Object Dependent Manifold Priors for Image Deconvolution

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Deblurring is an inverse problem which has traditionally been studied from a signal processing perspective. In this paper we consider the role of extra information in the form of prior knowledge of the object class to solve this problem. Specifically, we incorporate unlabeled image data of the object class, say natural images, in the form of a patch-manifold prior for the object class. The manifold is implicitly estimated from the given unlabeled data. We show how the patch manifold prior effectively exploits the availability of the sample class data for regularizing the deblurring problem.

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1. Introduction

Given a degraded image that is a convolution of an image with a linear time-invariant Point Spread Function (PSF) and then corrupted by additive noise, deconvolution aims to obtain an optimal estimate of the input image [2–4]. Examples of image degradation include blurring introduced by camera motion, defocusing as well as the noise introduced from the electronics of the system. The process of deconvolution is known to be an ill-posed problem [1, 6, 11, 14]. As a result, regularization is often used. The simplest and most common approach is to use quadratic functions of the unknown quantities, which leads to Tikhonov regularization [6]. These methods lead to computationally straightforward optimization problems, but they suppress useful features in the resulting image, such as edges. Recently, considerable effort has been spent in designing alternative, sparsity constraints which preserve such features. Methods based on these sparsity constraints have been successfully used for image deconvolution (c.f. [5, 7–10]).

In this paper, we consider the problem of exploiting extra information in the form of prior knowledge of the object class to regularize the inverse problem. Specifically, we use image data of the object class as the available information. This approach can broadly be termed as example-based image-enhancement [16]. We impose a patch-manifold prior for the object class [12], where the manifold is implicitly estimated from the given unlabeled data. We show how the patch manifold prior effectively exploits the availability of the example data for regularizing the deblurring problem. In what follows, we first define the problem of deconvolution. Then, a method based on a manifold prior is described. Finally, some numerical simulations are presented.

2. The Image Deblurring Problem

Since a digitally recorded image is on a finite discrete grid, an image deconvolution problem is formulated as a matrix inversion problem. Without loss of generality, assume the recorded arrays are of size $N \times N$. Let γ denote an $N \times N$ array of samples from a zero mean additive white Gaussian noise (AWGN) with variance σ^2. Given the $N \times N$ arrays y and x, representing the observed image and the image to be estimated, respectively, the matrix deconvolution problem can be described as

$$\mathbf{y} = H\mathbf{x} + \boldsymbol{\gamma}, \tag{1}$$

where $\mathbf{y}$, $\mathbf{x}$, and $\boldsymbol{\gamma}$ are $N^2 \times 1$ column vectors representing the arrays y, x, and γ lexicographically ordered, and H is the $N^2 \times N^2$ matrix that models the blur operator. In the case when H is a block-circulant-circulant-block matrix, the problem can be described as

$$y(n_1, n_2) = (x \circledast h)(n_1, n_2) + \gamma(n_1, n_2), \tag{2}$$

where $0 \leq n_1, n_2 \leq N - 1$, $\circledast$ denotes circular convolution, and h denotes the point spread function (PSF) of a linear space-invariant system.

3. Regularization with Manifold Model

We use the manifold prior model of [12] for regularizing the deblurring problem. Briefly, the prior states that given a class of images - say faces, or natural images – the set of all patches (e.g. 3×3) from the image live on a *manifold*. Let us denote the patch extracted from the image x, at location $q \in [0, 1]^2$ of width $\tau > 0$ by $p_q(x)(t) = x(i + t) \forall t \in [-\tau/2, \tau/2]^2$. The image

$x \in L^2[0,1]^2$, i.e. the set of 2-dimensional finite energy signals. The object dependent signal ensemble is then $\Theta \subset L^2[0,1]^2$. The patch manifold associated with this ensemble is $\mathcal{M} = \{p_q(x) | x \in \Theta\} \subset L^2[-\tau/2, \tau/2]^2$. Given an image, one needs a way to compute the closest point on the manifold prior. This is done in two stages. First, patches from an image are projected onto the patch manifold. This step is denoted by $c(q) = Proj_{\mathcal{M}}(p_q(x))$, which assigns closest patches from the manifold prior to the given image patches. From these projected patches, one reconstructs the global image by means of patch averaging. This operation is denoted by $Aver(c)$ which essentially averages out overlapping patches. Details of this procedure can be found in [12]. These two operations in conjunction are used to regularize the inverse problem as follows. The optimization problem for deblurring is now recast as finding an optimal x^*, given an observation y and the manifold prior as

$$(x^*, c^*) = \underset{x,c}{\arg\max} \|y - Hx\|^2 + \lambda \int_{[0,1]^2} \|p_q(x) - c(q)\|^2 dq \tag{3}$$

A stationary point is obtained by means of an iterative procedure that alternates between solving for x^* and c^*. Given the current estimate $x^{(k)}$, $c^{(k)}$ is obtained as $c^{(k+1)} = Proj_{\mathcal{M}}(p(x^{(k)}))$. Given $c^{(k+1)}$, we solve for $x^{(k+1)} = (H^T H + \lambda Id)^{-1}(H^T y + \lambda Aver(c^{(k+1)})$. This procedure is repeated till convergence.

Implementation Details: In actual implementation, we do not have an analytical characterization of the patch manifold. We instead learn the manifold using training examples of faces or natural images etc. The *Proj* operation then amounts to searching for the closest patch to a given patch in the training database. This is efficiently implemented using locality sensitive hashing [13]. Further, the matrix inversions involved in the optimization steps above are all implemented implicitly using the properties of the PSF matrix H [14].

4. Experimental Results

In this section, we present preliminary results of our proposed algorithm and compare them with a deconvolution method based on sparsity prior in a wavelet domain [15] and a Tikhonov method [14]. In the first experiment, a *Cameraman* image is blurred by a Gaussian blur with standard deviation 2.0. A comparison of different methods in terms of the improvement in signal-to-noise-ratio (ISNR) is shown in Fig. 2 (a)-(e). The ISNR is defined as $ISNR = 10 \log_{10} \left(\frac{\|x-y\|_2^2}{\|x-\hat{x}\|_2^2} \right)$.

The manifold-based method yields a value 5.48 dB which is better than the values obtained by any of the other methods. For this experiment, we had neglected the noise term $\boldsymbol{\gamma}$ in (1). Fig. 1 shows some of the images used to learn the patch manifold. In the second experiment, we compare the results of different methods on the same image in the presence of additive white Gaussian noise with standard deviation 0.01. We plot the ISNR values of different methods as a function of increase in the standard deviation of Gaussian blur kernel. Again, the manifold-based deconvolution algorithm outperforms the other methods in terms of ISNR.

Fig. 1. Some of the natural images used to learn the patch manifold of natural images.

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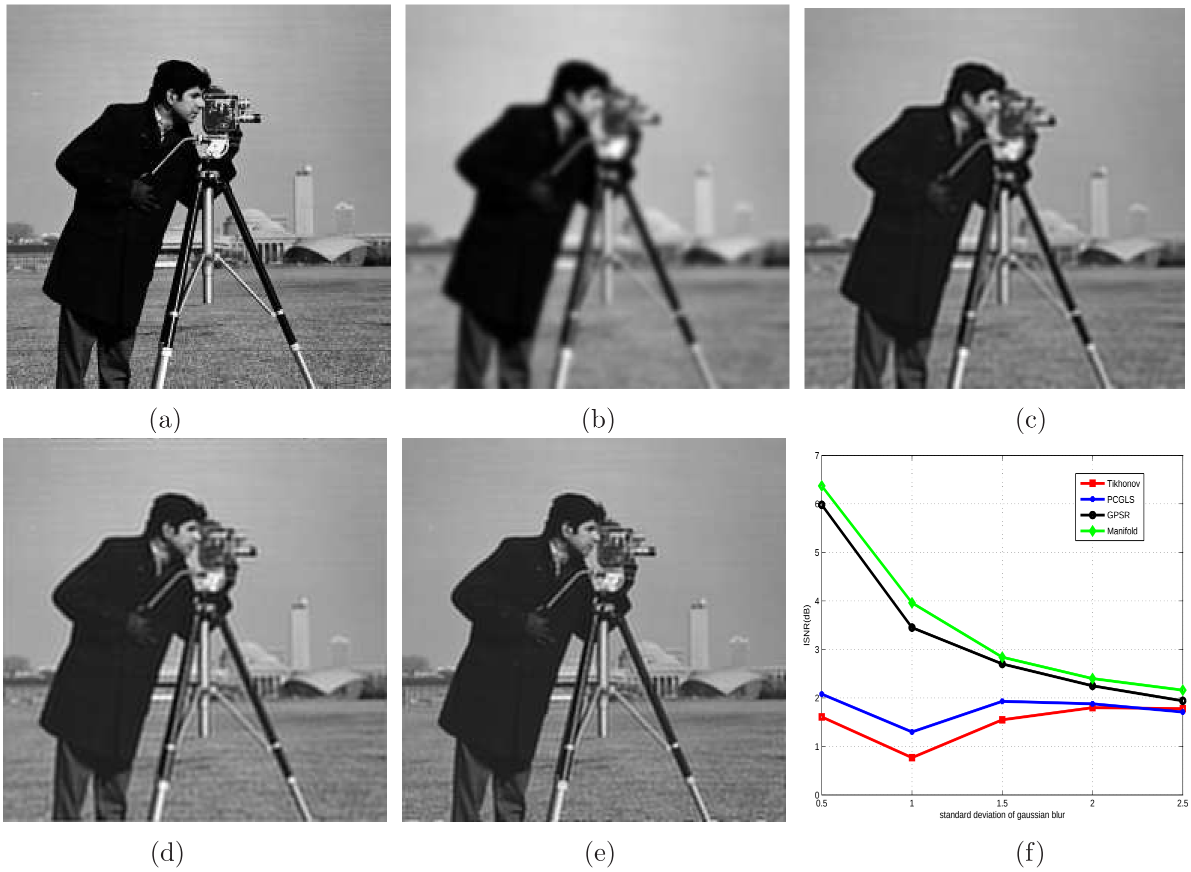

Fig. 2. An image deconvolution experiment with a Cameraman image. (a) Original image, (b) Noisy blurred image, (c) Tikhonov [14] estimate (ISNR 2.44 dB), (d) GPSR (Gradient projection for sparse reconstruction) [15] estimate (ISNR 3.77 dB), (e) Manifold based estimate (ISNR 5.48 dB), (f) ISNR performance of different methods as a function of increase in the standard deviation of Gaussian blur kernel [14, 15].

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