#From: A Course in Ordinary Differential Equations, 2nd Ed. by Stephen A. Wirkus & Randall J. Swift
#Chapter 1: Computer labs input and output
restart
#Maple Example 1

$f := x \rightarrow \cos(x) + e^{2 \cdot x} + \ln(x)$

$$x \rightarrow \cos(x) + e^{2x} + \ln(x) \quad \textbf{(1)}$$

diff$(f(x), x)$

$$-\sin(x) + 2\, e^{2x} + \frac{1}{x} \quad \textbf{(2)}$$

$\frac{d}{dx} f(x)$

$$-\sin(x) + 2\, e^{2x} + \frac{1}{x} \quad \textbf{(3)}$$

diff$(f(x), x\$2)$

$$-\cos(x) + 4\, e^{2x} - \frac{1}{x^2} \quad \textbf{(4)}$$

$\frac{d^2}{dx^2} f(x)$

#the second derivative symbol above was modified from the palette entry by inserting the power of 2

$$-\cos(x) + 4\, e^{2x} - \frac{1}{x^2} \quad \textbf{(5)}$$

diff$(f(x), t)$

$$0 \quad \textbf{(6)}$$

$\frac{d}{dt} f(x)$

$$0 \quad \textbf{(7)}$$

int$(f(x), x)$

$$\sin(x) + \frac{1}{2} e^{2x} + x \ln(x) - x \quad \textbf{(8)}$$

$\int f(x)\, dx$*#entered from Expression; if from Common Symbols, need to use* d *from there too*

$$\sin(x) + \frac{1}{2} e^{2x} + x \ln(x) - x \quad \textbf{(9)}$$

int$(f(x), x = 0 .. \mathrm{Pi})$

$$-\frac{1}{2} + \ln(\pi)\, \pi + \frac{1}{2} e^{2\pi} - \pi \quad \textbf{(10)}$$

$\int_0^{\pi} f(x)\, dx$

$$-\frac{1}{2} + \ln(\pi)\, \pi + \frac{1}{2} e^{2\pi} - \pi \quad \textbf{(11)}$$

$int(f(x), t)$

$$\left(\cos(x) + e^{2x} + \ln(x)\right) t \tag{12}$$

$\int f(x)\, dt$

$$\left(\cos(x) + e^{2x} + \ln(x)\right) t \tag{13}$$

$g := x \rightarrow \int_0^x f(t)\, dt$

$$x \rightarrow \int_0^x f(t)\, dt \tag{14}$$

$g(\pi)$

$$-\frac{1}{2} + \ln(\pi)\,\pi + \frac{1}{2} e^{2\pi} - \pi \tag{15}$$

$evalf(g(\pi))$

$$267.7005103 \tag{16}$$

#Maple Example 2
restart #checking a solution, plotting implicit solutions

$eq1 := \frac{d^2}{dx^2} y(x) + 2 \cdot \frac{d}{dx} y(x) + y(x) = 0$

$$\frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + y(x) = 0 \tag{17}$$

$eval(subs(y(x) = e^{-x}, eq1))$ *#note that* e *is entered from the palette*

$$0 = 0 \tag{18}$$

$eval(subs(y(x) = x \cdot e^{-x}, eq1))$

$$0 = 0 \tag{19}$$

$eval(subs(y(x) = e^{x}, eq1))$ *#this is NOT a solution*

$$4\, e^{x} = 0 \tag{20}$$

$eq2 := (x-4) \cdot y(x)^4 - x^3 \cdot (y(x)^2 - 3) \cdot \frac{d}{dx} y(x) = 0$

$$(x-4)\, y(x)^4 - x^3 \left(y(x)^2 - 3\right)\left(\frac{d}{dx} y(x)\right) = 0 \tag{21}$$

$sol2 := -\frac{1}{x} + \frac{2}{x^2} + \frac{1}{y} - \frac{1}{y^3} = C$

$$-\frac{1}{x} + \frac{2}{x^2} + \frac{1}{y} - \frac{1}{y^3} = C \tag{22}$$

$dsol2 := implicitdiff(sol2, y, x)$

$$\frac{y^4 (x-4)}{x^3 \left(y^2 - 3\right)} \tag{23}$$

$collect\left(expand\left(y(x)^4 \cdot x^3 \cdot dsol2\right), \frac{d}{dx} y(x)\right)$

$$\left(-y(x)^2 x^3 + 3x^3\right)\left(\frac{d}{dx} y(x)\right) + y(x)^4 x - 4y(x)^4 = 0 \quad \textbf{(24)}$$

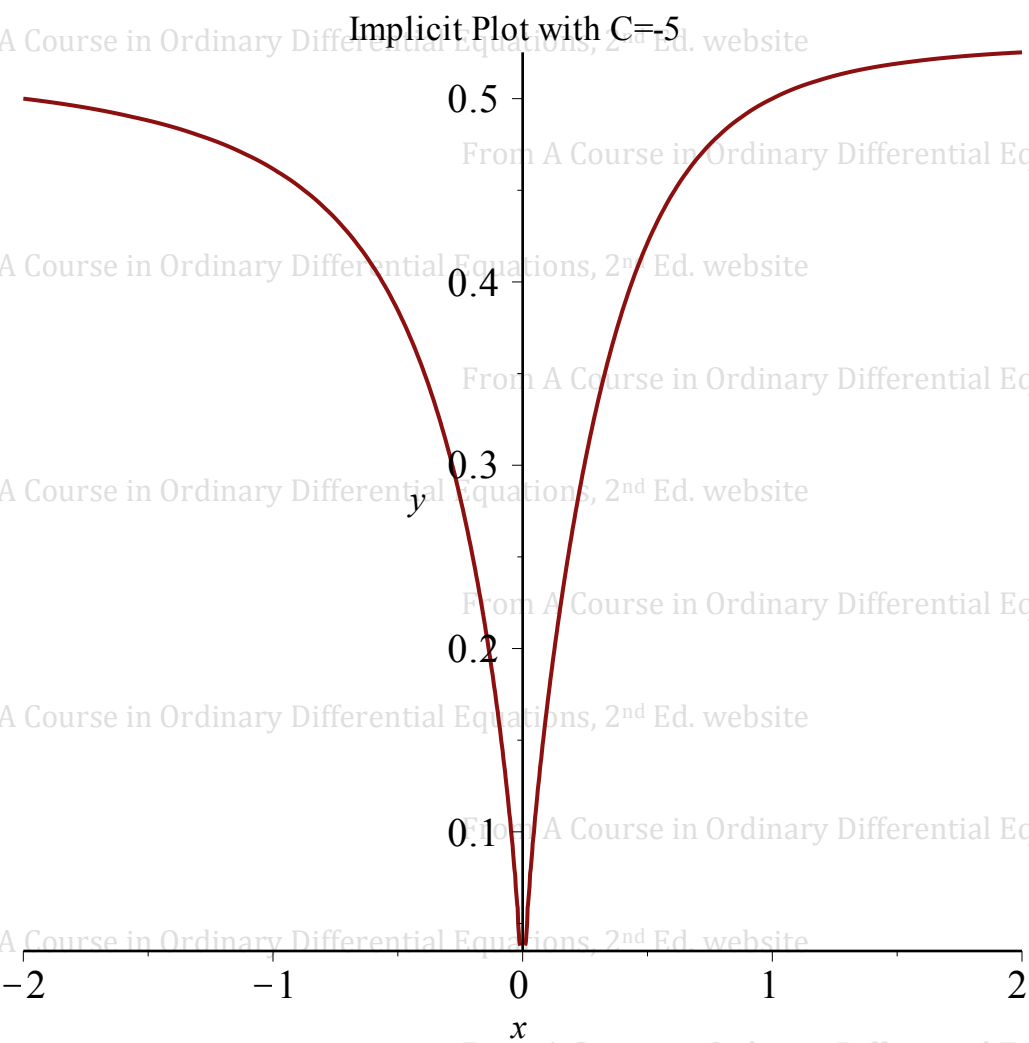
$solve\left(eq2, \frac{d}{dx} y(x)\right)$

$$\frac{y(x)^4 (x-4)}{x^3 \left(y(x)^2 - 3\right)} \quad \textbf{(25)}$$

$solve\left(dsol2, \frac{d}{dx} y(x)\right)$

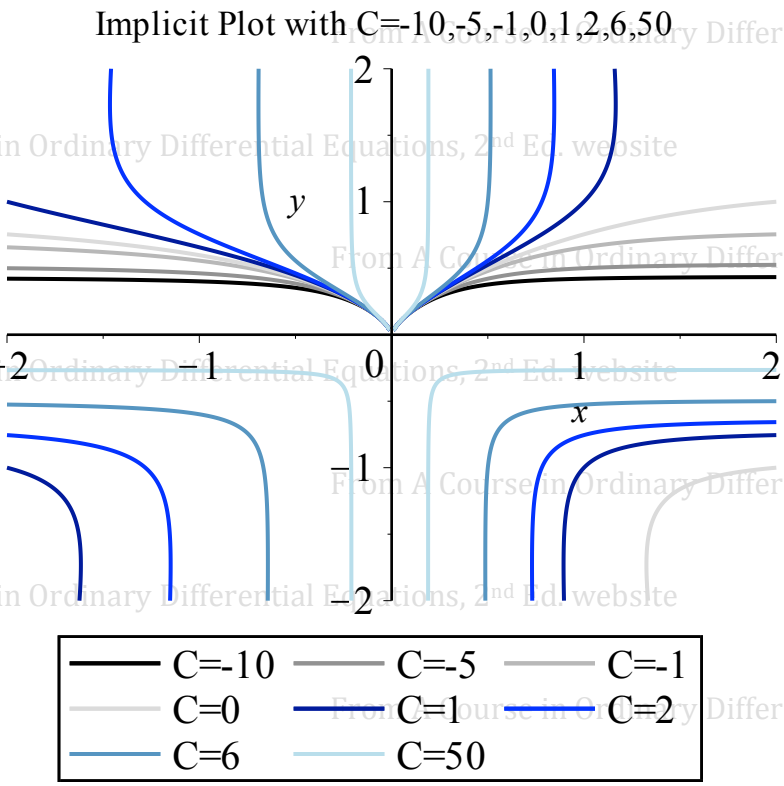
$$\frac{y(x)^4 (x-4)}{x^3 \left(y(x)^2 - 3\right)} \quad \textbf{(26)}$$

with(*plots*) : *#loads package for implicit plotting*
implicitplot(*subs*(*C* = −5, *sol2*), *x* = −2 ..2, *y* = −2 ..2, *gridrefine* = 4, *title* = "Implicit Plot with C=-5")



implicitplot([*subs*(*C* = −10, *sol2*), *subs*(*C* = −5, *sol2*), *subs*(*C* = −1, *sol2*), *subs*(*C* = 0, *sol2*), *subs*(*C* = 1, *sol2*), *subs*(*C* = 2, *sol2*), *subs*(*C* = 6, *sol2*), *subs*(*C* = 50, *sol2*)], *x* = −2 ..2, *y* = −2 ..2, *gridrefine* = 4, *color* = ["Black", "Gray", "DarkGray", "LightGray", "DarkBlue", "Blue",

"SteelBlue", "LightBlue"], *title* = "Implicit Plot with C=-10,-5,-1,0,1,2,6,50", *legend* = ["C=-10", "C=−5", "C=-1", "C=0", "C=1", "C=2", "C=6", "C=50"]) *#see plot,colornames*

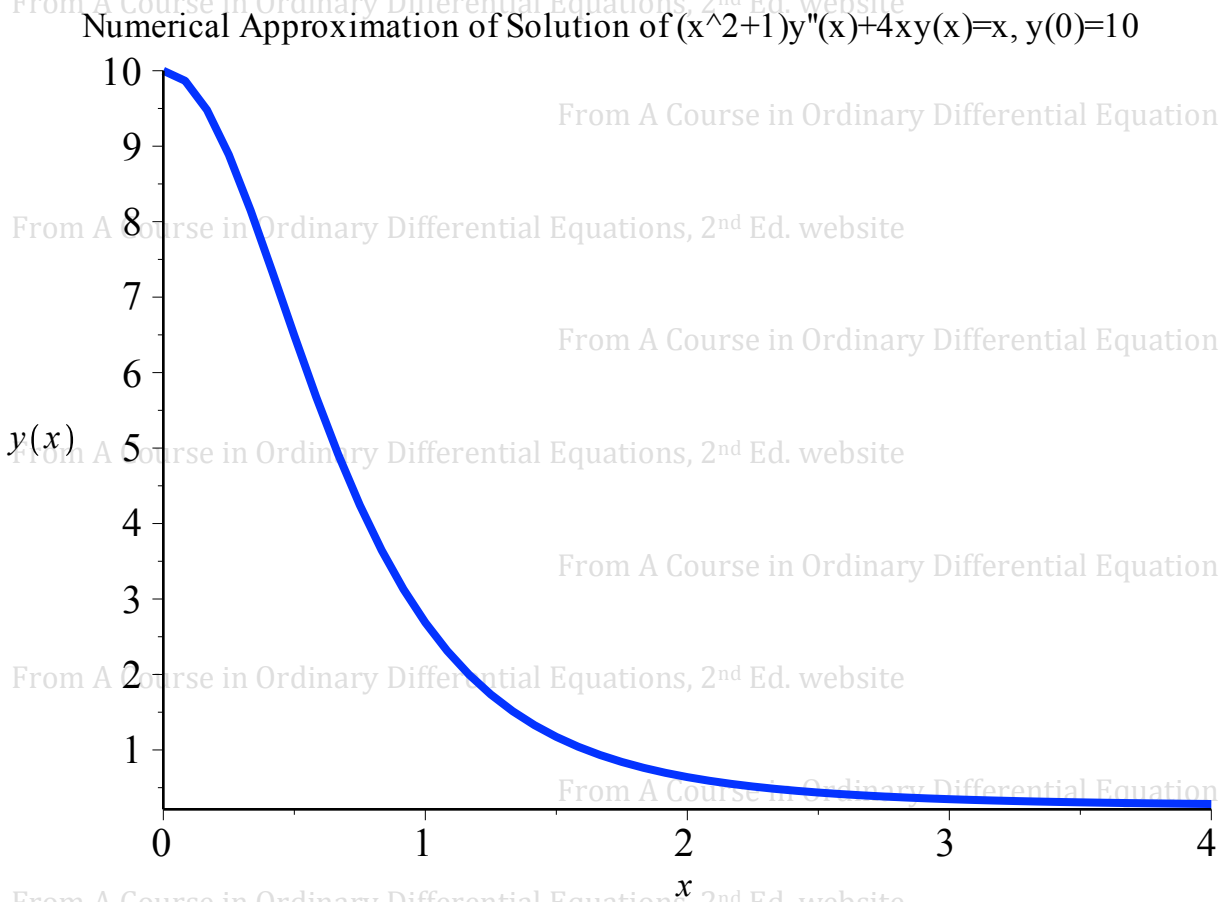


#Maple Example 3
restart
with(*DEtools*) :
with(*plots*) :

$$eq3 := \left(x^2+1\right)\cdot\frac{d}{dx}y(x)+4\cdot x\cdot y(x)=x$$

$$\left(x^2+1\right)\left(\frac{d}{dx}y(x)\right)+4\,x\,y(x)=x \qquad \textbf{(27)}$$

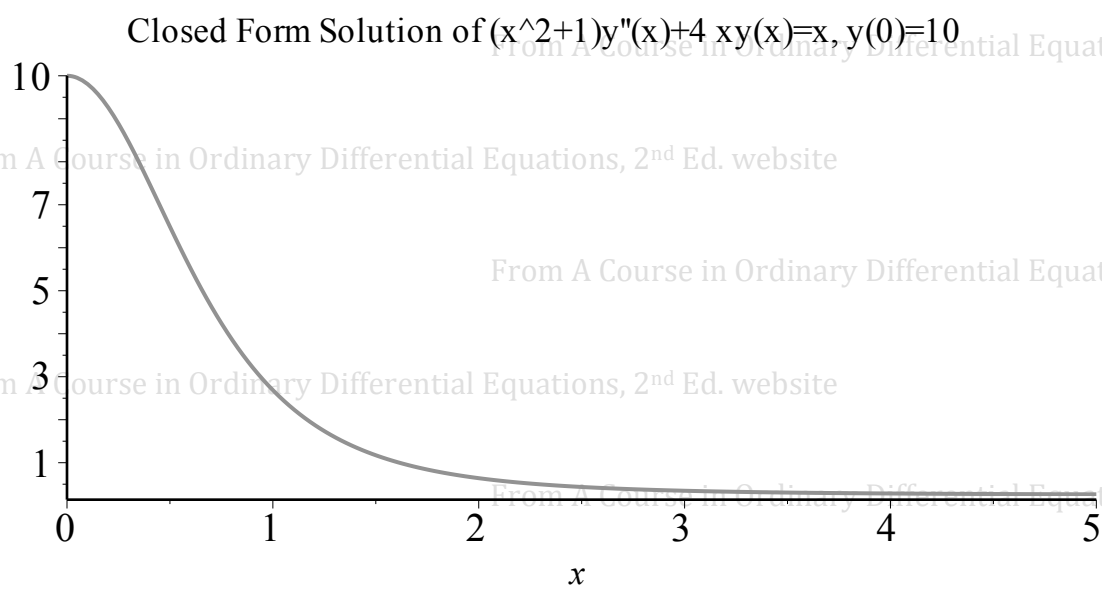
DEplot(*eq3*, *y*(*x*), *x* = 0 ..4, [*y*(0) = 10], *linecolor* = *blue*, *arrows* = *none*, *title* = "Numerical Approximation of Solution of (x^2+1)y\"(x)+4xy(x)=x, y(0)=10")



$sol3 := dsolve(\{eq3, y(0) = 10\})$

$$y(x) = \frac{\frac{1}{4}x^4 + \frac{1}{2}x^2 + 10}{(x^2+1)^2} \tag{28}$$

$plot(rhs(sol3), x = 0..5, color = \text{"Gray"}, title = \text{"Closed Form Solution of } (x\text{^}2 + 1)y\text{\"}(x) + 4\,xy(x) = x, y(0) = 10\text{"})$

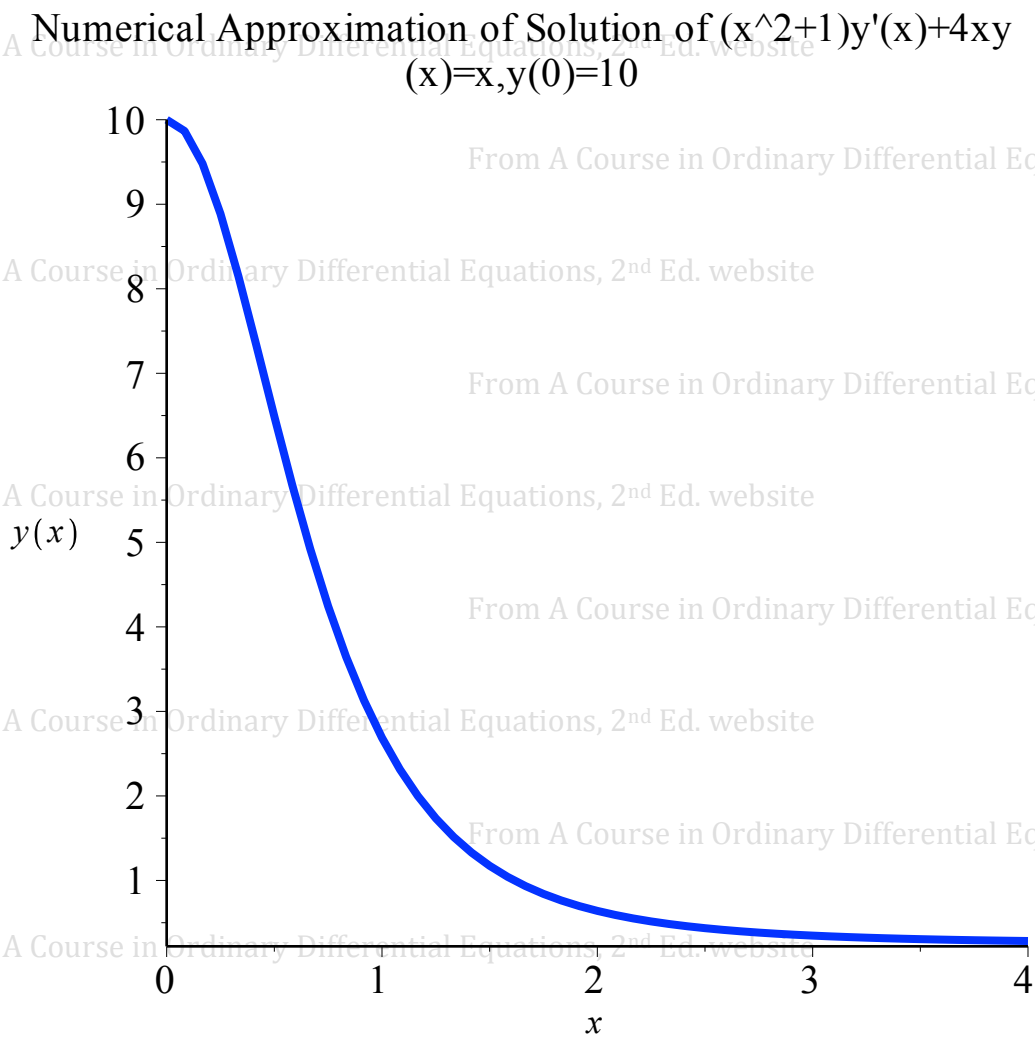


#Maple Example 4
restart
with(*DEtools*) :
with(*plots*) :

$$eq3 := (x^2+1)\cdot\frac{d}{dx}y(x) + 4\cdot x\cdot y(x) = x$$

$$(x^2+1)\left(\frac{d}{dx}y(x)\right) + 4\,x\,y(x) = x \tag{29}$$

DEplot(*eq3*, *y*(*x*), *x* = 0 ..4, [*y*(0) = 10], *linecolor* = *blue*, *arrows* = *none*, *title* = "Numerical Approximation of Solution of (x^2+1)y\'(x)+4xy(x)=x,y(0)=10")



sol3 := *dsolve*({*eq3*, *y*(0) = 10})

$$y(x) = \frac{\frac{1}{4}x^4 + \frac{1}{2}x^2 + 10}{(x^2+1)^2} \tag{30}$$

plot(*rhs*(*sol3*), *x* = 0 ..5, *color* = "Gray", *title* = "Closed Form Solution of (x^2+1)y\'(x)+4xy(x)=x,y(0)=10")

Closed Form Solution of (x^2+1)y'(x)+4xy(x)=x,y(0)=10