

*Building Credit History with Heterogeneously Informed Lenders**

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Abstract

This paper examines a novel mechanism of credit-history building as a way of aggregating information across multiple lenders. We build a dynamic model with multiple competing lenders, who have heterogeneous private information about a consumer's creditworthiness, and extend credit over multiple stages. Acquiring a loan at an early stage serves as a positive signal—it allows the borrower to convey to other lenders the existence of a positively informed lender (advancing that early loan)—thereby convincing other lenders to extend further credit in future stages. This signaling may be costly to the least risky borrowers for two reasons. First, taking on an early loan may involve cross-subsidization from the least risky borrowers to more risky borrowers. Second, the least risky borrowers may take inefficiently large loans relative to the symmetric-information benchmark. We demonstrate that, despite these two possible costs, the least risky borrowers often prefer these equilibria to those without information aggregation. Our analysis offers an interesting and novel insight into debt dilution. Contrary to the conventional wisdom, repayment of the early loan is more likely when a borrower subsequently takes on a larger rather than a smaller additional loan. This result hinges on a selection effect: larger subsequent loans are only given to the least risky borrowers.

Keywords: Credit History, Information Aggregation, Debt Dilution

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1 Introduction

Credit histories play an essential role in determining individuals' access to credit. It is important to understand not only how credit histories affect lending but also how borrowers can affect their credit histories. Existing literature treats credit histories as merely a way of keeping track of *public* information regarding an individual's risk profile. In contrast, we highlight the role of credit histories in aggregating *disperse* information among multiple (prospective) lenders. Specifically, we think of borrowers building their credit histories by taking on loans. The ability to qualify for a loan from one lender conveys that lender's positive information to other lenders. This mechanism is complementary to the more conventional story of signaling one's type via repayment of existing loans. Ours is the first paper to explicitly model how borrowers may affect this information aggregation through sequential borrowing.¹

We build a dynamic model with multiple competing lenders who have heterogeneous private information about a consumer's creditworthiness.² We explore how this private information is aggregated through lending that takes place over multiple stages. There are two key forces at play. Acquiring a loan at an early stage serves as a positive signal—it allows the borrower to convey to other lenders the existence of a positively informed lender (advancing that early loan)—thereby convincing other lenders to extend further credit in future stages. On the other hand, this signaling is costly to the least risky borrowers for two reasons. First, taking on an early loan may involve cross-subsidization from the least risky borrowers (those with all positive signals) to more risky borrowers (those with mixed signals). Second, either the presence of cross-subsidization or the threat of it makes the least risky borrowers may take inefficiently large loans relative to the symmetric-information benchmark. We demonstrate that despite these two possible costs, the least risky borrowers often prefer these equilibria to those without information aggregation. We interpret the mechanism of taking an early loan to signal their credit-worthiness to other lenders as *building a credit history*. It captures conventional wisdom present in consumer

¹Chatterjee et al. (2016) also analyze the dynamics of credit scores, but while they consider how *repayment* behavior affects a borrower's *reputation*, we think of borrowing itself as being a signal of the borrower's credit-worthiness. Kovbasyuk et al. (2018) also treat credit histories as records of repayment, and analyze how the length of such record affects access to credit.

²One way to interpret the assumption of heterogeneous information is to think about lenders observing the same credit history of the consumer but employing different, imperfectly correlated, models of credit risk to evaluate it. Alternatively, one can imagine lenders collecting information about the consumer in addition to that contained in the credit report.

credit markets, but absent in the academic discourse, that one way to quickly build a positive credit history is to take on a loan.

Our analysis offers interesting insights into debt dilution, where further lending dilutes existing loans by increasing the consumer’s probability of default, hence decreasing the probability that the consumer repays their original lender. Contrary to conventional wisdom, we show that in our model, consumers who take on larger additional loans are more likely to repay their incumbent lenders than those who take on smaller additional loans. Our analysis provides novel (potentially) testable implications on lending and repayment behavior in the presence of non-exclusivity in consumer credit markets.

In order to study these issues, we build a parsimonious dynamic model of consumer credit with heterogeneously informed lenders. Our model features risk-averse borrowers and competing, risk-neutral lenders. There are two periods, and the first period has two stages. Each borrower has zero income in the first period and uncertain income in the second period. In the beginning of the first period, lenders receive private signals about the distribution of the borrower’s income in the second period. For simplicity, we assume that signals are binary—positive or negative. Lenders can offer loan contracts—described by the loan size and price—to the borrower over the two stages of the first period. Lenders do not observe each others’ contracts, but they observe the contract that the borrower accepts.

We analyze Perfect Bayesian Equilibria in this environment. Our model features both ex-ante signaling and ex-post screening. Ex-ante signaling arises because borrowers recognize that their loan choices influence the information set of lenders. Ex-post screening may arise if lenders do not have complete information in the second stage. To emphasize the signaling nature of credit-history building, we analyze a limiting case of our model with impatient borrowers. This simplification eliminates ex-post screening and allows for clean, analytical results. Of course, as with most signaling models, ours features multiple equilibria and so we develop an equilibrium selection in the spirit of the Cho and Kreps (1987) intuitive criterion.

The mechanism of credit-history building is as follows. Borrowers who see offers from lenders in the first stage conclude that these lenders have positive signals about them since negatively-informed lenders do not make offers in the first stage. To transmit this information to other lenders, these borrowers accept an offer—i.e., take out a loan—from a positively informed lender in the first stage. Lenders who see that a borrower accepted

an offer conclude that this offer came from a lender with a positive signal, update their belief about the borrower’s creditworthiness upwards, and offer better contract terms in the second stage. Importantly, the signaling of the borrower’s creditworthiness comes at a cost. First, if the early loan offer is accepted by borrowers with different realizations of signals (by borrowers with all positive signals and by borrowers with some positive and some negative signals), then the price of the loan is such that the least risky borrowers (those with all positive signals) cross-subsidize the more risky borrowers (those some positive and some negative signals). Finally, either because of cross-subsidization or in order to avoid it, the least risky borrowers may take inefficiently large loans relative to the symmetric-information benchmark. We show that despite these costs, the least risky borrowers find it optimal to take on early loans to build a credit history. More specifically, the intuitive criterion picks the unique equilibrium (outcome) that is most preferred by the least risky borrowers, and it features credit-history building.

It is important to distinguish credit-history building from improving a credit score. Credit scores are meant to be a summary statistic for borrowers’ probability of default. Building a credit history in our model may actually lower a borrower’s credit score. Borrowers who take on early loans successfully communicate that they have a lower default probability *for a given loan size*, but they also end up with a higher default probability in equilibrium due to taking on a *larger* loan.³

While credit-history building increases the default probability in our model, surprisingly, the same may not be true regarding the extent of loan dilution. Our model yields a striking result that when the original lender faces uncertainty about how much his early loan will be diluted, he is actually *more* likely to be repaid when the borrower takes a *larger* additional loan. The reason is that large (additional) loans are only given to the least risky borrowers in equilibrium. And while taking out a larger loan—for a given quality borrower—increases the risk of default, it turns out that the least risky borrower is still more likely to repay a large loan than the more risky borrower is to repay a medium-size loan. We refer to this finding at the “more-dilution-is-better” result. It provides a new insight into the issue of debt dilution, where more dilution (a larger additional loan) is typically considered to decrease the probability of repayment on the original loan (as in, e.g., Bizer and DeMarzo (1992)). Our mechanism has this “dilution effect” as well: for

³Correspondingly, the mechanism we are highlighting is distinct and complementary to the idea of doctoring one’s credit score, as in, for example, Hu et al. (2017).

a borrower of a given risk/quality, a larger loan increases the probability of default. But there is also an additional, “selection effect”: less risky/better quality borrowers take out larger loans. This selection effect dominates the dilution effect. Importantly, information aggregation is key for this result: a larger top-up loan conveys positive information of the diluting lender.

We also explore how equilibrium outcomes and the costs of credit-history building change as we vary model parameters; specifically, the lenders’ signal quality. We find that cross-subsidization occurs for low enough and high enough values of the signal precision, and excessive borrowing occurs for intermediate values of the signal precision. The two costs can be present simultaneously or one at a time. We also illustrate that the “more-dilution-is-better” result is relevant for high enough values of the signal precision.

Our paper offers a new way of interpreting some findings of a growing empirical literature, including Liberman et al. (2017), who look at the effects of taking a payday loan on financial outcomes. The mechanism we are highlighting may help explain why taking on an additional (payday) loan does not lead to any additional financial distress for the borrowers with the lowest ex-ante credit scores.

A key feature of our model is non-exclusivity of relations between borrowers and lenders. Although a large literature has examined consumer credit markets, it has typically assumed exclusivity of debt contract—see, e.g., Chatterjee et al. (2007), Livshits et al. (2007), and surveys by Athreya (2005) and Livshits (2015). While debt dilution is a prominent feature of recent papers on defaultable debt in international finance—see, e.g., Chatterjee and Eyigungor (2012, 2015), and Arellano and Ramanarayanan (2012)—the questions studied in that literature are very different from those in the consumer credit literature. The idea of information aggregation among lenders is new to either literature and constitutes our central contribution.

Our paper also provides a theory of why borrowers take loans from multiple lenders. This important feature is absent, for example, from a seminal paper by Bizer and DeMarzo (1992), which shows that the anticipation of debt dilution leads to a too large loan at a too large interest rate, but the whole loan can as well be originated by a single lender. Parlour and Rajan (2001) provide a theory of borrowing from multiple lenders, but in their model borrowing is not sequential, and there is no credit-history building, which is the focus of our paper.

A recent empirical and theoretical literature has begun to investigate the role of information-

sharing across lenders in determining terms of credit. For example, studying firms’ access to credit, Sutherland (2018) finds empirically that, when lenders share information, their relationship with borrowers tend to dissolve more quickly. Hertzberg et al. (2011) also study how information sharing across lenders determines borrowers’ term of credit, highlighting the coordination role of the shared information. These papers treat the nature of information that is shared across lenders as exogenous, while we emphasize the borrowers’ incentives to *affect* the information that is shared.⁴

This rest of the paper is organized as follows. In section 2, we present the environment, which we model explicitly as a game, and define the equilibrium. We then narrow our attention to a subset of equilibria preferred by a specific class of borrowers and argue that this subset is the natural one to focus on. In section 3, we provide an illustrative simplification of the model, which facilitates establishing analytical results (and highlights the key mechanisms present in the more general setting). We establish a set of sufficient conditions for *information aggregation* to occur in equilibrium and highlight that the equilibrium which achieves this information aggregation features *credit-history building*. We show that credit-history building is associated with two types of costs, born by the (best) borrowers seeking to aggregate information. These costs are cross-subsidization (of other borrowers) and excessive borrowing. Depending on the parameter values, either, both or neither of the costs are present. Despite the possible cost, credit-history building is often preferred by the borrowers to the outcome of a single-stage game that yields no information aggregation (though not always). We provide sufficient conditions for credit-history building being preferable. We then offer a numerical example in section 4 that illustrates the richness of the model and demonstrates some of the possible equilibrium outcomes. We further establish some novel empirical predictions/insights into the nature of debt dilution. Section 5 concludes.

2 The Model

The Environment

There are two periods, I and II, and period I consists of two stages, 1 and 2. We study the

⁴Martin (2009) investigates a very different mechanism to address adverse selection. The key similarity to our paper is that the “good” borrowers in that model take loans over two stages from two (possibly distinct) lenders. Even though our explicit signaling motive is absent in Martin (2009), one of the loans undertaken by the “good” borrowers in that model does involve cross-subsidizing the “bad” borrowers.

interaction between a single borrower and multiple ($2 \times K$, $K \geq 2$) competing lenders.⁵ The borrower has no endowment in period I.⁶ Her endowment e in period II is stochastic, drawn from a finite support $\{e_\ell, e_m, e_h\}$, where $0 < e_\ell < e_m < e_h$. The probability distribution over these endowment realizations depends on the borrower’s unobservable “quality” state $s \in \{g, b\}$. Let $\pi(e, s)$ denote the probability that a borrower with quality s receives endowment e in period II. We assume that the endowment distribution of the g -borrowers first-order stochastically dominates that of the b -borrowers. The ex-ante probability that a borrower’s quality is g (and the share of g -borrowers in the population) is $\alpha \in (0, 1)$.

Each borrower (consumer) is risk averse and derives utility from consumption in each of the two periods according to the per-period utility function $u: [0, +\infty) \rightarrow \bar{\mathbb{R}}$. The function u is continuous, strictly increasing, and strictly concave. The borrower discounts period-II utility with the discount factor $\beta \in (0, 1)$. Note that there is no discounting across stages within period I. Lenders are risk neutral, have deep pockets, and discount period-II payoffs with the discount factor $\bar{q} = (1 + \bar{r})^{-1}$, where \bar{r} is the risk-free rate of interest.

The only financial instrument available in the economy is a non-contingent defaultable bond payable in period II. If the borrower defaults (fails to pay the full amount owed), she suffers a loss of fraction φ of her endowment. This cost of default is a dead-weight loss, as the lost portion of endowment is destroyed and not transferred to the lenders.

At the beginning of period I, each lender receives a private signal, σ , about the borrower’s ex-ante quality. The signals are binary, with support $\{A, B\}$. There are two equal-size classes of lenders, which differ only in the realization of the signal they receive.⁷ Within each class, lenders observe the same signal, while signals across the two classes are conditionally independent. For concreteness, we assume that the signal a lender receives is drawn from a distribution that depends on the unobservable quality state of the borrower: $\Pr(\sigma = A|s = g) = \Pr(\sigma = B|s = b) = (1 + \rho)/2$. We refer to $\rho \in (0, 1)$ as the *precision* of the signal.

Timing, Information, Actions, and Payoffs

In this section, we describe the interaction between the borrower and lenders as an extensive

⁵We can equivalently assume that there are many borrowers.

⁶The assumption of zero endowment is for expositional simplicity only. All of our analysis and results extend if we assume that the borrower has a positive but (relatively) small endowment in period I.

⁷The assumption of equal sizes of the two classes is only for concreteness. What is important is that there are at least two lenders in each class, and hence they will be competing.

form game. In each stage of period I, lenders simultaneously offer contracts to the borrower. A contract is a pair (x, q) , where x is the face value of the loan (equivalently, the amount of bonds the borrower sells) and q is the price. That is, a borrower who accepts a contract (x, q) from a given lender in a given stage of period I, receives qx from this lender in period I, and has a (defaultable) obligation to repay x to this lender in period II.

Let $O_t = \{(x_t^k, q_t^k, k)\}_k$ denote the set of offered contracts together with the identities of lenders offering these contracts in stage $t \in \{1, 2\}$. (A lender who is not offering a contract can be thought of making an offer $(0, 0)$.) After observing the set of offered contracts in a given stage, the borrower accepts at most one contract in that stage.⁸ That is, within a stage, contracts are exclusive. All lenders observe the terms of any contract accepted by the borrower in stage 1 as long as the loan size is no smaller than a minimal threshold \underline{x} . All lenders also observe the identity of the lender whose contract was accepted. Thus, the public history in the beginning of stage 2 is the borrower's stage-1 accepted contract, if any, and the identity of the lender whose contract was accepted. We will refer to this public history as the *credit history* of the borrower. Formally, the (public) credit history is $h_2^P = (x_1, q_1, j_1)$ if a contract (x_1, q_1) from lender j_1 was accepted in stage 1, and $h_2^P = (0, 0, 0)$ if no contract was accepted.

Suppose the consumer borrows x_1 at q_1 in stage 1 and x_2 at q_2 in stage 2. The borrower's consumption in period I is then $q_1x_1 + q_2x_2$, and the total loan balance carried into period II is $X := x_1 + x_2$. In period II, after observing the realized endowment, e , the borrower chooses whether to repay or default on her debt obligations. Repaying anything less than X is equivalent to defaulting and results in a dead-weight loss of fraction φ of the endowment. Implicitly, this way of modeling consumer default ensures that partial default is never optimal for the borrower.

If the borrower defaults in period II, her consumption in that period is $(1 - \varphi)e$, and that of her lenders is 0. If the borrower repays X , she consumes $e - X$, and the lenders who lent in stages 1 and 2 consume x_1 and x_2 , respectively. It follows immediately that the borrower will repay if and only if

$$e - y \geq (1 - \varphi)e. \tag{1}$$

⁸We assume that if the borrower is indifferent between multiple offers, she accepts each of these offers with equal probability.

This implies that the borrower’s payoff is

$$\pi^B = u(q_1x_1 + q_2x_2) + \beta u(\max\{e - x_1 - x_2, (1 - \varphi)e\}), \quad (2)$$

and the payoff to a lender associated with a contract (x, q) that he offers and that the borrower accepts in (one of the stages of) period I is

$$\pi^L = -qx + \bar{q}x\mathbb{1}_{[\phi e \geq X]}. \quad (3)$$

Equilibrium Definition

We will study Perfect Bayesian Equilibria (PBE) of the game described above. Among them we will focus on the one(s) that are preferred by *AA*-borrowers. We argue that this equilibrium selection puts an intuitive and natural restriction on the off-equilibrium beliefs, similar to the intuitive criterion of Cho and Kreps (1987), and the resulting equilibrium outcome has properties similar to Netzer and Scheuer (2014).⁹ Essentially, it rules out beliefs that early offers come from negatively rather than positively informed lenders. Because *AA*-borrowers use these early loans to build credit histories, it is intuitive that such a restriction on beliefs will pick equilibria favoring *AA*-borrowers.

In order to facilitate characterization of these equilibria, we define the sequence of problems faced by each agent in the order implied by backward induction. In the middle of stage 2, after lenders have made their stage-2 offers, the borrower has observed two sets of offers, O_1 and O_2 , and her own credit history $h_2^P = (x_1, q_1, j_1)$. Let $h_2^B = (O_1, h_2^P, O_2)$ denote this information set of the borrower. The borrower’s stage-2 action is to choose an offer from O_2 (or possibly reject all offers). She does so based in part on her posterior beliefs about her own quality state induced by the history (and her understanding of lenders’ strategies). We denote $\theta_2^B(e|h_2^B)$ the probability the borrower assigns in stage 2 to receiving endowment e in the second period. Note that this probability is a convolution of the posterior belief of the borrower regarding her underlying quality s and the probability distribution over outcomes implied by this quality. Of course, the borrower forms her posterior about her underlying quality based on public and private histories, as well as her understanding of lenders’ equilibrium strategies—on the equilibrium path, it is obtained using Bayes’ rule.

⁹The intuitive criterion of Cho and Kreps (1987) does not directly apply in our environment because of the richness of the strategic interactions that come *after* the signalling takes place in our model.

The borrower's stage-2 action maximizes her expected payoff under θ_2^B and so solves

$$V_2(h_2^B) = \max_{(x_2, q_2, j) \in O_2 \cup \{(0,0,0)\}} u(q_1 x_1 + q_2 x_2) + \beta \sum_e \theta_2^B(e|h_2^B) u(\max\{e - x_1 - x_2, (1 - \varphi)e\}). \quad (4)$$

At the beginning of stage 2, everyone has observed the public credit history of the borrower $h_2^P = (x_1, q_1, j_1)$. Additionally, each lender k knows his private signal about the borrower's state, σ_k , and his offer to the borrower in the first stage, (x_1^k, q_1^k) . Thus, the private history of the lender k is $h_2^k = (h_2^P, \sigma_k, (x_1^k, q_1^k))$. When choosing his second-stage offer, the k th lender forms expectations of other lenders' offers. Similar to the borrower, the lender forms his posterior belief $\mu_2(\sigma_{-k})$ regarding the other class's signal based in part on his understanding of equilibrium strategies. Equilibrium strategies imply a mapping from the vector of realized signals and the observed public history into an offer set O_2 , which will be faced by the borrower. For any (x, q) offered by the k th lender, denote by δ_2^k the probability of that offer being accepted (as perceived by the k th lender given the equilibrium strategies of the borrower and the other lenders).¹⁰ Then, the optimal offer made by lender k solves the following maximization problem:

$$W_2^k(h_2^k) = \max_{(x, q)} \sum_{\sigma_{-k}} \mu_2(\sigma_{-k}|h_2^k) \delta_2^k(x, q) \times \left[-qx - q_1 x_1 \mathbb{1}_{j_1=k} + \bar{q}(x + x_1 \mathbb{1}_{j_1=k}) \sum_e \theta_2^L(e|h_2^k, j_2 = k) \mathbb{1}_{[\varphi e \geq x_1 + x]} \right], \quad (5)$$

where $\theta_2^L(e|\cdot)$ is the lender's posterior probability that the borrower will receive endowment e conditional on the lender's information at the beginning of stage 2 and the fact that her offer was accepted by the borrower.

In stage 1, the borrower chooses among offers in the set O_1 (and the option of rejecting all offers) to maximize

$$V_1(O_1) = \max_{(x, q, k) \in O_1 \cup \{(0,0,0)\}} \mathbb{E}V_2(O_1, (x, q, k), O_2(x, q, k)). \quad (6)$$

¹⁰To be more precise, $\delta_2^k = \delta_2^k((x, q, k)|(x_1^k, q_1^k, k), O_1^{-k}(\sigma_k, \sigma_{-k}), h_2^P, O_2^{-k}(\sigma_k, \sigma_{-k}, h_2^P))$, where σ_k is the signal observed by the k -th lender, σ_{-k} is the signal observed by lenders of the other class, and O_i^{-k} is the offer set excluding the offer made by the k -th lender in stage $i = 1, 2$.

Note that the borrower understands that her choice of (x, q) influences not only her payoffs in V_2 directly but also the set of offers she will receive in stage 2, O_2 .

Similarly, lenders in stage 1 understand that the offer they make, if accepted, may influence the posteriors of other lenders in the second stage.¹¹ Having observed their signal, they make an offer that maximizes their expected profits:

$$W_1^k(\sigma_k) = \max_{(x,q)} \sum_{\sigma_{-k}} \mu_1(\sigma_{-k}|\sigma_k) \left[\delta_1^k(x, q) W_2^k((x, q, k), \sigma_k, (x, q)) \right. \\ \left. + (1 - \delta_1^k(x, q)) W_2^k((x_{-k}, q_{-k}, -k), \sigma_k, (x, q)) \right], \quad (7)$$

where δ_1^k and θ_1^L are defined similar to their stage-2 counterparts. Note that, if accepted, the lender's offer influenced her payoffs not only directly but also by affecting the offer set O_2 in the subsequent stage.

Definition 1. *A Perfect Bayesian Equilibrium consists of offer strategies for the lenders, acceptance strategies for the borrower, and posterior beliefs (for the lenders and the borrower) such that the lenders' and borrower's strategies are optimal and posterior beliefs satisfy Bayes' rule (where applicable).*

Finally, define a *symmetric-information* benchmark as a variant of our environment where signals are publicly observable. We will compare equilibrium outcomes in our model with those in the benchmark.

For notational convenience, we will refer to a lender who observes a signal realization A (a signal realization B) as an A -lender (a B -lender). We will refer to a borrower for whom the pair of signal realizations for the two lender classes are A and B as an AB -borrower. Similarly, the AA -borrowers (BB -borrowers) are those for whom both classes of lenders observe an A (a B) signal realization. Notice that whether a borrower is AA , AB , or BB is initially unknown to both the borrower and lenders. Whether borrowers or lenders may be able to infer this information will depend on the strategies these agents choose.

To facilitate our discussion, we will restrict attention to parameter values that generate equilibrium loan sizes in the set $\{\varphi e_L, \varphi e_M, \varphi e_H\}$. We will refer to the loans of these sizes as small, medium, and large, respectively. Intuitively, since for (total) loan sizes in each

¹¹In our setting, an individual lender's deviation does not change the borrower's posterior, since the borrower is facing many lenders. However, it may affect other lenders' posterior, since lenders do not observe the offer set O_1 , only the borrower's choice from that set.

of the intervals, $(0, \varphi e_L]$, $(\varphi e_L, \varphi e_M]$, $(\varphi e_M, \varphi e_H]$, the default probability is the same, the corresponding equilibrium loan prices will be constant as well. So if a borrower is sufficiently “hungry” (sufficiently willing to borrow), she will not choose an interior loan size but will prefer to be at the corner.¹² Furthermore, we also assume that the size of the smallest visible loan \underline{x} equals φe_L .

3 Limiting Case with Impatient Borrowers

In this section, we highlight the benefits and costs of credit-history building. To emphasize the role of signaling for credit-history building, we impose a simplifying assumption that eliminates screening in stage 2. Specifically, we assume that $\beta = 0$, i.e., borrowers only care about their period-I consumption.

We characterize equilibria with and without credit-history building. These equilibria reveal the benefits of credit-history building as well as two costs. We then show that for some parameter values, borrowers are able to build their credit history for free — in the absence of these costs. Finally, we show that there are parameter values for which the costs of credit-history building outweigh the benefits, and so the equilibrium most preferred by *AA*-borrowers features no credit-history building.

3.1 Simplifying Assumptions

To emphasize the role of signaling, in this section, we make the following simplifying assumptions. First, we assume that the borrower’s discount factor $\beta = 0$, which implies that all borrowers simply maximize the amount of consumption they receive in the first period. Importantly, this eliminates cream-skimming in our environment, as there is no difference in valuation of contracts across the borrower’s types. This significantly simplifies the equilibrium characterization.

Second, we assume that the borrowers are risk neutral. This is not particularly restrictive, given our assumption of $\beta = 0$. However, it simplifies analysis of situations when in equilibrium no action is taken in the first stage, and so the borrowers need to form expectations about their quality in the event of a lender’s deviation in the first stage.

¹²This requires a mild restriction on preferences and/or endowments of the borrower to ensure that interior debt levels are not preferred to these “corner” values.

Third, we assume that the endowment distribution of good and bad borrowers is such that bad borrowers can only receive low and medium endowment, while good borrowers can only receive medium and high endowment. Moreover, the probability of receiving the medium endowment is the same for both good and bad borrowers. Formally, $\pi(e_\ell, b) = \pi(e_h, g) = \delta$, $\pi(e_m, b) = \pi(e_m, g) = 1 - \delta$, $\pi(e_h, b) = \pi(e_\ell, g) = 0$. Third, we assume that there are equal ex-ante probabilities that the borrower is good vs. bad, i.e., $\alpha = 1/2$. These assumptions on the endowment distribution are not crucial for our analysis and are only made to simplify algebra.

3.2 Equilibrium with Credit-History Building

In the text, to shorten the exposition, we only describe on-the-equilibrium-path strategies. The full descriptions of all equilibria discussed in the text, including off-path strategies and beliefs, can be found in the Appendix.

Equilibrium with *lmh* outcome cross-subsidization. Consider the following equilibrium that features credit-history building. In stage 1, *B*-lenders make no offers, and *A*-lenders offer φe_ℓ at price q^A , defined as

$$q^A = \Pr(AA|A)q_h^{AA} + \Pr(AB|A)q_m^{AB}, \quad (8)$$

where

$$q_h^{AA} = \Pr(\text{repaying large loan}|AA)\bar{q} = \Pr(e = e_h|AA)\bar{q}, \quad (9)$$

$$q_m^{AB} = \Pr(\text{repaying medium loan}|AB)\bar{q} = \Pr(e \in \{e_m, e_h\}|AB)\bar{q}. \quad (10)$$

All borrowers with such an offer accept one. In stage 2, *A*-lenders whose offer was not accepted and who see that the accepted offer was made by a lender from the other class conclude that the borrower is *AA*. They offer a loan $\varphi(e_h - e_\ell)$ (i.e., top up to a large loan) at price q_h^{AA} . An *AA*-borrower accepts such an offer. *A*-lenders whose offer was accepted (or whose offer was not accepted, but the accepted offer came from a lender of the same class), or *B*-lenders who observed that an offer was accepted, offer $\varphi(e_m - e_\ell)$ at price q_m^{AB} . An *AB*-borrower accepts such an offer from one of those lenders. (Notice that *A*-lenders making such an offer correctly predict that only an *AB*-borrower would accept their offer.)

Finally, B -lenders who see that no offer was accepted conclude that this is a BB -borrower and offer her a risk-free small loan φe_ℓ at \bar{q} . A BB -borrower accepts such an offer.

Let us first comment on the price of the stage-1 loan given in (8). Notice that both AA - and AB -borrowers accept this loan, and hence AA -borrowers cross-subsidize AB -borrowers. In addition, the small stage-1 loan will be necessarily diluted in stage 2 to either a medium loan (for AB -borrowers) or a large loan (for AA -borrowers). Hence, the price q^A of the stage-1 loan is a weighted average of the price of a large loan that only AA -borrowers accept and a medium loan that only AB -borrowers accept.

Next, consider who learns what when in this equilibrium. Notice that because only A -lenders make offers in stage 1, the borrowers infer all the signals from seeing the stage-1 offers. Next, after observing that the borrower either did or did not accept an offer and the class of lenders that the accepted offer came from, the lenders from the other class learn the signal of the lender whose offer has been accepted. That is, if the borrower accepted (did not accept) an offer, the lenders from the other class conclude that the other class has an A (a B) signal. Finally, the lender whose offer the borrower accepted in stage 1—and all the lenders from the same class—do not learn anything new at that stage. Eventually, at the end of stage 2, those lenders will learn all the signals as well.

Borrowers understand how their actions affect their credit history and therefore the information that lenders have after stage 1. We refer to taking an early loan with the purpose of facilitating information aggregation as credit-history building. More formally, let $\lambda_{j,t}$ denote the probability that a class- j lender assigns to the borrower being of quality g at the beginning of stage t . We say that an equilibrium features *credit-history building* for a type $\omega \in \{AA, AB, BB\}$ if $\max_j \lambda_{j,2} > \max_j \lambda_{j,1}$. Under this definition, in the equilibrium described above, AA -borrowers build their credit history, while AB -borrowers do not. For AA -borrowers, in stage 2, the rejected class of lenders hold the most favorable beliefs about the borrower, having updated them from $\Pr(g|A)$ to $\Pr(g|AA)$. For AB -borrowers, the accepted class of A -lenders hold the most favorable beliefs in stage 2, but their beliefs do not improve from stage 1 to stage 2.

An improvement in beliefs associated with credit-history building yields benefits for the AA -borrowers. Conceptually, credit-history building allows AA -borrowers to persuade lenders to offer them better terms. Specifically, they can get a lower interest rate for any given size loan. Furthermore, in the equilibrium above, when the AA -borrowers face these improved interest rates, they choose to take on more credit.

On the other hand, building a credit history may come at a cost. We identify two possible costs of credit-history building: *cross-subsidization* and *excessive borrowing*. Cross-subsidization arises when *AA*-borrowers accept loan offers whose prices reflect the higher default risk of other borrower types. Excessive borrowing arises when the resulting loans of *AA*-borrowers are larger than what they would have taken on under symmetric information, i.e., in the environment where all lenders' signals are public information.^{13,14}

To better understand these costs, consider first cross-subsidization. As happens in the specific equilibrium above, *AA*-borrowers pay more than the actuarially fair interest rate on their first-stage loan as this loan is also accepted by *AB*-borrowers, whose equilibrium default probability is higher than that of the *AA*-borrowers in this equilibrium. As a result, the terms of the first-stage loan reflect the default risk of both *AA*- and *AB*-borrowers, as can be seen from equation (8). The condition for cross-subsidization in equilibrium is then simply

$$q_h^{AA} > q_m^{AB}, \quad (11)$$

which says that the *AA*-borrowers' actuarially fair price is higher (or the interest rate is lower) than that of the *AB*-borrowers (given the loans they receive in equilibrium). Condition (11) ensures that *AB*-borrowers are willing to accept the same first-stage loan as *AA*-borrowers. This condition is not surprising, as the only reason *AB*-borrowers accept the early loan is to take advantage of the cross-subsidization.

Condition (11) restricts the set of parameter values such that cross-subsidization can happen in equilibrium. Since $q_h^{AA} = \delta(1 + \rho)^2 / [2(1 + \rho^2)]$ is an increasing function of ρ , and $q_m^{AB} = 1 - \delta/2$ does not depend on ρ , it is immediate that cross-subsidization can only occur in equilibrium when ρ is sufficiently high. For large values of ρ — that is, when signals are precise — the default risk of *AA*-borrowers on a large loan is small, and so the interest rate on the first-stage loan is low. The first-stage loan is then attractive to *AB*-borrowers when ρ is high.

There could simultaneously exist a PBE similar to the one described above, but with a medium instead of a small stage-1 loan. Such an equilibrium features more cross-subsidization from *AA*- to *AB*-borrowers and is therefore more costly to *AA*-borrowers. As a result, *AA*-borrowers prefer the equilibrium described above to this PBE, and hence

¹³Note that multiple stages are irrelevant in this alternative environment. All equilibrium outcomes can be obtained by undertaking all loans in the last stage.

¹⁴We define excessive borrowing in terms of the face value of the loan X .

our equilibrium selection picks the former.

The second potential cost of credit-history building is excessive borrowing. It occurs in our equilibrium whenever the symmetric-information outcome has *AA*-borrowers ending up with a medium-size loan. Why would this over-borrowing occur? *AA*-borrowers' lack of self-control is the culprit. Once they undertake the first-stage loan (which they do to signal their type), they succumb to the temptation to top it up to the large loan, as opposed to limiting themselves to a medium-size loan. Formally, this is captured by the following two conditions:

$$q_h^{AA} e_h < q_m^{AA} e_m, \quad (12)$$

$$q_h^{AA} (e_h - e_\ell) > q_m^{AA} (e_m - e_\ell). \quad (13)$$

The first inequality guarantees that *AA*-borrowers choose the medium-size loan under symmetric information. The second inequality states that, after taking on a small loan in stage 1, *AA*-borrowers prefer topping up to the large loan, rather than the medium one.

Equations (12)-(13) can be rewritten as

$$\frac{e_m - e_\ell}{e_h - e_\ell} < \frac{q_h^{AA}}{q_m^{AA}} < \frac{e_m}{e_h}, \quad (14)$$

where q_h^{AA} and q_m^{AA} are independent of endowments. As e_ℓ becomes close to e_m , the first inequality in (14) is more likely to be satisfied. As e_ℓ moves close to e_m , the top-up to a medium loan in the second stage becomes smaller, and the *AA*-borrowers' temptation to top up to a larger loan becomes stronger.

The two costs of credit-history building, cross-subsidization and excessive borrowing, may be present in equilibrium for different model parameter values in any combination — both, one at a time, or none. The equilibrium described above features cross-subsidization and may or may not feature excessive borrowing depending on whether in the symmetric-information benchmark the *AA*-borrowers end up with a medium or a large loan. In the Appendix, we provide the conditions for the model parameters so that one or the other scenario occurs. Next, we describe an equilibrium without cross-subsidization.

Equilibrium with *lmh* outcome and no cross-subsidization. In stage 1, *B*-lenders make no offers, and *A*-lenders offer φe_ℓ at price q_h^{AA} . Only borrowers with two such offers

(i.e., the *AA*-borrowers) accept one. In stage 2, *A*-lenders whose offer was not accepted and who see that the accepted offer was made by a lender from the other class conclude that the borrower is *AA*. They offer a loan $\varphi(e_h - e_\ell)$ (i.e., top-up to a large loan) at price q_h^{AA} . An *AA*-borrower accepts such an offer. *A*-lenders whose offer was accepted (or whose offer was not accepted, but the accepted offer came from a lender of the same class), or *B*-lenders who observed that an offer was accepted, offer $\varphi(e_m - e_\ell)$ at price q_m^{AB} . An *AB*-borrower accepts such an offer from one of those lenders. (Notice that *A*-lenders making such an offer correctly predict that only an *AB*-borrower would accept their offer.) Finally, *B*-lenders who see that no offer was accepted conclude that this is a *BB*-borrower and offer her a risk-free small loan φe_ℓ at \bar{q} . A *BB*-borrower accepts such an offer.

This equilibrium without cross-subsidization may or may not feature excessive borrowing depending on parameter values (see the Appendix). Notice that when there is no excessive borrowing, this equilibrium achieves first best (i.e., symmetric-information equilibrium) payoffs for all players. Thus, credit-history building is costless in this case.

When credit-history building does come at a cost for the *AA*-borrowers, they will weigh the costs — cross-subsidization and/or excessive borrowing — with the benefits. The benefits, as we mentioned earlier, come from getting improved loan terms as a result of being identified as an *AA*-borrower. To make the assessment of benefits more formal, consider the following candidate equilibrium, in which heterogeneous information of the lenders is not aggregated.

Equilibrium without information aggregation. No lender makes an offer in stage 1. In stage 2, *A*-lenders offer a medium loan φe_m at $q_m^A = \Pr(AA|A)q_m^{AA} + \Pr(AB|A)q_m^{AB}$. All borrowers with such an offer accept it. *B*-lenders offer a small loan φe_ℓ and \bar{q} . All borrowers with only such offers (i.e., *BB*-borrowers) accept one.¹⁵

When a perfect Bayesian equilibrium with credit-history building results in excessive borrowing, *AA*-borrowers may prefer the equilibrium outcome without information aggregation. To illustrate this point, consider such a case with ρ very close to 1. Since the fraction of *AB*-borrowers (the probability of the pair of signals being *AB*) shrinks to 0 as ρ approaches 1, the no-information-aggregation outcome approaches that in the symmetric-information environment. In contrast, the equilibrium with credit-history building still

¹⁵The assumption of $\beta = 0$ is crucial for existence of such an equilibrium as it eliminates lenders' ability to cream-skim in stage 2.

features excessive borrowing, the cost of which does not shrink to 0 as ρ tends to 1.

Between a PBE with credit-history building and the PBE without information aggregation (which can coexist for some parameter values), our equilibrium selection picks the one preferred by *AA*-borrowers. Notice that when the two equilibria yield them exactly the same payoffs (which happens on a measure-zero set of parameter values), both of them survive our selection. In that case, these two equilibria are Pareto-ranked, because *BB*-borrowers get the same (small risk-free) loan in both equilibria, while *AB*-borrowers strictly prefer the equilibrium without information aggregation. While, in the equilibrium without information aggregation, *AB*-borrowers receive exactly the same loan and thus the same utility as *AA*-borrowers, in the equilibrium with credit-history building, they are necessarily worse off than *AA*-borrowers. Since *AA*-borrowers are indifferent between the two equilibria, *AB*-borrowers must prefer the equilibrium without information aggregation.

The mechanism of information aggregation in our model highlights the important distinction between credit-history building and improving one's credit score. Since the purpose of a credit score is to proxy a borrower's probability of repayment, and information aggregation leads to larger and hence riskier loans, the credit-history building that emerges in these equilibria would result in *lower* credit scores. Taking on an early loan communicates positive information to other lenders, lowering the posterior probability of default on a given loan size. But, since information aggregation induces borrowers to take on a *larger* loan, the resulting probability of default is increased (relative to those borrowers who do not take on early loans).

4 Numerical Analysis with Patient Borrowers

The key insights derived above carry over to the general model specified in Section 2 with $\beta > 0$. We now use a numerical example to illustrate possible equilibrium outcomes and to illustrate key comparative statics. We will then point out a novel insight on debt dilution that our model predicts.

4.1 Comparative Statics of Equilibrium Outcomes and Credit-History Building Costs

In this section, we now describe how equilibrium outcomes vary with parameter values by examining comparative statics with respect to the signal precision ρ . To illustrate the costs of credit-history building that arise in our model, we fully describe equilibrium strategies for a set of high values of the signal precision. We then show which costs are present in equilibrium at various levels of the signal precision.

Figure 1 illustrates (from the bottom to the top) equilibrium outcomes under symmetric information, equilibrium outcomes in our game, and the costs of credit-history building. For equilibrium outcomes, we only report the total loan sizes, using the following notation: xyz , with $x, y, z \in \{\ell, m, h\}$, meaning that BB -borrower's total loan is φe_x , AB 's is φe_y , and AA 's is φe_z . In the figure, we assume that the equilibrium outcome with uninformative signals would be mmm , i.e., a medium loan for all borrowers. Moreover, we assume that with arbitrarily informative signals (ρ close to one), under symmetric information there is full separation by loan size, i.e., we get the lmh outcome.¹⁶

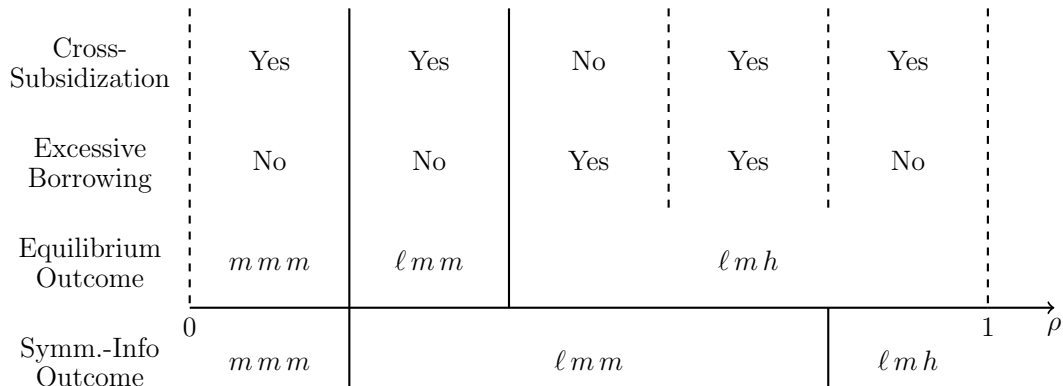


Figure 1: **Comparative statics with respect to the signal precision ρ .** Notation: lmh means φe_ℓ to BB -borrowers, φe_m to AB -borrowers, φe_h to AA -borrowers.

Consider how equilibrium outcomes change as ρ falls from 1 to 0. For ρ high enough, the equilibrium depicted in the figure (columns 4 and 5) is the lmh equilibrium with cross-subsidization described in Section 3. (We verified numerically that this is still an

¹⁶Specifically, the parameter values in the illustrative example are $e_\ell = 3$, $e_m = 8$, $e_h = 15$, $\varphi = 0.3$, $\alpha = 0.2$, $\pi(e_\ell, g) = 0$, $\pi(e_m, g) = 0.4$, $\pi(e_h, g) = 0.6$, and $\pi(e_\ell, b) = 0.7$, $\pi(e_m, b) = 0.3$, $\pi(e_h, b) = 0$, $u(c) = \ln c$, $\beta = \bar{q} = 1/1.05$.

equilibrium when $\beta > 0$.) When ρ is sufficiently close to one, there is no excessive borrowing in this case as *AA*-borrowers take on a large loan under symmetric information (column 5). Moreover, because the probability of receiving mixed signals (*AB*) is close to zero, the cost of cross-subsidization becomes arbitrarily small as ρ tends to 1. Hence, the costs of credit-history building are small when ρ is sufficiently large.

As ρ decreases, the size of the loan that an *AA*-borrower takes in the symmetric-information equilibrium falls from a large loan to a medium loan (column 4). The reason is that as ρ declines, an *AA*-borrower's perceived probability of receiving a high endowment in period II declines. That is, *AA*-borrowers become more pessimistic about their endowment process and choose to borrow less in period I (see the bottom row of Figure 1).

Consider what happens in our environment with private signals as ρ falls. Note that the switch from a large to medium loan by an *AA*-borrower does not happen at the same value of ρ in the asymmetric-information environment. Since *AA*-borrowers cross-subsidize *AB*-borrowers on the stage-1 loan, their period-I consumption is lower than they would obtain in the symmetric-information benchmark for the same size loan. To increase consumption in period I, *AA*-borrowers end up with a large instead of a medium loan. This is the excessive-borrowing feature that we have discussed earlier.

Note that with $\beta > 0$, there is an extra force that drives excessive borrowing in addition to the lack of self control mentioned in the previous section. When $\beta > 0$, the lower the borrower's proceeds from the stage-1 loan, the higher the desire to over-borrow in stage 2 because of diminishing marginal utility. Of course, cross-subsidization impacts the proceeds borrowers receive from stage-1 loans and therefore influences whether over-borrowing occurs or not.

As ρ decreases further, the likelihood that an *AA*-borrower repays a large loan falls and with it the price, q_h^{AA} . On the other hand, q_m^{AB} remains unchanged. This leads to a violation of the cross-subsidization equation (11) for sufficiently low signal precision. That is, for low enough ρ , *AB*-borrowers would no longer receive a subsidy if they were to take a stage-1 loan and thus prefer to wait for an actuarially fair-priced loan in stage 2. Hence we switch to an equilibrium without cross-subsidization, where only *AA*-borrowers accept an early loan—column 3 in the figure.

A further decrease in ρ makes *AA*-borrowers' endowment prospects less and less favorable, which causes their price of a large loan to fall. Ultimately, these borrowers prefer to switch from a large to a medium-size loan (column 2) in equilibrium. Of course, as that hap-

pens, AB -borrowers start accepting the early loan, and we again have cross-subsidization.

Finally, as ρ gets sufficiently close to zero, the information content of the signals vanishes. As a consequence, borrowers with different signal combinations have sufficiently similar endowment prospects. In equilibrium (as well as in the symmetric-information benchmark), all borrowers obtain a medium loan. Note that cross-subsidization only happens between AA - and AB -borrowers.

As Figure 1 illustrates, cross-subsidization takes place for large enough and small enough values of the signal precision, while excessive borrowing occurs for intermediate values of signal precision. Moreover, the two costs can occur simultaneously or one at a time.

4.2 More Dilution is Better

Our model generates a novel prediction, as can be seen in the equilibrium depicted in columns 4 and 5 of Figure 1. Notice that in this equilibrium, there is uncertainty for the stage-1 lender about how much his loan will be diluted in stage 2. If the borrower turns out to be an AA -borrower, the stage-1 loan will be diluted to a large loan, and if the borrower turns out to be an AB -borrower, the loan will be diluted to a medium loan. Although the lender earns zero profit ex ante, in which of these two scenarios is he better off? In other words, in which of the two cases is the probability of being repaid higher? The answer immediately follows from equation (11) and the definitions of q_h^{AA} and q_m^{AB} (equations (9) and (10)) — in this equilibrium, an AA -borrower is more likely to repay a large loan than an AB -borrower to repay a medium loan. That is, the incumbent lender is more likely to be repaid if he is diluted by more. We refer to this implication as the “more-dilution-is-better” result.

The result is contrary to the conventional wisdom that more dilution increases the probability of default. Indeed, for a borrower of a given risk/quality, a larger loan increases the probability of default (dilution effect). But here, less risky/better quality borrowers take out larger loans (selection effect). This selection effect dominates the dilution effect in the considered equilibrium. It is important to note that information aggregation is key for the “more-dilution-is-better result”: a larger top-up loan conveys positive information of the diluting lender.

Can more dilution ever be worse in this model? The answer is no. To see this, consider what happens if (11) is violated. In that case, AB -borrowers do not find it optimal to

accept the stage-1 loan, and hence only *AA*-borrowers will accept it. As a result, there is no heterogeneity in the size of the top-up loan. As Figure 1 illustrates, the “no-dilution-is-better” result is relevant for large enough values of the signal precision parameter ρ .

5 Conclusions

We have put forward a mechanism of credit-history building (by taking on loans) as a way of aggregating information across heterogeneously informed lenders. We illustrate this mechanism in a parsimonious model, which allows us to analyze costs and benefits associated with credit-history building and yields testable empirical implications. One particularly striking model implication concerns debt dilution. The standard mechanism, which is present in our model, implies that when a borrower of a given quality increases her overall loan size, she also increases her probability of default. On the other hand, the novel information-aggregation channel present in our model suggests that larger loans are chosen by higher quality (or less risky) borrowers. Hence, in our model, a lender prefers to see his borrower taking on a larger, rather than a smaller, additional loan from a competing lender. We plan on testing this empirically by using individual loan-level data.

Another testable implication is the very notion that taking out a loan makes a borrower more likely to be approved for other loans in the future. While we obviously do not expect this to be universally true (as taking on a loan may also signal an onset of a negative income or expense shock), we expect to find evidence of this channel when informational heterogeneity across lender is particularly salient. We are currently working on obtaining data that will enable us to test these model predictions.

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Appendix

A Symmetric Information Outcomes

To establish a benchmark for our analysis, consider the variant of our model environment, in which all lenders' signals are public information. The multi-stage nature of period I is irrelevant in this setting, as there is no need to aggregate any information. We can thus simply restrict attention to equilibria where all borrowing occurs in the last stage of the period, which avoids any concerns of debt dilution. All loans are then competitively priced, and we can simply think of the borrowers as choosing their preferred loan size, given actuarially fair interest rates appropriate for the specific type of the borrower.

All of the examples in the paper share one key feature of their symmetric information benchmark. Namely, the equilibrium outcome of in the limiting case as ρ approaches 1 features full separation in loan sizes between the three borrower types. I.e., for ρ arbitrarily close to 1, BB -borrowers take on a small loan, AB -borrowers choose a medium loan, and AA -borrowers get a large loan in the equilibrium of the symmetric information environment.¹⁷ The restrictions on the parameter values that yield this outcome, which we will sometimes refer to as lmh , are:

Assumption 1. *Assume that parameter values satisfy the following conditions:*

$$\left(1 - \frac{\delta}{2}\right) e_m > e_l \tag{15}$$

$$\frac{\delta}{2} e_h < \left(1 - \frac{\delta}{2}\right) e_m \tag{16}$$

$$\delta e_h > e_m \tag{17}$$

$$e_l > (1 - \delta) e_m \tag{18}$$

Proposition 1. *If parameter values satisfy Assumption 1, then symmetric-information equilibrium outcome is:*

1. *for ρ arbitrarily close to 1, BB -borrowers take on $(\phi e_l, \bar{q})$, AB -borrowers choose $(\phi e_m, q_m^{AB})$, and AA -borrowers get $(\phi e_h, q_h^{AA})$;*

¹⁷The only reason we are not referring to this limiting case as “full information” environment is that the latter does not have AB -borrowers.

2. for $\rho = 0$, all borrowers receive a medium-size loan.

Proof. First, note that the structure of the signals is such that the posterior regarding the underlying state of AB -borrowers is the same as uninformed prior and thus does not depend on the precision of the signal. Hence, conditions (15) and (16) guarantee that AB -borrowers choose the medium-size loan under actuarially fair loan pricing. But these same conditions then guarantee that all borrowers choose medium-size loans when signals are completely uninformative.

Condition (17) guarantees that, when signals are perfectly informative, AA -borrowers take on a large loan, if all prices are actuarially fair. Condition (15) guaranteed that BB -borrowers in this situation choose the small loan.

Note that this set of conditions also ensures that AA -borrowers do not choose the small loan. To see this, note that the condition for AA to prefer a medium loan to a small one is $(1 - \delta + \theta_h^{AA})e_m > e_l$, where θ_k^ω denotes the probability that a type- ω borrower will get endowment realization e_k in period II . Note that $\theta_h^{AA} > \delta/2$ whenever $\rho > 0$. Hence, since $(1 - \delta + \theta_h^{AA}) \geq 1 - \delta/2$ and our previous assumption ensures $(1 - \delta/2)e_m > e_l$, we know that AA -borrowers prefer a medium loan to a small one. \square

Proposition 2. *If parameter values satisfy Assumption 1, then*

1. *There exists $\rho^{BB} \in (0, 1)$ such that BB -borrowers take on $(\phi e_l, \bar{q})$ in the symmetric-information equilibrium whenever $\rho < \rho^{BB}$, and they choose $(\phi e_m, q_m^{BB})$ whenever $\rho > \rho^{BB}$.*
2. *There exists $\rho^{AA} \in (0, 1)$ such that AA -borrowers take on $(\phi e_h, q_h^{AA})$ in the symmetric-information equilibrium whenever $\rho > \rho^{AA}$, and they choose $(\phi e_m, q_m^{AA})$ whenever $\rho < \rho^{AA}$.*

Proof. Since borrowers are impatient, they simply maximize the size of the loan advance they receive in period I . The medium-size loan yields $\phi e_m q_m^\omega$ to a type- ω borrower, where $q_m^\omega = \bar{q}(1 - \delta Pr(N|\omega))$.

To establish the first part of the Proposition, simply note that $Pr(N|BB)$ is increasing in ρ , and thus q_m^{BB} is monotonically decreasing in ρ . On the other hand, the advance on the safe loan $(\phi e_l, \bar{q})$ is not affected by ρ . Since the medium-size loan is preferred by BB -borrowers for $\rho = 0$, and the small loan is preferred when $\rho = 1$ (as was established

in Proposition 1), there must be an interior ρ^{BB} , as described in the statement of this Proposition.

The advance on the large loan is given by $\phi e_h q_h^\omega$, where $q_h^\omega = \bar{q} \delta Pr(G|\omega)$. Just like in the case above, it is straightforward to show that $(\phi e_h q_h^{AA} - \phi e_m q_m^{AA})$ is monotonically increasing in ρ . And since the advance to an AA -borrower from a large loan is greater than that from a medium-size loan when $\rho = 1$, and since the opposite is true when $\rho = 0$ (both premises are guaranteed by Proposition 1), there must exist an interior ρ^{AA} described in the statement of this Proposition. Making this argument more explicit, the large loan yield a (weakly) larger loan advance whenever

$$\frac{\delta (1 + \rho)^2}{2 (1 + \rho^2)} e_h \geq \left(1 - \frac{\delta (1 - \rho)^2}{2 (1 + \rho^2)} \right) e_m.$$

This defines a quadratic equation in ρ that is strictly negative at $\rho = 0$ and strictly positive at $\rho = 1$. Under the above assumptions (specifically the second inequality), this quadratic equation is concave, implying that there is a unique root between 0 and 1. We will denote this root ρ_{AA} . □

B Equilibrium and Equilibrium Outcomes

In this section, we construct Perfect Bayesian equilibrium, maintaining the assumption that $\beta = 0$. The following formulae will prove useful throughout this section.

Preliminaries: Note that, since $\alpha = 1/2$, we have

$$Pr(G|A) = (1 + \rho)/2, \quad Pr(G|B) = (1 - \rho)/2$$

and

$$Pr(G|AA) = \frac{1 (1 + \rho)^2}{2 (1 + \rho^2)}, \quad Pr(G|AB) = \frac{1}{2}, \quad Pr(G|BB) = \frac{1 (1 - \rho)^2}{2 (1 + \rho^2)}.$$

Recall that we restrict stage-1 offers to ϕe_1 for $e_1 \in E$, and given a history ϕe_1 , we restrict stage-2 offers to $\phi(e_2 - e_1)$ for $e_2 \in E$ and $e_2 > e_1$.

B.1 Equilibrium Outcome 1: lmh with Cross-Subsidization

We construct an equilibrium with terminal loans $(BB, AB, AA) = (l, m, h)$, in which both AA - and AB -borrowers accept loans in the first stage. We then establish a set of sufficient and necessary conditions for it to be an equilibrium. We construct the equilibrium as follows:

- Stage 1:
 - A -lenders offer $\phi e_l @ q^A = Pr(AA|A)q_h^{AA} + Pr(AB|A)q_m^{AB}$
 - B -lenders offer nothing
 - AA - and AB -borrowers accept one A offer
- Stage 2:
 - Rejected A -lenders who sees stage-1 credit of $(\phi e_l, q^A)$ from the other class of lender offer $\phi(e_h - e_l) @ q_h^{AA}$
 - Accepted-class A -lenders and B -lenders who sees stage-1 credit of $(\phi e_l, q^A)$ offer $\phi(e_m - e_l) @ q_m^{AB}$
 - B -lender who sees no credit offers $\phi e_l @ q = 1$
 - Rejected A -lender who sees no credit history offers $\phi e_m @ q_m^{AB}$
 - Other off-equilibrium-path behavior is specified below, as we discuss the off-equilibrium-path beliefs
- Off-equilibrium-path beliefs and strategies: In stage 2, a stage-1-rejected lender who observes a credit record (from the other class of lenders) forms the following beliefs (about the signal observed by the other class of lenders):
 - For credit histories, $(\phi e_l, q)$, $\sigma^- = A$ if $q \geq q^A$ and $\sigma^- = B$ if $q < q^A$.
The strategies upon observing $q > q^A$ are:
 - * A -lenders of the accepted class and B -lenders of the other class offer $(\phi(e_m - e_l), q_m^{AB})$
 - * A -lenders of the rejected class offer $(\phi(e_h - e_l), q_h^{AA})$
 - * B -lenders of the accepted class offer $(\phi(e_m - e_l), q_m^{BB})$

The strategies upon observing $q < q^A$ are:

- * A -lenders of either class offer $(\phi(e_m - e_l), q_m^{AB})$
- * B -lenders of either class offer $(\phi(e_m - e_l), q_m^{BB})$
- For credit histories, $(\phi e_m, q)$, $\sigma^- = A$ if $q \geq \tilde{q}_m$ and $\sigma^- = B$ if $q < \tilde{q}_m$ where

$$\tilde{q}_m : \quad \tilde{q}_m e_m + q_h^{AA}(e_h - e_m) = q^A e_l + q_h^{AA}(e_h - e_l).$$

The strategies upon observing $q \geq \tilde{q}_m$ are:

- * A -lenders of the accepted class and B -lenders of the other class offer $(\phi(e_h - e_m), q_h^{AB})$
- * A -lenders of the rejected class offer $(\phi(e_h - e_m), q_h^{AA})$
- * B -lenders of the accepted class offer $(\phi(e_h - e_m), q_h^{BB})$

The strategies upon observing $q < \tilde{q}_m$ are:

- * A -lenders of either class offer $(\phi(e_h - e_m), q_h^{AB})$
- * B -lenders of either class offer $(\phi(e_h - e_m), q_h^{BB})$
- For credit histories, $(\phi e_h, q)$, $\sigma^- = A$ if $q \geq \tilde{q}_h$ and $\sigma^- = B$ if $q < \tilde{q}_h$ where $\tilde{q}_h \in [\tilde{q}_h^B, \tilde{q}_h^A]$ and

$$\begin{aligned} \tilde{q}_h^A : \quad \tilde{q}_h^A e_h &= q^A e_l + q_h^{AA}(e_h - e_l) \\ \tilde{q}_h^B : \quad \tilde{q}_h^B e_h &= q^A e_l + q_m^{AB}(e_m - e_l). \end{aligned}$$

No lenders make any offers after observing this credit history.

- Off-equilibrium-path strategies of the borrowers in stage 1 (we don't need to worry about their beliefs, as a deviation from a single lender does not change their information set — borrowers still learn their “type”):

- AA -borrowers
 - * accept ϕe_l loans at prices $q \geq q^A$, reject them at prices $q < q^A$
 - * accept ϕe_m loans at prices $q > \tilde{q}_m$, reject them at prices $q \leq \tilde{q}_m$
 - * accept ϕe_h loans at prices $q \geq \tilde{q}_h$, reject them at prices $q < \tilde{q}_h$
- AB -borrowers

- * accept ϕe_l loans at prices $q \geq q^A$, reject them at prices $q < q^A$
- * accept ϕe_m loans at prices $q \geq \hat{q}_m$, reject them at prices $q < \hat{q}_m$, where

$$\hat{q}_m : \quad \hat{q}_m e_m + q_h^{AB}(e_h - e_m) = q^A e_l + q_m^{AB}(e_m - e_l)$$

- * accept ϕe_h loans at prices $q \geq \hat{q}_h$, reject them at prices $q < \hat{q}_h$, where

$$\hat{q}_h : \quad \hat{q}_h e_h = q^A e_l + q_m^{AB}(e_m - e_l)$$

– *BB*-borrowers

- * accept ϕe_l loans at prices $q \geq \min\{\tilde{q}_l, q^A\}$, reject them at prices $q < \min\{\tilde{q}_l, q^A\}$, where

$$\tilde{q}_l : \quad \tilde{q}_l e_l + \max\{q_h^{BB}(e_h - e_l), q_m^{BB}(e_m - e_l)\} = e_l$$

If for some reason $\tilde{q}_l > q^A$, then the threshold above should be q^A , since it allows *BB*-borrowers to fool some lenders.

- * accept ϕe_m loans at prices $q \geq \hat{q}_m^B$, reject them at prices $q < \hat{q}_m^B$, where

$$\hat{q}_m^B : \quad \hat{q}_m^B e_m + q_h^{BB}(e_h - e_m) = e_l$$

- * accept ϕe_h loans at prices $q \geq \frac{e_l}{e_h}$, reject them at prices $q < \frac{e_l}{e_h}$

Under our above assumptions, it is useful to note a few relationships between the various thresholds characterizing the off-equilibrium-path beliefs.

Lemma 1. 1. $\tilde{q}_m \geq q^A$.

2. If $q_h^{AA} \geq q_m^{AB}$, then $\tilde{q}_h^A > \tilde{q}_h^B$.

Proof. The first claim follows from the fact that the definition of \tilde{q}_m may be written as

$$\tilde{q}_m e_m = q^A e_l + q_h^{AA}(e_m - e_l) \geq q^A e_m.$$

The second part of the claim follows since

$$\begin{aligned}
q^A e_l + q_h^{AA}(e_h - e_l) &= q^A e_l + q_h^{AA}(e_h - e_m + e_m - e_l) \\
&\geq q^A e_l + q_h^{AA}(e_h - e_m) + q_m^{AB}(e_m - e_l) \\
&\geq q^A e_l + q_m^{AB}(e_m - e_l) \\
&= \tilde{q}_h^B e_h.
\end{aligned}$$

□

B.1.1 Equilibrium Conditions

Incentives. We now prove that no agent (borrower or lender) has any incentives to deviate from the prescribed equilibrium. Consider first deviations by borrowers.

1A. An *AA*-borrower could reject the stage-1 loan. Accepting is optimal as long as

$$q^A e_l + q_h^{AA}(e_h - e_l) \geq q_m^{AB} e_m.$$

1B. An *AB*-borrower could reject the stage-1 loan. Accepting is optimal as long as

$$q^A e_l + q_m^{AB}(e_m - e_l) \geq q_m^{AB} e_m.$$

Note that the *AB*-borrower's incentive constraint is satisfied if and only if $q_h^{AA} \geq q_m^{AB}$ (implicitly in the definition of q^A). Moreover, if this incentive constraint is satisfied, then the *AA*-borrower's incentive constraint is also satisfied.¹⁸ Note, $q_h^{AA} \geq q_m^{AB}$ holds when

$$\frac{\delta}{2} \frac{(1 + \rho)^2}{1 + \rho^2} \geq 1 - \frac{\delta}{2}.$$

This is a quadratic equation in ρ . For it to have a solution (e.g., for ρ large enough to exist such that this is satisfied), we require $\delta \geq 2/3$.

¹⁸To see this, write her incentive constraint as

$$q^A e_l + q_h^{AA}(e_m - e_l) + q_h^{AA}(e_h - e_m) \geq q_m^{AB} e_l + q_m^{AB}(e_m - e_l).$$

That $q_h^{AA} \geq q_m^{AB}$ implies that the *AA*-borrower receives a higher price on the first and second part of the loan and she receives even more on the final top-up to e_h .

1C. An AB -borrower could reject the stage-1 loan and take the B offer. This requires

$$q^A e_l + q_m^{AB} (e_m - e_l) \geq e_l.$$

Notice, though, that by 1B, we know

$$q^A e_l + q_m^{AB} (e_m - e_l) \geq q_m^{AB} e_m,$$

where $q_m^{AB} = 1 - \delta/2$, and by the symmetric info assumptions, we know $(1 - \delta/2)e_m > e_l$, hence 1B implies 1C. Moreover, 1C implies an AA -borrower would not pursue this strategy also.

We now consider lender deviations (and borrowers' best responses). Consider first stage-2 deviations by lenders.

2A. B -lenders who see a credit record and A -lenders whose peers' offer was accepted could offer $\phi(e_h - e_l) @ q_h^{AB}$. For this to be unprofitable, it must be that AB -borrowers in stage 2 do not want to accept this over their fair top-up to a medium loan. Or,

$$q_m^{AB} (e_m - e_l) \geq q_h^{AB} (e_h - e_l).$$

This is equivalent to requiring

$$\left(1 - \frac{\delta}{2}\right) e_m - \frac{\delta}{2} e_h \geq (1 - \delta) e_l.$$

2b. A -lenders whose stage-1 offer was rejected could offer $\phi(e_m - e_l) @ q_m^{AA}$. For this to be unprofitable, AA informed borrowers must not accept, or

$$q_h^{AA} (e_h - e_l) \geq q_m^{AA} (e_m - e_l)$$

or

$$\frac{\delta}{2} \frac{(1 + \rho)^2}{1 + \rho^2} (e_h - e_l) \geq \left(1 - \frac{\delta}{2} \frac{(1 - \rho)^2}{1 + \rho^2}\right) (e_m - e_l).$$

We know this is satisfied at $\rho = 1$ under our previous assumptions (namely $\delta e_h > e_m$).

2C. B -lenders who do not see a credit history could offer $\phi e_m @ q_m^{BB}$. For this to be

unprofitable, we require

$$\phi e_l \geq q_m^{BB} \phi e_m$$

or

$$e_l \geq \left(1 - \delta + \frac{\delta(1 - \rho)^2}{2(1 + \rho^2)}\right) e_m.$$

We next consider deviations by lenders in stage 1. Consider first A -lenders.

- 3A. An A -lender could offer ϕe_l at a price below q^A . No borrower would accept this. An A -lender could offer ϕe_l at a higher price ($q > q^A$). This offer would be accepted by both AA s and AB s but necessarily loses money since it will be topped up in stage 2 as the equilibrium offer. ($\beta = 0$ makes this last claim trivial.)
- 3B. An A -lender could offer ϕe_m at some price q . Note that for any q , the subgame payoff to an AB from accepting this offer is

$$q\phi e_m + q_h^{AB}\phi(e_h - e_m).$$

- **Proof.** The critical step in the proof is to show that the class of A -lenders whose deviating offer was accepted will offer a stage-2 top-up to $\phi(e_h - e_m)$ @ $q = q_h^{AB}$. Why? Note that if $q \geq \tilde{q}_m$, then a rejected (class) of A -lenders will infer the borrower is an AA type and offer the high top-up @ q_h^{AA} . A (class) of B -lenders will make the same inference and offer an AB -priced top-up. As a result, the deviating class of A -lenders know that, for $q \geq q_h^{AA}$, they necessarily lose money (above q_h^{AA} they must earn negative profits, at q_h^{AA} they get AA s and AB s with positive probability and lose). And for $q > q_h^{AB}$, they would only attract AB s with positive probability and lose money on those borrowers. Hence, they bid the price up to q_h^{AB} . If instead $q < \tilde{q}_m$, rejected A -lenders would infer the borrower is AB , and B -lenders would infer the borrower is BB , so that the best price in the market would be q_h^{AB} . Since AA s would not accept this deviation (by construction of \tilde{q}_m), the deviating class of lenders knows the borrower is AB and therefore offers at most a top up to ϕe_h @ q_h^{AB} .

We now show such deviations by A -lenders are unprofitable.

- Case 1: $q \geq \tilde{q}_m$. For any such q , both AA - and AB -borrowers will accept the

deviation offer. AA s accept by construction of \tilde{q}_m . AB s accept because $\tilde{q}_m \geq q^A$, and therefore

$$q\phi e_m \geq q^A\phi e_m \geq q^A\phi e_l + q_m^{AB}\phi(e_m - e_l).$$

(Notice, AB s would accept independent of the top-up they receive in stage 2.) Since in this subgame AA s and AB s accept and top-up to a large loan and $q \geq \tilde{q}_m > q^A$, this loan must lose money (q^A is the actuarially fair price when AA s and AB s accept and AB s top up to a medium loan while AA s top up to a large loan).

- Case 2: $q < \tilde{q}_m$. By construction of \tilde{q}_m , AA s do not accept. Note that for all $q < q_h^{AB}$, AB s also do not accept since their subgame payoff would satisfy

$$qe_m + q_h^{AB}(e_h - e_m) < q_h^{AB}e_h < q_m^{AB}e_m < q^Ae_l + q_m^{AB}(e_m - e_l).$$

Hence, if AB s accept, the price is larger than q_h^{AB} and the deviator must lose money.

- 3C. An A -lender could offer ϕe_h at some q . Exactly as for ϕe_m deviations, offers $q \geq \tilde{q}_h^A$ will attract AA s and AB s and lose money. Below \tilde{q}_h^A , at best only AB s will accept. Offers below q_h^{AB} will be rejected, and offers above, if accepted, lose money.

Consider finally stage-1 deviations by B -lenders.

- 4A. A B -lender could offer ϕe_l at some q . If $q \geq q^A$, then AB s accept and top-up to a large loan (since rejected A -lenders now believe the borrower is an AA type and $q_h^{AA}(e_h - e_l) > q_m^{AA}(e_m - e_l)$). Similarly, BB s accept and top-up to a medium loan at price q_m^{AB} . As a result, the deviation must earn negative profits. If $q < q^A$, then at best only BB -borrowers accept.

- If $q_h^{BB}(e_h - e_l) > q_m^{BB}(e_m - e_l)$, then if the BB -borrower accepts the deviation, she will top up to a large loan. In this case,

$$qe_l + q_h^{BB}(e_h - e_l) \geq e_l > q_h^{BB}e_h$$

implying $q > q_h^B$ and so the deviation earns negative profit.

- If $q_m^{BB}(e_m - e_l) > q_h^{BB}(e_h - e_l)$, then if the BB -borrower accepts the deviation, she will top up to a medium loan. In this case,

$$qe_l + q_m^{BB}(e_m - e_l) \geq e_l > q_m^{BB}e_m$$

implying $q > q_m^{BB}$, and so the deviation earns negative profit.

4B. A B -lender could offer ϕe_m at some q . If $q \geq \tilde{q}_m$, both AB s and BB s accept (in the following subgame, AB s behave like AA s and BB s behave like AB s), and at these prices, an A -lender would lose money, so such deviations must lose money as well. If $q \in [\tilde{q}_m^B, \tilde{q}_m^A]$, then AB s accept and top-up to a large loan. But, since $\tilde{q}_m^B > q_h^{AB}$, which is the actuarially fair price, the deviation loses money on the AB -borrower. If BB s also accept, the deviation loses money on them as well. Finally, if $q < \tilde{q}_m^B$, then only BB s may accept. When $q \leq q_h^{BB}$, BB s do not accept, and above q_h^{BB} , they may accept, but such a deviation earns negative profit.

4C. A B -lender could offer ϕe_h at some price q . If $q < \tilde{q}_h^B$, at best only BB s accept, and since they reject when $q \leq q_h^{BB}$, any time they accept, such loans earn negative profit. When $q \geq \tilde{q}_h^B$, the deviation earns negative profits on AB s, since $\tilde{q}_h^B > q_h^{AB}$.¹⁹ Furthermore, such an offer also attracts BB -borrowers and loses money on them as well.

B.1.2 Excessive Borrowing

We can establish conditions such that over-borrowing occurs with cross-subsidization. To see this, note that we need AA to take a medium-size loan in the symmetric outcome. This requires $\rho < \rho^{AA}$. Second, we need the conditions for the lmh equilibrium to exist to hold, which, other than a restriction on δ , are conditions that ρ be large enough. I want to prove that the lower bounds on ρ that arise from equilibrium are below ρ^{AA} . To do so, it's useful to summarize the posterior probability that AA receives high income at these thresholds.

Let $\theta_h = \frac{\delta}{2}(1 + \rho)^2 / (1 + \rho^2)$. Recall that $\theta_h(\rho)$ is monotone increasing in ρ .

¹⁹This follows from

$$\tilde{q}_h^B e_h = q^A e_l + q_m^{AB}(e_m - e_l) > q_m^{AB} e_m > q_h^{AB} e_h.$$

- ρ_{AA} is defined as the ρ such that

$$\theta_h e_h = (1 - \delta + \theta_h) e_m$$

or

$$\theta_h^1 = (1 - \delta) \frac{e_m}{e_h - e_m}.$$

- For lmh equilibrium, we need $q_h^{AA} \geq q_m^{AB}$, which requires

$$\theta_h \geq 1 - \frac{\delta}{2}.$$

Hence, suppose $\theta_h^1 > 1 - \delta/2$. (Computationally, such δ exist.)

- For lmh equilibrium, we also need the AA -lender to not top-up to a medium loan. This incentive is satisfied as long as

$$\theta_h(e_h - e_l) \geq (1 - \delta + \theta_h)(e_m - e_l)$$

or

$$\theta_h \geq (1 - \delta) \frac{e_m - e_l}{e_h - e_m}.$$

Clearly, $\theta_h^1 > (1 - \delta)(e_m - e_l)/(e_h - e_m)$.

Hence, as long as we can find δ that satisfy the initial assumptions and

$$(1 - \delta) \frac{e_m}{e_h - e_m} \geq 1 - \frac{\delta}{2},$$

we can sustain over-borrowing. This inequality can be written as

$$(1 - \delta)e_m \geq e_h - e_m - \frac{\delta}{2}(e_h - e_m)$$

or

$$\delta(e_h - 3e_m) \geq 2(e_h - 2e_m).$$

Note that if $e_h \geq 3e_m$, then this requires $\delta \geq 1$. So we will impose $e_h \leq 3e_m$, and this

inequality is then an upper bound on δ :

$$\delta \leq \frac{2(e_h - 2e_m)}{e_h - 3e_m}.$$

Since we need $\delta \geq 0$, we must also therefore have $e_h < 2e_m$. So we re-write the upper bound as

$$\delta \leq \frac{2(2e_m - e_h)}{3e_m - e_h}.$$

Combining all of the necessary inequalities, we need

$$\max \left\{ \frac{e_m}{e_h}, \frac{e_m - e_l}{e_m}, \frac{2}{3} \right\} \leq \delta \leq \min \left\{ \frac{2(e_m - e_l)}{e_m}, \frac{2e_m}{e_m + e_h}, \frac{2(e_m - e_l)}{e_m + e_h - 2e_l}, \frac{2(2e_m - e_h)}{3e_m - e_h} \right\}.$$

Summary. We have shown if $e_m \geq 2e_h/3$ and the above restrictions on δ hold, there must exist $\rho \leq \rho_{AA}$ such that lmh is an equilibrium outcome while $(l/m)mm$ is the symmetric info outcome so that over-borrowing occurs.

B.2 Equilibrium Outcome 2: lmh with No Cross-Subsidization

We construct an equilibrium with terminal loans $(BB, AB, AA) = (l, m, h)$, in which only AA -borrowers accept the first-stage loan, and establish a set of sufficient and necessary conditions for it to be an equilibrium. We construct the equilibrium as follows:

- Stage 1:
 - A -lenders offer $\phi e_l @ q_h^{AA}$
 - B -lenders offer nothing
 - AA -borrowers accept one A offer
 - AB -borrowers reject their one A offer
- Stage 2:
 - Rejected A -lenders who sees stage-1 credit of $(\phi e_l, q_h^{AA})$ from the other class of lender offer $\phi(e_h - e_l) @ q_h^{AA}$
 - Rejected A -lenders who see no credit history offer $(\phi e_m, q_m^{AB})$
 - Accepted-class A -lenders offer $\phi(e_m - e_l) @ q_m^{AB}$

- B -lender who sees no credit offers $\phi e_l @ q = 1$
 - B -lender who sees stage-1 credit of $(\phi e_l, q_h^{AA})$ from the other class of lender offers $\phi(e_m - e_l) @ q_m^{AB}$
 - B -lender who sees stage-1 credit of $(\phi e_l, q_h^{AA})$ from their own class of lender offers $\phi(e_m - e_l) @ q_m^{BB}$
 - Other off-equilibrium-path behavior is specified below, as we discuss the off-equilibrium-path beliefs
- Off-equilibrium-path beliefs and strategies: In stage 2, a lender who observes a credit record (from the other class of lenders) forms the following beliefs (about the signal observed by the other class of lenders):

- For credit histories, $(\phi e_l, q)$, $\sigma^- = A$ if $q \geq q_h^{AA}$ and $\sigma^- = B$ if $q < q_h^{AA}$.

The strategies upon observing $q > q_h^{AA}$ are:

- * A -lenders of the accepted class and B -lenders of the other class offer $(\phi(e_m - e_l), q_m^{AB})$
- * A -lenders of the rejected class offer $(\phi(e_h - e_l), q_h^{AA})$
- * B -lenders of the accepted class offer $(\phi(e_m - e_l), q_m^{BB})$

The strategies upon observing $q < q_h^{AA}$ are:

- * A -lenders of either class offer $(\phi(e_m - e_l), q_m^{AB})$
- * B -lenders of either class offer $(\phi(e_m - e_l), q_m^{BB})$

- For credit histories, $(\phi e_m, q)$, $\sigma^- = A$ if $q \geq q_h^{AA}$ and $\sigma^- = B$ if $q < q_h^{AA}$. The strategies upon observing $q \geq q_h^{AA}$ are:

- * A -lenders of the accepted class and B -lenders of the other class offer $(\phi(e_h - e_m), q_h^{AB})$
- * A -lenders of the rejected class offer $(\phi(e_h - e_m), q_h^{AA})$
- * B -lenders of the accepted class offer $(\phi(e_h - e_m), q_h^{BB})$

The strategies upon observing $q < q_h^{AA}$ are:

- * A -lenders of either class offer $(\phi(e_h - e_m), q_h^{AB})$
- * B -lenders of either class offer $(\phi(e_h - e_m), q_h^{BB})$

- For credit histories, $(\phi e_h, q)$, $\sigma^- = A$ if $q \geq q_h^{AA}$ and $\sigma^- = B$ if $q < q_h^{AA}$. No lenders make any offers after observing this credit history.
- Off-equilibrium-path strategies of the borrowers in stage 1 (we don't need to worry about their beliefs, as a deviation from a single lender does not change their information set — borrowers still learn their “type”):

– *AA*-borrowers

- * accept ϕe_l loans at prices $q \geq q_h^{AA}$, reject them at prices $q < q_h^{AA}$
- * accept ϕe_m loans at prices $q > q_h^{AA}$, reject them at prices $q \leq q_h^{AA}$
- * accept ϕe_h loans at prices $q \geq q_h^{AA}$, reject them at prices $q < q_h^{AA}$

– *AB*-borrowers

- * accept ϕe_l loans at prices $q \geq \hat{q}_l$, reject them at prices $q < \hat{q}_l$, where

$$\hat{q}_l : \quad \hat{q}_l e_l + \max\{q_h^{AB}(e_h - e_l), q_m^{AB}(e_m - e_l)\} = q_m^{AB} e_m$$

- * accept ϕe_m loans at prices $q \geq \hat{q}_m$, reject them at prices $q < \hat{q}_m$, where

$$\hat{q}_m : \quad \hat{q}_m e_m + q_h^{AB}(e_h - e_m) = q_m^{AB} e_m$$

- * accept ϕe_h loans at prices $q \geq \hat{q}_h$, reject them at prices $q < \hat{q}_h$, where

$$\hat{q}_h : \quad \hat{q}_h e_h = q_m^{AB} e_m$$

– *BB*-borrowers

- * accept ϕe_l loans at prices $q \geq \min\{\tilde{q}_l, q_h^{AA}\}$, reject them at prices $q < \min\{\tilde{q}_l, q_h^{AA}\}$, where

$$\tilde{q}_l : \quad \tilde{q}_l e_l + \max\{q_h^{BB}(e_h - e_l), q_m^{BB}(e_m - e_l)\} = e_l$$

If for some reason $\tilde{q}_l > q_h^{AA}$, then the threshold above should be q_h^{AA} , since it allows *BB*-borrowers to fool some lenders.

- * accept ϕe_m loans at prices $q \geq \hat{q}_m^B$, reject them at prices $q < \hat{q}_m^B$, where

$$\hat{q}_m^B : \quad \hat{q}_m^B e_m + q_h^{BB}(e_h - e_m) = e_l$$

* accept ϕe_h loans at prices $q \geq \frac{e_l}{e_h}$, reject them at prices $q < \frac{e_l}{e_h}$

B.2.1 Equilibrium Conditions

Incentives. We now establish the set of condition that ensure no agent (borrower or lender) has an incentive to deviate from the prescribed equilibrium. Consider first deviations by borrowers:

1A. An *AA*-borrower could reject the stage-1 loan. Accepting is optimal as long as

$$q_h^{AA} e_h \geq q_m^{AB} e_m.$$

That is

$$\frac{\delta (1 + \rho)^2}{2 (1 + \rho^2)} e_h \geq \left(1 - \frac{\delta}{2}\right) e_m.$$

1B. An *AB*-borrower could accept the stage-1 loan. Rejecting is optimal as long as

$$q_h^{AA} e_l + q_m^{AB} (e_m - e_l) \leq q_m^{AB} e_m.$$

Note that the *AB*-borrower's incentive constraint is satisfied if and only if $q_h^{AA} \leq q_m^{AB}$.

Note, $q_h^{AA} \leq q_m^{AB}$ holds when

$$\frac{\delta (1 + \rho)^2}{2 (1 + \rho^2)} \leq 1 - \frac{\delta}{2}.$$

1C. An *AB*-borrower could reject the stage-2 loan and take the *B* offer. This requires

$$q_m^{AB} e_m \geq e_l.$$

That is,

$$\left(1 - \frac{\delta}{2}\right) e_m \geq e_l,$$

which is condition 15. Note that 1A and 1C imply that an *AA*-borrower would not pursue this strategy also.

We now consider lender deviations (and borrowers' best responses). Consider first stage-2 deviations by lenders.

2A. *A*-lenders, who see (equilibrium) credit offer of the other class accepted, could offer a top-up to a medium-size loan. For this to be unprofitable, *AA*-borrowers must prefer the large loan:

$$q_h^{AA}(e_h - e_l) \geq q_m^{AA}(e_m - e_l).$$

2B. *A*-lenders, who see no credit history, could offer a large loan at *AB* price. For this to be unprofitable, *AB*-borrowers must prefer medium-size loan:

$$q_m^{AB} e_m \geq q_h^{AB} e_h.$$

2C. *B*-lenders (who see no credit history) could offer a medium-size loan at *BB* price. For this to be unprofitable, *BB*-borrowers must prefer small loan:

$$e_l \geq q_m^{BB} e_m.$$

We don't need to worry about a large loan offer — if *AB*-borrowers prefer medium to large, then *BB*-borrowers will too.

2D. *B*-lenders could offer loans at prices better than *BB*-prices, but winners curse makes that unprofitable.

B.3 Equilibrium Outcome 3: No Credit-History Building

We construct an equilibrium with no information aggregation and with terminal loans $(B, AA) = (l, m)$. No offers are made (or accepted) in this equilibrium in stage 1. We establish a set of sufficient and necessary conditions for it to be an equilibrium. We construct the equilibrium as follows:

- Stage 1:
 - Lenders make no offers
 - Borrowers have no action to take on equilibrium path at this stage
 - Borrowers do not learn anything at this stage on equilibrium path
- Stage 2:

- A -lenders offer $\phi e_m @ q_m^A = \bar{q} (1 - \theta_l^A)$
- B -lenders offer $(\phi e_l, \bar{q})$
- Borrowers simply accept the contract that yields the largest loan advance, regardless of whether that contract is on- or off-equilibrium-path
- Other off-equilibrium-path behavior is specified below, as we discuss the off-equilibrium-path beliefs

- Off-equilibrium-path beliefs and strategies:

In stage 2, a lender who observes a credit record (from the other class of lenders) forms the following beliefs (about the signal observed by the other class of lenders):

- For credit histories $(\phi e_l, q)$, $\sigma^- = A$ if $q \geq \hat{q}_l$ and $\sigma^- = B$ if $q < \hat{q}_l$, where \hat{q}_l is defined below, by equation (19) or (20), depending on the relevant case (parameter values).

The strategies upon observing $q \geq \hat{q}_l$ are:

- * A -lenders of the accepted class and B -lenders of the other class offer $(\phi(e_m - e_l), q_m^{AB})$
- * A -lenders of the rejected class offer $(\phi(e_h - e_l), q_h^{AA})$
- * B -lenders of the accepted class offer $(\phi(e_m - e_l), q_m^{BB})$

The strategies upon observing $q < \hat{q}_l$ are:

- * A -lenders of either class offer $(\phi(e_m - e_l), q_m^{AB})$
- * B -lenders of either class offer $(\phi(e_m - e_l), q_m^{BB})$

- For credit histories $(\phi e_m, q)$, $\sigma^- = A$ if $q \geq \hat{q}_m$ and $\sigma^- = B$ if $q < \hat{q}_m$, where \hat{q}_m is defined by:

$$\hat{q}_m e_m + \left(\frac{1 + \rho^2}{2} q_h^{AA} + \frac{1 - \rho^2}{2} q_m^{AB} \right) (e_h - e_m) = q_m^A e_m$$

The strategies upon observing $q \geq \hat{q}_m$ are:

- * A -lenders of the accepted class and B -lenders of the other class offer $(\phi(e_h - e_m), q_h^{AB})$
- * A -lenders of the rejected class offer $(\phi(e_h - e_m), q_h^{AA})$
- * B -lenders of the accepted class offer $(\phi(e_h - e_m), q_h^{BB})$

The strategies upon observing $q < \hat{q}_m$ are:

- * A -lenders of either class offer $(\phi(e_h - e_m), q_h^{AB})$
- * B -lenders of either class offer $(\phi(e_h - e_m), q_h^{BB})$
- For credit histories $(\phi e_h, q)$, $\sigma^- = A$ if $qe_h \geq q_m^A e_m$ and $\sigma^- = B$ if $qe_h \geq q_m^A e_m$.
No lenders make any offers after observing this credit history.

- Off-equilibrium-path beliefs and strategies of the borrowers in stage 1 (we now do need to worry about their beliefs about the signal of the proposing lender, as the borrowers were not expecting to learn anything at this stage):

- For offers of $(\phi e_l, q)$, we have to consider two possible cases:

Case 1 arises when (parameter values are such that) an AB -borrower faced with actuarially fair prices would top up a small loan from the first stage to a medium-sized loan in the second stage, i.e., $q_m^{AB}(e_m - e_l) \geq q_h^{AB}(e_h - e_l)$.

Case 2 is when the above inequality is reversed, i.e., when an AB -borrower prefers the top-up to a large loan.

Define the following cutoff values of prices, \hat{q}_l^1 and \hat{q}_l^2 , for these two cases:

$$\hat{q}_l^1 e_l + \frac{1 + \rho^2}{2}(e_h - e_l)q_h^{AA} + \frac{1 - \rho^2}{2}(e_m - e_l)q_m^{AB} = q_m^A e_m \quad (19)$$

$$\hat{q}_l^2 e_l + \left(\frac{1 + \rho^2}{2}q_h^{AA} + \frac{1 - \rho^2}{2}q_m^{AB} \right) (e_h - e_l) = q_m^A e_m \quad (20)$$

These cutoffs identify the willingness of the borrower to accept the first-stage offer, if that offer were to be interpreted as having come from an A -lender. The borrower's equilibrium strategy in response to the first-stage offer (deviation) is thus

- * accept ϕe_l loans at prices $q \geq \hat{q}_l$, reject them at prices $q < \hat{q}_l$

Define also the cutoffs for the profitability of a possible stage-1 deviation by an A -lender in the two cases:

$$\check{q}_l^1 = \frac{1 + \rho^2}{2}q_h^{AA} + \frac{1 - \rho^2}{2}q_m^{AB} = \frac{(1 + \rho)^2 \delta}{4} + \frac{(1 - \rho^2)(2 - \delta)}{4} \quad (21)$$

$$\check{q}_l^2 = \frac{1 + \rho^2}{2}q_h^{AA} + \frac{1 - \rho^2}{2}q_h^{AB} \quad (22)$$

- For offers of $(\phi e_m, q)$, the borrower’s strategy is to accept the loan at prices $q \geq \hat{q}_m$, and reject at prices $q < \hat{q}_m$
- For offers of $(\phi e_h, q)$, the borrower accepts if $q e_h \geq q_m^A e_m$ and rejects otherwise

C Equilibrium Selection

Bayesian Perfect equilibria with and without credit-history building co-exist for a non-trivial set of parameter values. Our selection criterion generically selects a unique equilibrium outcome across these equilibria — we simply pick the equilibrium that delivers greater utility (i.e., larger period- I loan advance) to AA -borrowers.

Proposition 3. *If parameter values satisfy Assumption 1, then*

- *If $q_1 e_l + q_h^{AA}(e_h - e_l) > q_m^A e_m$, then the unique equilibrium outcome is that of the equilibrium with credit-history building*
- *If $q_1 e_l + q_h^{AA}(e_h - e_l) < q_m^A e_m$, then the unique equilibrium outcome is that of the equilibrium without credit-history building*
- *If $q_1 e_l + q_h^{AA}(e_h - e_l) = q_m^A e_m$, then there multiple equilibria that satisfy our selection criterium, one of which features credit-history building, while another one has no information aggregation. AA - and BB -borrowers are indifferent between the two equilibria. AB -borrowers are strictly better off in the equilibrium with no credit-history building.*

In the expressions above, q_1 is the price of the stage-1 loan in the credit-history building equilibrium, which, depending on parameter values, is equal to either q^A (defined in Section B.1) or q_h^{AA} . On the other hand, $q_m^A = \bar{q} \left(1 - \frac{\delta}{2}(1 - \rho)\right)$ is the price of the (pooled) second-stage loan offered by A -lenders in the no-information-aggregation equilibrium.

The following proposition, while less general, gives a clearer insight into the economic mechanisms behind the equilibrium selection:

Proposition 4. *If the candidate equilibrium with credit-history building does not feature excessive borrowing, then it is the one that survives the equilibrium selection. I.e., that equilibrium’s outcome is preferred by AA -borrowers to the outcome of the (potential) equilibrium with no information aggregation.*