optimal as long as

$$q^A e_\ell + q_m^{AB}(e_m - e_\ell) \geqslant \max\{q_m^{AB} e_m, \bar{q} e_\ell\}. \tag{25}$$

Under (24), this incentive constraint reduces to $q^A \geqslant q_m^{AB}$ or $q_h^{AA} \geqslant q_m^{AB}$, which holds by Condition 2 (i).

Off-equilibrium path strategies specified above are constructed to be optimal for the borrower given prescribed continuation strategies.

**Lenders' Stage-2 Deviations.** We now analyze possible deviations of lenders in stage 2 and show that they are not profitable given Condition 2 (ii)-(iv).

1. *Small Loans*
   - Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from a G-class lender.
     - When G-class lenders have signal $\sigma^G = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
       * Any top-up to medium loan with $q \neq q_m^{AB}$ is either not accepted or unprofitable. Note, loans with $q > q_m^{AB}$ would be accepted with strictly positive probability but would earn negative expected profits. The reason is that for such prices with $q < q_h^{AA}$, only AB-borrowers accept making the loan unprofitable and at or above $q_h^{AA}$, both $AA$ and $AB$'s may accept making the loan unprofitable. Moreover, since borrowers in stage 2 accept the loan with the large $qx$, loan offers $(\varphi(e_m - e_\ell), q)$ with $q < q_m^{AB}$ are not accepted.

* Any weakly profitable loan with a top up to $e_h$—and hence priced at most $q_h^{AB}$—is not accepted under Condition 2 (ii) which implies

$$q_m^{AB}(e_m - e_\ell) \geqslant q_h^{AB}(e_h - e_\ell).$$

– When G-class lenders have signal $\sigma^G = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$;

  * Any top-up to a medium loan with $q \neq q_m^{BB}$ is either not accepted or unprofitable.
  * Any weakly profitable loan with a top up to $e_h$ is not accepted under Condition 2 (ii) which also implies

$$q_m^{BB}(e_m - e_\ell) \geqslant q_h^{BB}(e_h - e_\ell).$$

– Suppose F-class lenders have signal $\sigma^F = A$. If $q \geqslant q^A$, they offer $(\varphi(e_h - e_\ell), q_h^{AA})$ and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.

  * When $q \geqslant q^A$, any top-up to a large loan with $q \neq q_h^{AA}$ is either not accepted or unprofitable. Under Condition 2 (iii),

$$q_h^{AA}(e_h - e_\ell) \geqslant q_m^{AA}(e_m - e_\ell),$$

  which implies any top-up to a weakly profitable medium loan is not accepted.
  * When $q < q^A$, any top-up to medium loan with $q \neq q_m^{AB}$ is either not accepted or unprofitable. Any weakly profitable loan with a top up to $e_h$ again is not accepted under Condition 2 (ii).

– Suppose F-class lenders have signal $\sigma^F = B$. If $q \geqslant q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$, and if $q < q^A$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.

  * When $q \geqslant q^A$, any top-up to a medium loan $q \neq q_m^{AB}$ is either not accepted or unprofitable. Condition 2 (ii) implies that any weakly profitable top-up to a large loan is not accepted.
  * When $q < q^A$, any top-up to medium loan with $q \neq q_m^{BB}$ is either not accepted or unprofitable. Any weakly profitable loan with a top up to $e_h$ again is not accepted under Condition 2 (ii).

• Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ from an F-class lender.

– Suppose G-class lenders have signal $\sigma^G = A$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_h -$

$e_\ell), q_h^{AA})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.

  - Suppose G-class lenders have signal $\sigma^G = B$. If $q \geqslant q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$ and if $q < q_m^{AB}$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.
  - When F-class lenders have signal $\sigma^F = A$, they offer $(\varphi(e_m - e_\ell), q_m^{AB})$.
  - When F-class lenders have signal $\sigma^F = B$, they offer $(\varphi(e_m - e_\ell), q_m^{BB})$.

  These strategies are optimal under Conditions 2 (ii)-(iii). The arguments are analogous to those for when the borrower accepted a small loan from a G-class lender.

2. *Medium Loans*

   Medium loans only occur off the equilibrium path. Note that all strategies in these histories involve topping up the borrower to a large loan that earns zero profits under the specified beliefs. Therefore, any deviation would either not be accepted or be unprofitable.

3. *Large Loans*

   Trivially, offering any further loans would result in non-repayment and negative profits.

4. *No Loans*

   - If a lender has signal $\sigma = A$, they offer $(\varphi e_m, q_m^{AB})$;
     - Any medium loan with $q \neq q_m^{AB}$ is unprofitable or not accepted.
     - Under Condition 2 (ii), any weakly profitable large loan is not accepted.
     - Under Assumption 1, any weakly profitable small loan is not accepted.
   - If a lender has signal $\sigma = B$, they offer $(\varphi e_\ell, 1)$.
     - Any small loan with $q \neq 1$ is unprofitable or not accepted.
     - Under Condition 2 (iv), $\bar{q}\varphi e_\ell \geqslant q_m^{BB}\varphi e_m$, so that weakly profitable medium loans are not accepted.
     - Condition 2 (ii) and (iv) together imply any weakly profitable large loan is not accepted.

**Lenders' Stage-1 Deviations.** We now present conditions such that for each signal, lenders' stage-1 strategies are optimal.

1. G-class lender with an *A* Signal.

- Any small loan at $q \neq q^A$ is either unprofitable or not accepted. At prices below $q^A$, borrowers do not accept. At prices above $q^A$, the offer would be accepted by both $AA$s and $AB$s. Such a loan necessarily loses money since it will be topped up in stage 2 in the same way as happens on the equilibrium path. Since the lender breaks even in equilibrium, offering higher prices must lose money.
- From Lemma 1 $\tilde{q}_{m1} > \tilde{q}_{m3}$. As a result, medium loans with $q \leqslant \tilde{q}_{m3}$ are not accepted. Medium loans with $q \in (\tilde{q}_{m3}, \tilde{q}_{m1})$ are accepted only by $AB$-borrowers. Condition 2 (ii) and $q^A > q_m^{AB}$ imply $q^A e_\ell + q_m^{AB}(e_m - e_\ell) > q_h^{AB} e_h$ or $\tilde{q}_{m3} > q_h^{AB}$. As a result, such a loan is accepted and yields negative expected profits. Medium loans with $q \geqslant \tilde{q}_{m1}$ are accepted by both $AA$ and $AB$-borrowers. From Lemma 1, $\tilde{q}_{m1} > \Pr(AA|A) q_h^{AA} + \Pr(AB|A) q_h^{AB}$. As a result, any such loan must yield negative expected profits.
- From Lemma 1, large loans with $q < \tilde{q}_{h3}$ are not accepted. Large loans with $q \in [\tilde{q}_{h3}, \tilde{q}_{h1}]$ are accepted by only $AB$-borrowers. Since $\tilde{q}_{h3} > q_h^{AB}$ (same condition as for medium loans), such loans must be unprofitable. Large loans with $q \geqslant \tilde{q}_{h1}$ are accepted by both $AA$ and $AB$-borrowers. Analogous to medium loans, $\tilde{q}_{h1} > \Pr(AA|A) q_h^{AA} + \Pr(AB|A) q_h^{AB}$ so that any such loan must yield negative expected profits.

2. F-class lender with an $A$ Signal.

   - Any small loan at $q \neq q_m^{AB}$ is either unprofitable or not accepted. At prices below $q_m^{AB}$, borrowers do not accept. At prices above $q_m^{AB}$ but below $q^A$, only $AB$-borrowers accept and the loan is unprofitable. At prices above $q^A$, both $AA$- and $BA$-borrowers will accept and the loan is unprofitable.
   - From Lemma 1 $\tilde{q}_{m1} > \tilde{q}_{m4}$. Medium loans with $q \leqslant \tilde{q}_{m4}$ are not accepted. Medium loans with $q \in (\tilde{q}_{m4}, \tilde{q}_{m1})$ are accepted only by $AB$-borrowers. Condition 2 (ii) implies $\tilde{q}_{m4} > q_h^{AB}$ so that such loans are unprofitable. Medium loans with $q \geqslant \tilde{q}_{m1}$ are unprofitable as described when a G-class lender with an $A$ signal offers such a loan.
   - From Lemma 1, large loans with $q < \tilde{q}_{h2}$ are not accepted. Large loans with $q \in (\tilde{q}_{h2}, \tilde{q}_{h1})$ are accepted by only $AB$-borrowers. Condition 2 (ii) implies $\tilde{q}_{h2} > q_h^{AB}$ so that such loans must be unprofitable. Large loans with $q \geqslant \tilde{q}_{h1}$ are accepted by both $AA$- and $AB$-borrowers. Analogous to large loans, $\tilde{q}_{h1} > \Pr(AA|A) q_h^{AA} + \Pr(AB|A) q_h^{AB}$ so that any such loan must yield negative expected profits.

3. Lender with a B Signal.

   - Any small loan with $q < q_{l2}$ is accepted by no borrowers. Any small loan with $q \in [q_{l2}, q_m^{AB})$ (if such an interval exists) is accepted only by a BB-borrower and must lose money (whether the BB type tops up to a large or a medium loan) because $q_{l2} > q_m^{BB}$, which follows from Condition 2 (iv). For any $q > q_m^{AB}$, the loan is accepted by BB-borrowers only or by AB- and BB-borrowers and these borrowers obtain a top up to a medium loan (or more) in stage 2. Such loans must earn negative profits.
   - Medium loans attract BB-borrowers if $q > \min\{\tilde{q}_{m4}, \tilde{q}_{m6}\}$. The lowest price loans that attract AB-borrowers satisfy $q > \tilde{q}_{m4}$. If the loan only attracts BB-borrowers, since $\min\{\tilde{q}_{m4}, \tilde{q}_{m6}\} > q_h^{BB}$, the loan must earn negative profits. If the loan attracts both BB and AB-borrowers, since $\tilde{q}_{m4} > q_h^{AB} > \Pr(AB|B)q_h^{AB} + \Pr(BB|B)q_h^{BB}$, the loan must earn negative profits.
   - Since $e_\ell > q_h^{BB} e_h$, large loans that attract only BB-borrowers must be unprofitable. Large loans must offer $q > \tilde{q}_{h2}$ to attract AB-borrowers. Since $\tilde{q}_{h2} > q_h^{AB}$, the loan earns negative profits (it attracts AB- and BB-borrowers).

This completes our characterization of this equilibrium.

## D.6 Equilibrium Outcome 3: No Credit-History Building

We construct an equilibrium with no information aggregation. No offers are made (or accepted) in this equilibrium in stage 1. We then establish a set of sufficient conditions for it to be an equilibrium. We construct the equilibrium as follows.

### D.6.1 On-Path Actions

- Stage 1:
  - Lenders make no offers (borrowers have no available action).
- Stage 2:
  - G-class A-lenders offer $\varphi e_m$ at $q_m^A = \Pr(AA|A)q_m^{AA} + \Pr(AB|A)q_m^{AB}$;
  - F-class A-lenders offer $\varphi e_m$ at $q_m^{AB}$;
  - B-lenders offer $\varphi e_\ell$ at $\bar{q}$;
  - Borrowers accept the contract that yields the largest loan advance.

### D.6.2 Equilibrium Payoffs

The payoffs to borrowers in equilibrium are as follows:

- $AA$-borrowers: $\varphi e_m q_m^A$;
- $AB$-borrowers with $A$ signals from G-class lenders: $\varphi e_m q_m^A$;
- $AB$-borrowers with $A$ signals from F-class lenders: $\varphi e_m q_m^{AB}$;
- $BB$-borrowers: $\varphi e_\ell \overline{q}$.

### D.6.3 Equilibrium Conditions

Before we proceed with construction of beliefs and (off-path) strategies, we state necessary conditions on the model parameters so that our constructed equilibrium candidate is indeed an equilibrium. We later show that these conditions together with Assumption 1 are sufficient to ensure that relevant incentive constraints are satisfied.

**Condition 3** *Suppose that the model parameters satisfy*

*(i)*

$$\frac{\delta\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2} \leqslant 1-\delta(1-\alpha),$$

*or, equivalently,* $q_h^{AA} \leqslant q_m^{AB}$. *Note that this is the same condition as part (i) of Condition 1.*

*(ii)*

$$[1-\delta(1-\alpha)](e_m-e_\ell) \geqslant \delta\alpha(e_h-e_\ell),$$

*or, equivalently,* $q_m^{AB}(e_m-e_\ell) \geqslant q_h^{AB}(e_h-e_\ell)$. *Note that this is the same condition as part (ii) of Conditions 1 and 2.*

*(iii)*

$$\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2+(1-\alpha)(1-\rho)^2}(e_h-e_\ell) \geqslant \left[1-\delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2}\right](e_m-e_\ell),$$

*or, equivalently,* $q_h^{AA}(e_h-e_\ell) \geqslant q_m^{AA}(e_m-e_\ell)$. *Note that this is the same condition as part (iii) of Conditions 1 and 2.*

*(iv)*

$$e_\ell \geqslant \left(1-\delta+\delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2+(1-\alpha)(1+\rho)^2}\right)e_m,$$

*or, equivalently,* $\bar{q}e_\ell \geqslant q_m^{BB}e_m$*. Note that by Assumption 1 (ii) and Corollary 1 we also have* $q_m^{BB}e_m \geqslant q_h^{BB}e_h$*. Combining, we have* $\bar{q}e_\ell \geqslant \max\left\{q_m^{BB}e_m, q_h^{BB}e_h\right\}$*. Note that this is the same condition as part (iv) of Conditions 1 and 2.*

*(v)*

$$q_m^A(e_m - e_\ell) \geqslant q_h^A(e_h - e_\ell), \tag{26}$$

*where*

$$q_m^A = \Pr(AA|A)\left[1 - \delta\frac{\alpha(1-\rho)^2}{\alpha(1-\rho)^2 + (1-\alpha)(1+\rho)^2}\right] + \Pr(AB|A)[1-\delta(1-\alpha)], \tag{27}$$

$$\begin{aligned} q_h^A &= \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_h^{AB} \\ &= \Pr(AA|A)\frac{\alpha(1+\rho)^2}{\alpha(1+\rho)^2 + (1-\alpha)(1-\rho)^2} + \Pr(AB|A)\delta\alpha, \end{aligned} \tag{28}$$

*and* $\Pr(AA|A)$ *and* $\Pr(AB|A)$ *are given by* (10)–(11)*. Notice that* (26) *implies*

$$q_m^A e_m > q_h^A e_h. \tag{29}$$

*(vi)*

$$\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right]e_m \geqslant q_h^A e_h,$$

*where* $q_m^{AB} = 1 - \delta(1-\alpha)$*,* $q_m^A$ *and* $q_h^A$ *are given by* (27)–(28)*, and* $\Pr(AA|A)$ *and* $\Pr(AB|A)$ *are given by* (10)–(11)*.*

### D.6.4 Beliefs

**Beliefs of borrowers after offers are made in stage 1.**

- If the borrower observes an offer from a G-class lender, then the borrower believes $\Pr(\sigma^G = A) = 1$.
- If the borrower observes an offer from an F-class lender, then the borrower believes $\Pr(\sigma^F = A) = 1$.

**Beliefs of lenders at the end of stage 1.** Note, for any accepted deviation loan in stage 1, beliefs of lenders in the same class as the lender who made the loan do not change. That is, for this class of lenders, beliefs are $\Pr(\sigma^- = A) = \Pr(\sigma^- = A|\sigma)$.

1. *Small loans.*

- Suppose the borrower has accepted a loan $(\varphi e_{\ell}, q)$ from a G-class lender.
  - Beliefs of F-class lenders are $\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^A, \\ 0 & \text{if } q < q_m^A. \end{cases}$
- Suppose the borrower has accepted a loan $(\varphi e_{\ell}, q)$ from an F-class lender.
  - Beliefs of G-class lenders are $\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^{AB}, \\ 0 & \text{if } q < q_m^{AB}. \end{cases}$
- Lenders do not update their beliefs about the other class' signal if the offer of a lender from their class was accepted.

2. *Medium loans.*

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from a G-class lender.
  - Beliefs of F-class lenders are $\Pr(\sigma^G = A) = \begin{cases} 1 & \text{if } q \geqslant q_m^A, \\ 0 & \text{if } q < q_m^A. \end{cases}$
- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ from an F-class lender.
  - Beliefs of G-class lenders are

$$\Pr(\sigma^F = A) = \begin{cases} 1 & \text{if } q \geqslant \Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}, \\ 0 & \text{if } q < \Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}. \end{cases}$$

- Lenders do not update their beliefs about the other class' signal if the offer of a lender from their class was accepted.

3. *Large loans.* Suppose the borrower has accepted a loan $(\varphi e_h, q)$. Then lenders' beliefs in this scenario going forward are irrelevant.

#### D.6.5 Strategies.

**Stage-1 strategies of borrowers after offers are made in stage 1.**

1. *Small loans.*

- Suppose the borrower observes an offer $(\varphi e_{\ell}, q)$ from an G-class lender.
  - The borrower accepts if $q \geqslant q_m^A$, rejects otherwise.

- Suppose the borrower observes an offer $(\varphi e_\ell, q)$ from an F-class lender. Define $\check{q}_\ell$ by

$$\begin{aligned}\check{q}_\ell e_\ell \quad &+ \Pr(AA|A) q_h^{AA}(e_h - e_\ell) + \Pr(AB|A) q_m^{AB}(e_m - e_\ell) \\ &= \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m.\end{aligned}$$

  - The borrower accepts if $q \geqslant \max\left\{\check{q}_\ell, q_m^{AB}\right\}$, rejects otherwise.

  Note that $\check{q}_\ell > q_m^{AB}$ and thus $\max\left\{\check{q}_\ell, q_m^{AB}\right\} = \check{q}_\ell$. Indeed, by Condition 3 (iii) and $q_m^{AA} > q_m^A$,

$$\check{q}_\ell e_\ell + \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right)(e_m - e_\ell) > \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m,$$

$$\check{q}_\ell > \Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB} > q_m^{AB}.$$

2. *Medium loans.*

- Suppose the borrower observes an offer $(\varphi e_m, q)$ from a G-lender.
  - The borrower accepts the offer if and only if $q \geqslant \check{q}_m^G$, where

$$\check{q}_m^G e_m + q_h^A(e_h - e_m) \quad = \quad q_m^A e_m.$$

- Suppose the borrower observes an offer $(\varphi e_m, q)$ from a F-lender.
  - The borrower accepts the offer if and only if $q \geqslant \check{q}_m^F$, where

$$\check{q}_m^F e_m + q_h^A(e_h - e_m) \quad = \quad \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m.$$

3. *Large loans.*

- Suppose the borrower observes an offer $(\varphi e_h, q)$ from a G-class lender. The borrower accepts the loan if and only if $q e_h > q_m^A e_m$.
- Suppose the borrower observes an offer $(\varphi e_h, q)$ from an F-class lender. The borrower accepts the loan if and only if $q e_h > \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right) e_m$.

**Stage-2 strategies of lenders.**

1. *Small loans.*

- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q \geqslant q_m^A$ from a G-class lender.

- G-class A-lenders offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
- G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{BB})$ or nothing;
- F-class A-lender offers $(\varphi(e_h - e_\ell), q_h^{AA})$;
- F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{AB})$.

- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q < q_m^A$ from a G-class lender.
  - G-class A-lenders offer $(\varphi(e_m - e_\ell), q_m^{A})$;
  - G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{BB})$;
  - F-class A-lender offers $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{BB})$.
- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q \geqslant q_m^{AB}$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_\ell), q_h^{AA})$;
  - G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - F-class A-lender offers $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{BB})$ or nothing.
- Suppose the borrower has accepted a loan $(\varphi e_\ell, q)$ with $q < q_m^{AB}$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_m - e_\ell), q_m^{AB})$;
  - G-class B-lenders offer $(\varphi(e_m - e_\ell), q_m^{BB})$;
  - F-class A-lender offers $(\varphi(e_m - e_\ell), q_m^{A})$;
  - F-class B-lender offers $(\varphi(e_m - e_\ell), q_m^{BB})$.

2. *Medium loans.*

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q \geqslant q_m^A$ from a G-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_m), q_h^{AB})$;
  - G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{BB})$ or nothing;
  - F-class A-lender offers $(\varphi(e_h - e_m), q_h^{AA})$;
  - F-class B-lender offers $(\varphi(e_h - e_m), q_h^{AB})$.
- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q < q_m^A$ from a G-class lender.

- G-class A-lenders offer $(\varphi(e_h - e_m), q_h^A)$;
- G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{BB})$;
- F-class A-lender offers $(\varphi(e_h - e_m), q_h^{AB})$;
- F-class B-lender offers $(\varphi(e_h - e_m), q_h^{BB})$.

- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q \geqslant \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right)$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_m), q_h^{AA})$;
  - G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{AB})$;
  - F-class A-lender offers $(\varphi(e_h - e_m), q_h^{AB})$;
  - F-class B-lender offers $(\varphi(e_h - e_m), q_h^{BB})$ or nothing.
- Suppose the borrower has accepted a loan $(\varphi e_m, q)$ with $q < \left(\Pr(AA|A) q_m^A + \Pr(AB|A) q_m^{AB}\right)$ from an F-class lender.
  - G-class A-lenders offer $(\varphi(e_h - e_m), q_h^{AB})$;
  - G-class B-lenders offer $(\varphi(e_h - e_m), q_h^{BB})$;
  - F-class A-lender offers $(\varphi(e_h - e_m), q_h^A)$;
  - F-class B-lender offers $(\varphi(e_h - e_m), q_h^{BB})$.

3. *Large loans.* Suppose the borrower has accepted a loan $(\varphi e_h, q)$ in stage 1. Then lenders make no offers in stage 2.

#### D.6.6 Incentives.

**Borrower's Stage-1 Deviations.** Strategies of borrowers after offers are made in stage 1 are optimal given the specified continuation strategies of the lenders.

1. Suppose the borrower observes an offer $(\varphi e_\ell, q)$ from a G-class lender.

   - It is optimal for the borrower to accept the offer if $q \geqslant q_m^A$. After doing so in stage 1, she believes she will receive (and accept) either an offer $(\varphi(e_h - e_\ell), q_h^{AA})$ from an F-class A-lender or an offer $(\varphi(e_m - e_\ell), q_m^{AB})$ from an F-class B-lender (or a G-lender). These follow from Condition 3 (ii) and (iii). Should she reject this stage 1 offer, she believes G-class lenders have an $A$ signal and therefore will offer $(\varphi e_m, q_m^A)$. Condition 3 (iii) ensures that for all $q \geqslant q_m^A$, the borrower optimally accepts this stage 1 offer.

- It is optimal for the borrower to reject the offer if $q < q_m^A$. Should she accept this stage 1 offer, G-class lenders (who she believes have an $A$ signal) will offer $(\varphi(e_m - e_\ell), q_m^A)$. The fact that these lenders will top her up to a medium rather than a large loan follows from Condition 3 (v). Instead, if she rejects the offer, the same G lenders will offer $(\varphi e_m, q_m^A)$ in the second stage. Since $q < q_m^A$, the borrower optimally rejects this stage 1 offer.

2. Suppose the borrower observes an offer $(\varphi e_\ell, q)$ from an F-class Lender.

   - It is optimal for the borrower to accept the offer if $q \geqslant \check{q}_\ell$. After doing so in stage 1, she believes she will receive (and accept) either an offer $(\varphi(e_h - e_\ell), q_h^{AA})$ from a G-class A-lender or an offer $(\varphi(e_m - e_\ell), q_m^{AB})$ from a G-class B-lender (or an F-lender). These follow from Condition 3 (ii) and (iii). Should she reject this stage 1 offer, she believes F-class lenders have an $A$ signal and therefore will offer $(\varphi e_m, q_m^{AB})$, and if G-class lenders also have an $A$ signal, she will receive and accept an offer $(\varphi e_m, q_m^A)$. Thus the definition of $\check{q}_\ell$ ensures for any $q \geqslant \check{q}_\ell$, the borrower optimally accepts this stage 1 offer.
   - It is optimal for the borrower to reject the offer if $q < \check{q}_\ell$. For any $q \in [q_m^{AB}, \check{q}_\ell)$, the borrower's continuation payoffs from accepting or rejecting this stage 1 offer are the same as the case when $q \geqslant \check{q}_\ell$. Hence, by the definition of $\check{q}_\ell$, she optimally rejects such offers. Continuation payoffs from accepting this stage 1 offer when $q < q_m^{AB}$ are even lower, and it is thus optimal to reject such offers. Specifically, if a borrower accepts a stage 1 loan with $q < q_m^{AB}$ from an F-class lender, she believes she will receive an offer $(\varphi(e_m - e_\ell), q_m^{AB})$ in the second stage (with no chance of receiving an offer $(\varphi(e_h - e_\ell), q_h^{AA})$ which yields larger payoffs under Condition 3 (iii)).

3. Suppose the borrower observes an offer $(\varphi e_m, q)$ from a G-class lender.

   - It is optimal for the borrower to accept the offer if $q \geqslant \check{q}_m^G$. Note the definition of $\check{q}_m^G$ immediately implies $\check{q}_m^G \leqslant q_m^A$. After accepting the offer in stage 1, if $q \in [\check{q}_m^G, q_m^A)$, then the borrower believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^A)$ from a G-class lender. If instead $q \geqslant q_m^A$, she believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^{AA})$ from an F-class lender with an $A$ signal or an offer $(\varphi(e_h - e_m), q_h^{AB})$ from an F-class lender with a B signal or a G-class lender. In either case, her expected payoff from accepting the stage 1 loan is $\varphi q e_m + \varphi q_h^A(e_h - e_m)$. For $q \geqslant \check{q}_m^G$, this payoff is larger than $\varphi q_m^A e_m$, which is her expected payoff from rejecting the stage 1 offer, and hence, it must be optimal for the borrower to accept the stage 1 offer.

- It is optimal for the borrower to reject the offer if $q < \check{q}_m^G$. Since $\check{q}_m^G < q_m^A$, should the borrower accept this stage 1 loan, she believe she will receive a payoff equal to $q\varphi e_m + q_h^A\varphi(e_h - e_m)$. When $q < \check{q}_m^G$ this payoff is smaller than the payoff she receives from rejecting the loan, $q_m^A\varphi e_m$ and hence rejecting the offer is optimal.

4. Suppose the borrower observes an offer $(\varphi e_m, q)$ from an F-class lender.

   - It is optimal for the borrower to accept the offer if $q \geqslant \check{q}_m^F$. Note that the definition of $\check{q}_m^F$ immediately implies $\check{q}_m^F \leqslant \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$. After accepting the offer in stage 1, if $q \in \left[\check{q}_m^G, \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right)$, then the borrower believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^A)$ from an F-class lender. If instead $q \geqslant \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$, she believes she will receive (and accept) an offer $(\varphi(e_h - e_m), q_h^{AA})$ from a G-class lender with an $A$ signal or an offer $(\varphi(e_h - e_m), q_h^{AB})$ from a G-class lender with a $B$ signal or an F-class lender. In either case, her expected payoff from accepting the stage 1 loan is $\varphi q e_m + \varphi q_h^A(e_h - e_m)$. For $q \geqslant \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$, this payoff is larger than $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right]\varphi e_m$, which is her expected payoff from rejecting the stage 1 offer, and hence, it is optimal for the borrower to accept the stage-1 offer.
   - It is optimal for the borrower to reject the offer if $q < \check{q}_m^F$. Since $\check{q}_m^F < \Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}$, should the borrower accept this stage 1 loan, she believes she will receive a payoff equal to $q\varphi e_m + q_h^A\varphi(e_h - e_m)$. When $q < \check{q}_m^F$ this payoff is smaller than the payoff she receives from rejecting the loan, $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right]\varphi e_m$, and hence rejecting the offer is optimal.

5. Suppose the borrower observes an offer $(\varphi e_h, q)$ from a G-class lender.

   - If the borrower accepts such an offer, she will receive no offers in the second stage. If she rejects, she believes she will receive an offer $(\varphi e_m, q_m^A)$ from G-lenders in the second stage. Hence, it is optimal to accept if and only if $q\varphi e_h > q_m^A\varphi e_m$.

6. Suppose the borrower observes an offer $(\varphi e_h, q)$ from an F-class lender.

   - If the borrower accepts such an offer, she will receive no offers in the second stage. If she rejects, she with either receive an offer $(\varphi e_m, q_m^A)$ from G-lenders or an offer

$(\varphi e_m, q_m^{AB})$ from F-lenders in the second stage. Hence, it is optimal to accept if and only if $q\varphi e_h > \left(\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right)\varphi e_m$.

**Lenders' Stage-1 Deviations.**

1. *Small loans.*

   - A G-class lender has no incentive to offer $\varphi e_\ell$.
     - Any offer below $q_m^A$ is not accepted. Any offer above $q_m^A$ is accepted. If F-lenders have an A signal, the borrower will be topped up to a large loan by Condition 3 (iii) and if they have a B signal the borrower will be topped up to a medium loan. This is implies that the probability of repayment of the loan is smaller than that priced into $q_m^A$. Hence, offers with prices $q \geqslant q_m^A$ earn negative expected profits.
   - An F-class lender has no incentive to offer $\varphi e_\ell$.
     - Any offer $q < \check{q}_\ell$ is not accepted. Any offer $q \geqslant \check{q}_\ell$ is accepted and earns negative expected profits. If the F-class lender has an A signal, then the expected payoff (per dollar of face value) is $-q + \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB} \leqslant -q + q_m^{AB} \leqslant 0$, where the first inequality follows from Condition 3 (i) and the second inequality follows from $\check{q}_\ell \geqslant q_m^{AB}$. If the F-class lender has a B signal, then the expected payoff (per dollar of face value) is

$$-q + \Pr(AB|B)q_h^{AB} + \Pr(BB|B)q_m^{BB} \leqslant -q + \Pr(AB|B)q_m^{AB} + \Pr(BB|B)q_m^{AB}$$
$$\leqslant -q + q_m^{AB} \leqslant 0.$$

2. *Medium loans.*

   - A G-class lender has no incentive to offer $\varphi e_m$.
     - Any offer $q < \check{q}_m^G$ is not accepted. Offers with $q \geqslant \check{q}_m^G$ yield negative expected profits. If the G-class lender has an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. Since $q \geqslant \check{q}_m^G$, the definition of $\check{q}_m^G$ implies $(q - q_h^A)e_m \geqslant q_m^A e_m - q_h^A e_h$. Condition 3 (v) implies $q_m^A e_m \geqslant q_h^A e_h$ and thus $-q + q_h^A \leqslant 0$. If the G-class lender has a B signal, their expected profits are weakly lower and hence also negative.
   - An F-class lender has no incentive to offer $\varphi e_m$.

- Any offer $q < \check{q}_m^F$ is not accepted. Offers with $q \geqslant \check{q}_m^F$ yield negative expected profits. If the F-class lender has an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. Since $q \geqslant \check{q}_m^F$, the definition of $\check{q}_m^F$ implies $(q - q_h^A)e_m \geqslant \left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m - q_h^A e_h$. By Condition 3 (vi),
  $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m \geqslant q_h^A e_h$ and thus $-q + q_h^A \leqslant 0$. If the F-class lender has a B signal, their expected profits are weakly lower and hence also negative.

3. *Large loans.*

- A G-class lender has no incentive to offer $\varphi e_h$.
  - Any offer with $qe_h \leqslant q_m^A e_m$ is not accepted. Offers with $qe_h > q_m^A e_m$ would be accepted by all borrowers and yield negative expected profits. If the G-class lender as an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. Since $qe_h > q_m^A e_m$ and Condition 3 (v) implies $q_m^A e_m \geqslant q_h^A e_h$, it follows that $qe_h > q_h^A e_h$. As a result, $-q + q_h^A < 0$. If the G-class lender has a B signal, their expected profits are weakly lower and hence also negative.
- An F-class lender has no incentive to offer $\varphi e_h$.
  - Any offer with $qe_h \leqslant \left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m$ is not accepted. Offers with $qe_h > \left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m$ would be accepted by all borrowers and yield negative expected profits. If the F-class lender has an A signal, expected profits (per dollar of face value) of the stage 1 deviation loan are $-q + q_h^A$. By Condition 3 (vi),
    $\left[\Pr(AA|A)q_m^A + \Pr(AB|A)q_m^{AB}\right] e_m \geqslant q_h^A e_h$, which implies that $qe_h > q_h^A e_h$ so that the stage 1 offer earns negative expected profits. If the F-class lender has a B signal, their expected profits are weakly lower and hence also negative.

**Borrower's Stage-2 Deviations.** Trivially, accepting on-path stage-2 offers gives borrowers a strictly higher payoffs than rejecting them (recall that the lenders break even).

**Lenders' Stage-2 Deviations.** Stage-2 strategies of the lenders are optimal.

1. G-class lender with an A signal offers $(\varphi e_m, q_m^A)$.

- The lender cannot improve profits by offering a loan $(\varphi e_m, q)$ with $q \neq q_m^A$. If the lender offers a price $q > q_m^A$, all borrowers would accept the offer but it would yield negative expected profits (since $q_m^A$ yields zero expected profits). If the lender offers a price $q < q_m^A$, the borrower would accept the loan.
- The lender cannot improve profits by offering a loan $(\varphi e_h, q)$. The highest price that a lender is willing to offer on a large loan is $q_h^A$. Condition 3 (v) implies that the borrower prefers $(\varphi e_m, q_m^A)$ to $(\varphi e_h, q_h^A)$, and so the lender does not have incentives to offer a large loan.
- The lender cannot improve profits by offering a loan $(\varphi e_\ell, q)$. Assumption 1 (i) implies that the borrower prefers $(\varphi e_m, q_m^A)$ to $(\varphi e_\ell, \bar{q})$, and since offering $q \geqslant \bar{q}$ can never be profitable, the lender does not have incentives to offer the small loan.

2. F-class lender with an $A$ signal offers $(\varphi e_m, q_m^{AB})$.

   - The lender cannot improve profits by offering a loan $(\varphi e_m, q)$ with $q \neq q_m^{AB}$. If $q < q_m^{AB}$, the offer will not be accepted. If $q \in (q_m^{AB}, q_m^A)$, the offer would only be accepted by an $AB$ borrower and hence earns negative expected profits ($q_m^{AB}$ is the zero profit price for these borrowers). If $q \geqslant q_m^A$, all borrowers would accept the offer but it would yield negative expected profits (since $q_m^A$ yields zero expected profits when all borrowers accept).
   - The lender cannot improve profits by offering a loan $(\varphi e_\ell, q)$ or $(\varphi e_h, q)$. The arguments are identical for those used for the G-class lender with an $A$ signal.

3. Lenders with a B signal offer $(\varphi e_\ell, \bar{q})$. By Condition 3 (iv), the borrower prefers $(\varphi e_\ell, \bar{q})$ to $(\varphi e_m, q_m^{BB})$, and $q_m^{BB}$ is the largest price that a lender with a B signal is willing to offer. So the lender has no incentives to deviate.

This completes our characterization of this equilibrium.