Product Service Outsourcing: Impact of Environment Uncertainty and Partial Observability

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Problem definition: Product service plays a crucial role for brands to retain customers and spur revenue growth. It is, however, often outsourced to a third-party provider, driven by cost savings and the ability to focus on core businesses. While there is a large body of literature studying service outsourcing, the impact of service environment uncertainty (i.e., changing customer needs and shifting resource requirements) has received sparse attention in the past but is becoming a major concern due to increased market turbulence. This research explores how environment uncertainty in service provision influences a brand’s intent to out-source, and, if the brand decides to outsource, how it can retain the potential cost advantages offered by a third-party provider. Methodology/results: This research develops a normative model to explore key drivers that impact service outsourcing outcomes under environment uncertainty and partial observability. We find that environment uncertainty can accelerate a brand’s propensity to outsource, and a brand typically benefits from outsourcing initially. Yet, we show that such benefits can dissipate over time due to partial observability. Monitoring efforts help to mitigate the adverse impact of environment uncertainty and partial observability, but cannot attain anticipated outsourcing benefits unless monitoring is costless. In contrast, nudging service providers to self-report cost of resources is effective even if monitoring cost is high. Managerial implications: Brands should carefully consider environment uncertainty, partial observability, and monitoring ability when deciding whether to outsource product services to third-party providers. A heuristic monitoring policy can be effective when the monitoring cost is very high or very low but can perform poorly when the monitoring cost is in the intermediate range. Thus, outsourcing is more attractive when environment uncertainty is significant, but the value of outsourcing can only be realized when (a) partial observability is insignificant, (b) monitoring is inexpensive, or (c) provider self-reporting can be nudged. If none of the conditions hold, then the brand can suffer significant losses from the anticipated benefits of product service outsourcing.

Key words: service outsourcing, environment uncertainty, partial observability, operations strategy

1. Introduction

Product services consist of implicit and explicit service components related to the sale of a product, such as warranty services, maintenance services, and customization services. It is well recognized that product services play an important role for brands to retain customers and spur revenue growth (Guajardo et al. 2016, Deshpande and Pendem 2022). Yet, many brands outsource product services to third-party providers, motivated by cost savings with service guarantees and the ability to focus on core businesses. Gottfredson et al. (2005), for example, note that “finding more-qualified partners to provide critical functions usually allows companies to enhance the core capabilities that drive competitive advantage in their industries” (p.133). Oftentimes, however, the decision...
to outsource is based on anticipated benefits without considering the implications of service environment uncertainty where both customer needs and service provisions can change with increased uncertainty (Murphy-Reuter 2020). Furthermore, with outsourced product services, environment uncertainty can lead to partial observability; that is, brands become less in touch with how the service environment affects cost efficiencies and resource requirements for service provisions (Burt 2002, Li and Choi 2009). As a result, the outcome of the outsourced product services is often inconsistent (Lacity et al. 2017). For example, a recent survey shows 82% of consumers are upset and disappointed with brands due to ever-changing and increasingly uncertain customer expectations in product services (PR Newswire 2019).

From a brand’s perspective, the consequence of poorly managed product services is significant, costing businesses more than $75 billion a year (Hyken 2018). There is a plethora of practical discussions surrounding the negative impact of environment uncertainty and partial observability on outsourced product services. For example, Austin (2011) argues that managing environment uncertainties (e.g., material and operational changes) is critical as otherwise the providers’ services will become stale and outdated, failing to address customer concerns. This is echoed by McCray (2022), who points out that “if you underestimate the importance of change management [due to environment uncertainty] and governance [monitoring] in outsourcing, success will elude you.”

According to the Technocrati and Gartner research, nearly 70% of brands felt forced to cut the scope and remit of the [outsourcing] agreement, because they suspected [due to partial observability] their vendor was overcharging them for base services (BPG 2022, p.07). One of the key reasons is that “the economics of the outside world are constantly changing [i.e., environment uncertainty], making it hard to pin down the outcomes your organisation is trying to achieve” (BPG 2022, p.18).

In the airline industry, for example, planemakers consider developing the jetliner programs as a core business; on the other hand, assembling parts/servicing airlines is a relatively low value-added activity and, hence considered a peripheral business that can be outsourced. The Economist (2017) noted that “surprisingly little of the work is done by Boeing and Airbus. By outsourcing the most complex parts of their aircraft, Airbus and Boeing lost control [and knowledge] of what turned out to be a highly lucrative market for servicing aircraft, with airlines as customers (suppliers made profit margins of between 14% and 17%, compared with 9% for planemakers)” (para. 1). This created tensions when some suppliers decided to merge (e.g., UTC’s merger with Rockwell Collins). Interestingly, after the UTC’s merger with Collins in 2017, a subsequent customer satisfaction survey shows that “Collins, which has consistently been known for highly ranked customer service, fell from 7.1 in 2019 to 6.4 in this survey - dropping most in terms of parts costs, parts availability and OEM service center performance” (Shay 2020).
The above example epitomizes some of the concerns over the negative consequences of product service outsourcing. While the core business of Boeing and Airbus is to sell airplanes, the outsourcing of product services (and components) may hurt their profitability. Thus, the outsourcing of product services can hurt the brand’s business, even though the original intention of outsourcing is to focus on and to improve the core business. A key question we explore is how environment uncertainty and partial observability influence a brand’s product service performance over time, where we define environment uncertainty as shifting external conditions that affect cost efficiencies and resource requirements for service provision, and partial observability as a brand’s gradual loss of visibility of true cost efficiencies and resource requirements of the outsourced product service.¹ This research does not attempt to capture all the interactions discussed above, but instead focuses on the performance of outsourced product services. Part of the reason is that the impact of product service performance on core business is analytically intuitive but can be empirically subtle and thus better left for field studies. Nevertheless, it is useful to consider a typical outsourcing decision sequence and illustrate how our research fits into such a general sequence of events.

Consider the following sequence of decisions on product service outsourcing: (1) Environment uncertainty complicates the process of providing service (Buell et al. 2020), leading to an increased desire to outsource. (2) As the outsourcing relationship moves forward, the challenge of providing service will be transferred to a third-party service provider. Product service performance should improve, at least in the near term, because the service provider just focuses on service provision (e.g., not making airplanes in the airline example) and hence can provide better service than the brand itself. Moreover, product performance should also improve as the brand can focus more on the core business. (3) Over time, however, as the service environment shifts (customer needs or technologies change which induces different resource mixes to provide service), the brand will be less in touch with the service resource cost, which may compromise its ability to deliver for the customer — possibly in terms of product R&D (e.g., misalignment of product value and customer needs), service provision (e.g., misaligned products create more service issues), or aggravated information asymmetry regarding the cost of service provision. (4) To overcome the degradation of product services, the brand can exert monitoring efforts to stay better informed about the changing cost of resources. However, the effort required to stay informed can be expensive when the target market and the third-party service provider are in far-flung regions.

This research captures the essential features of the above four steps but does not explicitly consider the dependence of service outcome and product alignment (which can create a feedback

¹Environment uncertainty captures variations not only of customer service requests within a particular time period and over time but also of the cost of resources required to render such services. Variations of both customer service requests and cost of resources are also referred to as market/environment dispersion (Buell et al. 2020).
loop on product service as described in step (3). We simplify such dependence by assuming that product value is enhanced by a fixed amount when product service is outsourced since the brand can focus more on its core business (Doig et al. 2001, Ellram et al. 2008, Dolgui and Proth 2013). This simplification creates an upper bound on the possibly enhanced product value by focusing on core business via product service outsourcing. Irrespective of any dependence between product alignment and service provision, understanding the cost implications of outsourced product service under environment uncertainty is still valuable from a business perspective, because, even when outsourced, product services still carry significant cost implications and thus warrant brands’ attention. Motivated by the preceding examples and observations, we setup an analytical model to capture key considerations when a brand firm outsources the product service to a third-party service provider, such that

- Brand is the principal and the service provider is the agent;
- Service provider has a lower cost than the brand;
- Product value improves due to outsourcing;
- Brand specifies a target service level;
- Brand dictates the level of resource that the service provider should maintain; and
- Fixed setup cost to switch from direct service to an outsourcing model and vice versa.

Our analysis reveals some nuanced insights into the relationship between environment uncertainty and product service outsourcing performance. We show that environment uncertainty increases a brand’s intent to outsource, and yet over time, environment uncertainty can significantly reduce the benefit of outsourcing. Nevertheless, if a third-party provider offers a significantly more efficient process, then the brand may still benefit from outsourcing (as compared with in-house service provision), especially at the beginning of the outsourcing relationship. Such benefits, however, can deviate significantly from the anticipated benefits of product service outsourcing due to partial observability. We show that monitoring efforts to close the knowledge gap can potentially help the brand retain the service outsourcing benefit. A critical challenge with monitoring is that the optimal policy will depend on reasonable estimations of environment uncertainty, which creates implementation difficulties. We thus propose a heuristic policy that does not depend on the estimation of environment uncertainty. The heuristic policy performs well when the monitoring cost is very low or very high but can perform poorly when the monitoring cost is in the intermediate range. This offers a cautionary note on the effectiveness of often discussed monitoring mechanisms in the practice literature to safeguard product outsourcing benefits. In contrast, we find nudging service providers to self-report true cost of resources can be effective to retain outsourcing benefits even if the monitoring cost is high. Thus, brands should balance monitoring versus nudging efforts carefully to maintain the benefits of product service outsourcing.
While there is a large body of research that focuses on service outsourcing, most of the existing studies focus on snapshot analysis without considering environment uncertainty over time. The existing studies point out important factors, such as motivation, capability, and human capital, that influence the outcome of service outsourcing, but the dynamic risk aspect of service outsourcing, in terms of environment uncertainty, has received sparse attention (Wuyts et al. 2015). To the best of our knowledge, this research is the first to study product service outsourcing under environment uncertainty and partial observability. A key contribution of our research is to illustrate that environment uncertainty and partial observability can significantly diminish the benefit of product service outsourcing, and that even an optimal monitoring policy may not be able to maintain the anticipated benefits of service outsourcing. Departing from the existing literature, we show that nudging providers to self-report true cost of resources can be effective even when the monitoring cost is high. Our research, therefore, helps to refine the cost and benefit analysis of product service outsourcing, and thus ties conceptual insights from the existing literature to operational insights that brands can leverage to manage product services more effectively over the long run.

2. Related Literature

The existing literature on service outsourcing can be broadly classified as works that focus on why brands outsource or how to manage outsourcing more effectively. The early literature largely focused on factors that influence a brand’s intention to outsource, such as Grover et al. (1996), Ellram et al. (2008), Cachon and Harker (2002), Balakrishnan et al. (2008), and Amaral and Tsay (2009). Grover et al. (1996) explore the widespread adoption of business process outsourcing and explain the rationale for outsourcing based on the theory of transaction cost economics. In a similar vein, Ellram et al. (2008) leverage transaction cost economics theory to examine brands’ motivation to outsource business processes. From an operational perspective, Cachon and Harker (2002) show that brands may choose to outsource even if a provider’s capability is no better than a brand’s internal capability, and the key driver is not transaction cost theory but rather a simple result of scale economies. With task uncertainty, Balakrishnan et al. (2008) examine business process outsourcing with a focus on tasks where agents are unsure about their difficulty. They show that brands have an incentive to outsource when a provider has better knowledge, such as when a provider serves multiple brands. From a behavioral angle, Amaral and Tsay (2009) document a simulated game setting that incorporates behavioral aspects of outsourcing relations. Given that a brand outsources, they demonstrate that a win-win outcome is not guaranteed even if every member of the supply chain is better off with a more effective outsourcing design. All these studies focus on a snapshot analysis where a provider and a brand make decisions based on the current understanding of the service requirements. They do not consider future environment uncertainty.
that can influence service cost, nor do they consider the limited visibility of service cost due to environment uncertainty in a product outsourcing relationship.

A large body of conceptual and empirical studies explore the strategies to effectively manage outsourcing. Valk and Iwaarden (2011) point out that as the subcontractor interacts on a daily basis with the end customer, they will accumulate end customer intelligence and adapt the service based on specific end customers’ preferences. As a result, the service delivered may not be completely the same as the service contracted. This usually results in role stress for the subcontractor, since on the one hand their intrinsic motivation to perform their job well leads them to try and satisfy the end customer without getting paid for it, while on the other hand doing what the buyer contracted for reduces the subcontractor’s sense of job satisfaction. In the end, since the subcontractor will not want to jeopardize compensation, we expect the subcontractor to act according to the targets linked to compensation. (p. 201)

This literature motivates us to model the contract between brands and third-party providers that are driven by resource provisions and payments. Furthermore, Plugge et al. (2013) point out that a brand should carefully assess whether a provider is willing to adapt to changes in customer requirements (i.e., whether the provider has the right motivation and capability). Along this same dimension, Wuyts et al. (2015) note that “rapidly changing end customer preferences can quickly turn previously acquired customer insights obsolete” and that “market volatility reduces a provider’s ability to effectively deploy its internal processes to address customer needs.” (p. 43) Somewhat related to the uncertainty of product service requirements, Perdikaki et al. (2015) study an interesting outsourcing setting where an e-tailer may outsource either front-end or back-end services, or both. Using empirically obtained data, they find that outsourcing the back-end services is typically more fruitful than outsourcing the front-end services. The reason is that the front-end service directly interacts with customers and it can be challenging for the e-tailer to coordinate front-end and back-end services to achieve a smooth service delivery, making it harder to achieve high customer satisfaction. Our research complements the above literature by explicitly incorporating environment uncertainty and partial observability on product service outcomes.

From a control point of view, Aron et al. (2008) investigate how buyers should monitor service outcomes when outsourcing business processes. They show that, interestingly, complex business processes are monitored less by the brand because the monitoring cost is higher for more complex processes. They do point out, however, that a more complex service may also have higher value, which countervails the monitoring cost, so whether a more complex service is monitored more or less is case dependent. Broadening the theory, Valk and Iwaarden (2011) conceptualize the design of contract type (i.e., outcome-based vs. behavior-based contracts) in ensuring service quality. For
complex service types, however, contracting on outcomes might not be feasible. Hagiu and Wright (2019) study how a firm may control service delivery when service is provided by independent contractors, and find that the trade-off involved with control versus enabling differs from that with classical make or buy decisions. Our research highlights a related factor that influences monitoring intensity: the uncertainty of service environment. Environment uncertainty may or may not be correlated with process complexity, and so the incentive to monitor is different from the above literature.

Call centers offer an interesting angle to examine service outsourcing. Ren and Zhou (2008) study contracting methods to achieve optimal staffing and service quality. They point out that service quality is difficult to contract, because “each service encounter is unique (and sometimes unobservable), it may not be possible to contract directly on service quality.” (p. 370) They propose a partnership contract to achieve system optimal outcome, but the partnership contract is quite complex and hence can be challenging to implement in practice. Hasija et al. (2008) study a similar setting with staffing level and service rate contracting design to coordinate a call center. Their focus is on information asymmetry in terms of call center productivity. Information asymmetry means that, while the brand can contract on a staffing level, it is difficult to contract on productivity. They study a variety of different contract terms and explain that pay per time (PPT) or pay per call (PPC) are widespread in practice because they can help the brand overcome this information asymmetry. In a recent study, Feng et al. (2019) explore capacity versus quality cost in designing service outsourcing contracts. They find that the brand may achieve the first-best result even if the provider has private information. They point out that the capacity cost versus quality cost can be positively or negatively correlated. Further, they distinguish between capacity and quality; that is, a higher capacity makes service quicker but that itself does not mean the quality is good. We take a similar conceptual approach by considering both service quality and service level. All of the above studies assume the mean demand rate is known and constant (which only partially reflects the changing customer needs). The current work complements the above stream of research by considering dynamically changing customer requirements over time.

In closing, despite the extensive studies on product service outsourcing, many brands continue to experience poor product service outcomes. We suspect this is partly due to the lack of a long-term view of the product service outsourcing relationship, where environment uncertainty and partial observability can be significant. An outsourcing arrangement that appears attractive now may not turn out to be so in the long run. It is therefore important to understand how service environment uncertainty and partial observability affect product outsourcing performance over time. This study is a first step to refining the cost and benefit analysis of outsourcing under dynamic
environment conditions so that brands can take a more informed, longer-term view when making service outsourcing decisions.

The remainder of this paper is organized as follows. Section 3 describes the model setup with some preliminary observations, and Section 4 characterizes basic properties of direct service and outsourced third-party service. In Section 5 we study the impact of partial observability due to service environment uncertainty and investigate the properties of optimal and heuristic monitoring policies. Section 6 contains detailed discussions on our findings and further research directions. Some supplementary results and all proofs can be found in the Appendix.

3. The Model
Consider a brand that sells a physical product that requires after-sale product service (e.g., support centers, maintenance, and repair operations). The outcome of a service engagement depends on two factors: the resource level of the service provider (e.g., staffing, training, information technology infrastructure, equipment type, and capacity) and the cost of resources in providing customer services (which can be influenced by the service environment, such as the nature of customer requests and the availability of local talents). While some aspects of the resource level (e.g., staffing level) may be controlled or monitored, the service environment (e.g., macroeconomic conditions, shifting customer requests, technology changes, and local labor supplies) is often uncertain and beyond the brand’s control. Hence, the service cost and outcome can be random even when the resource level is held constant. We next formalize the relationship between resource level and service outcome, the evolution of service environment, the phenomenon of partial observability, and brand’s monitoring option.

3.1. Service Provision
Let $k$ denote the invested resource level and $\theta(k) = 1 - e^{-\lambda k}$ denote the expected (average) customer satisfaction level, where $1/\lambda > 0$ measures (among other possible metrics) the average number of interactions a typical customer seeking product services would experience. Since $\theta(k)$ decreases in $1/\lambda$, this reflects settings where customers prefer fewer interactions to resolve product issues. Thus, if $1/\lambda$ is large, then all else being equal customers are less likely to be satisfied (i.e., satisfaction decreases with an increasing number of interactions to resolve the product service issue).

The resource level $k$ (e.g., staffing and training) should be viewed as a standardized measurement against a reference training level. This is important because typically customer satisfaction not

\[ \theta(k) \] captures the essential feature that customer satisfaction is correlated with waiting/sojourn times, which are convex decreasing in resource level (Whitt 1993, Feng et al. 2019). The specific form is inspired by the fact that the probability of sojourn time exceeding a threshold is exponentially distributed, convex decreasing in resource level while increasing in service requests. See (1.3.5) in Medhi (2002).
only depends on the headcounts of employees but also on their training level (e.g., diplomas and certificates) related to the specific product services. For example, a higher headcount with a low level of training can exhibit a lower \( k \) than a low headcount with a higher level of training. Therefore, standardizing the resource level \( k \) allows a fair comparison among different resource mixes. With standardized \( k \), it is then natural and consistent to interpret \( \theta(k) \) as being concave increasing in \( k \); that is, all else being equal, a higher \( k \) increases the chance that a typical customer request is resolved satisfactorily, although the marginal benefit of resource \( k \) is declining.

Let \( c(k, d) = C \cdot k(1 + \beta d) \) denote the cost of maintaining a standardized resource level \( k \), where \( C \) is the base unit cost, \( \beta \) is a sensitivity parameter that captures how service environment impact resource cost, and finally \( d \) captures the general environment for product services. The environment parameter \( d \) can be interpreted as factors affecting the cost efficiency of service provisions (e.g., changing needs of customers, entrance or exits of competitors, economic environment, availability of resources, technology changes, and government regulations). We adopt the convention that an increasing \( d \) reflects a more challenging service environment that, all else being equal, increases the cost of maintaining the same resource level. For example, an increased \( d \) may reflect higher customer expectations, a more expensive labor market, or tighter regulation for service provision, whereas a decreased \( d \) may reflect more efficient internal processes, better deployment of technologies, or lower wages for skilled labor (via tapping talents from lower cost regions).

Note that the parameter \( \beta \geq 0 \) captures the marginal impact of service environment on the resource level \( k \). If \( \beta = 0 \), then the cost of carrying resource level \( k \) does not depend on environment (e.g., service provision requires basic, routine knowledge). In contrast, as \( \beta \) increases, the cost of carrying resource level \( k \) is increasingly more expensive in more challenging environments (e.g., service provision requires a mix of specialized, tacit knowledge). The cost of maintaining service resources is thus jointly increasing and supermodular in \( k \) and \( d \). As a concrete example, suppose the prevailing wage for a basic certified technician is $50 per hour and the desired headcount is 10 technicians, then \( C = $50/\text{hour and } k = 10 \). The service provision can be partially influenced by the broad service environment (e.g., availability of certified technicians). Suppose the environment is such that there is a shortage of certified technicians so a 30% market premium is needed to retain the technicians, then \( d = 1.3 \). However, the service provider is only partially influenced by the environment (e.g., the provider has better working conditions or reputations), and it only needs to pay a fraction, say 60%, of the market premium, then \( \beta = 0.6 \). Putting all together, the service cost is given by \( c(k, d) = c(10, 1.3) = $50 \times 10 \times (1 + 0.6 \times 1.3) = $890 \) per hour.

On the revenue side, a customer’s expected utility is driven by two factors: the value of the product itself and the expected service level should the needs arise in the future. Let \( u = v - \pi(1 - \theta(k)) + \epsilon \) denote a customer’s utility, where \( v \) is the product value (e.g., utility minus price), \( \pi \)
is the expected penalty cost of unsatisfied service encounter, and $\epsilon$ is uniformly distributed with mean zero that captures a customer’s idiosyncratic perception. Normalizing the customer’s outside option to zero, the customer will purchase the product when $\epsilon \geq \pi(1 - \theta(k)) - v$. The brand’s expected revenue can be scaled as

$$R(k) = v - \pi(1 - \theta(k)),$$

and the expected profit is, therefore, $R(k) - c(k, d)$. Note that in (1) providing product services is treated as a cost center (i.e., penalties occur for unsatisfied customers), but a superb product service may increase the core product’s value (as hinted in the airline example in the introduction section). We can modify (1) to accommodate such a situation by redefining

$$\tilde{R}(k) = \tilde{v} - \tilde{\pi}(1 - \theta(k)) + \tilde{s}\theta(k),$$

where $\tilde{s}\theta(k)$ captures the enhanced revenue generated by satisfied customers. Notice that $\tilde{R}(k)$ can be simplified to

$$\tilde{R}(k) = (\tilde{v} + \tilde{s}) - (\tilde{\pi} + \tilde{s})(1 - \theta(k)),$$

which is equivalent to $R(k)$ by setting $v = \tilde{v} + \tilde{s}$ and $\pi = \tilde{\pi} + \tilde{s}$. As such, the formulation in (1) is amenable to more general cases of value-enhancing product services.

The optimal resource level $k$ and hence the expected profit will depend on service environment $d$, which can vary over time for diverse reasons. For example, customer needs can shift over time (Morgan 2016), labor market conditions can soften or worsen, and resource mix can vary between basic and advanced skills (Feng et al. 2019). These different reasons can be viewed as random events affecting $d$ over time. As such, we next describe the evolution of the service environment.

### 3.2. Evolution of Service Environment

Let $i = 1, 2, \ldots, T$ denote the time period, where time is indexed forward with $T$ representing the planning horizon (which is infinite when $T \to \infty$). It is useful to treat each time period as reasonably long (e.g., each period represents a quarter or a fiscal year), as such are typical time intervals to review performance or contracts. Let $e_i$ be a random variable that aggregates all exogenous factors that impact service environment in period $i$, then the service environment in the next period will transit to $e_i d_i$. Denote the initial service environment as $d_0$, then the environment at the beginning of period $i$ can be expressed as $d_i = \prod_{j=0}^{i-1} e_j d_0$. Note that $e_i$ can be simply set to 1 if no exogenous events affect the service environment in period $i$.

**Assumption 1**

The random events $e_i$, $i = 1, 2, \ldots, T$ are independent, log-normally distributed with $E[e_i] = 1$.

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3 Let $\epsilon \sim U(-a, a)$ with $v - a \leq \pi(1 - \theta(k)) \leq v + a$, and $\omega$ be unit profit associated with a customer. The expected revenue is $\omega P(\epsilon \geq \pi(1 - \theta(k)) - v) = \omega \left( \frac{1}{2} - \frac{1}{2(\pi(1 - \theta(k)) - v)} \right) \approx \frac{1}{2} - \frac{1}{2 \pi(1 - \theta(k)) - v} \propto v - \pi(1 - \theta(k))$. Without loss of generality, we normalize the customer mass to be 1.

4 According to the Centre for Outsourcing Research and Education (CORE), the average term of outsourcing agreements is trending down and is less than 5 years in 2010 (Austin 2011).
The above assumption implies that the environment \( d_i \) follows the multiplicative martingale model of forecast evolutions (MMFE), which has strong empirical support in fitting random shocks to general market conditions (Oh and Özer 2013). A key property of MMFE is that the expected future environment is the same as the current period environment—given all the information available up to the current period. Thus, the service environment is expected to remain the same until it is disturbed by some random events. This property fits the settings where the brand does not foresee a persistent shift in service environment that makes service provision more (or less) challenging over its planning horizon.

3.3. Partial Observability

The brand’s knowledge of service environment (and hence cost) depends on whether it provides product services in-house or outsources it to a third-party provider. If it provides product services in-house, the brand is in direct touch with end customers (i.e., observes or experiences random events \( e_i \)) and hence knows the service environment \( d_i \). If it outsources the product services, the brand may not observe or experience random events and hence can be less certain about \( d_i \). For example, the brand may not (a) observe knowledge developments (e.g., training and human capital development) invested by the provider and (b) assess root causes of customer dissatisfaction (e.g., lack of provider knowledge, deficiency in product design, or incorrect use of the product). In effect, outsourcing erodes the brand’s visibility of resource requirements due to environment uncertainty.

With partial observability, the brand and the service provider hold two series of asymmetric beliefs on the service environment. These beliefs are correlated and partially updated over time. To incorporate such evolutions of belief processes, we adapt the martingale model of asymmetric forecast evolutions (MMAFE) developed in Oh and Özer (2013). The MMAFE is a general framework that allows multiple asymmetric correlated beliefs to jointly evolve in a martingale fashion, thus facilitating the explicit formulation of each party’s beliefs at any time period. Moreover, as noted in their study, there is strong empirical support that multiplicative martingale fits well with the impact of event shocks on general market conditions. We next describe the basic setup using MMAFE.

Recall that \( e_i \) is a random variable that aggregates all exogenous factors impacting service environment in period \( i \). Let \( I_i = \{\xi_{i,1}, \xi_{i,2}, \ldots, \xi_{i,n_i}\} \) denote the set of exogenous factors in period \( i \), where each \( \xi_{i,j} \) is independent, lognormally distributed random variable with \( E[\xi_{i,j}] = 1 \). The aggregate effect \( e_i \) can then be derived as \( e_i = \Pi_{k \in I_i} \xi_{i,k} \), which is also lognormally distributed with \( E[e_i] = 1 \) (except for \( e_0 \), which is known to both the brand and the service provider at the beginning of the planning horizon). If \( I_i \) is an empty set for some \( i \), then we set \( e_i = 1 \); that is, the environment does not shift in period \( i \).
If the brand provides product service in-house, then it observes the service environment $d_i$ in each period, specifically $d_i = \prod_{k=0}^{i-1} e_k d_0$. In contrast, if the brand outsources product service to a third-party provider, it only learns some of the exogenous factors that influence the environment. Let $I_i^b \subseteq I_i$ denote the subset of factors the brand learns in period $i$, then $e_i^b = \prod_{k \in I_i^b} \xi_{i,k}^b$. Note we use superscript $b$ to denote the brand. With martingale property, the brand’s estimation of service environment is unbiased (i.e., $E[e_i^b] = E[e_i]$), but the variance of $e_i^b$ is less than the true event variance $e_i$ since a subset of exogenous factors is missing in $e_i^b$.

Suppose the brand outsourced product service in period $i^o$, then its belief about service environment in period $i > i^o$ is given by $d_b^i = \prod_{k=0}^{i^o-1} e_k \prod_{k=i^o}^{i-1} e_k d_0$. Further, let $\xi_i^b = \prod_{k=i^o}^{i-1} e_k / e_k$ denote the cumulative random events that the brand does not observe after period $i^o$ (i.e., after the product service is outsourced), and $s_i$ denote the estimated variance parameter of $\xi_i^b$. Although the brand does not observe $\xi_i^b$ directly, it could leverage periodical revelations over time to estimate the distribution of $\xi_i^b$ and hence the variance parameter $s_i$. We study heuristic policies that do not require knowledge of $s_i$ in §5.3. The service provider, on the other hand, observes these events and knows the actual environment $d_i$. Note that by the MMFE property, we have $d_i = \xi_i^b d_b^i$.

### 3.4. Reporting and Belief Updating

Under product service outsourcing—given that the brand can only partially observe service environment, whereas the service provider can fully observe the true environment (including $\xi_i^b$, the set of information observed by the brand)—the service provider may choose to update the brand by reporting true environment if such reporting is in its interest. The reason is that the provider’s compensation is based on the brand’s belief in the service environment’s cost of resources, and the compensation is higher if the brand believes the service environment indicates a higher cost of resources. All else being equal, therefore, in each period the service provider’s objective is to maximize the brand’s belief in the cost of resources. As such, when the true service environment indicates that the cost of resources is higher than the brand’s belief, the service provider will have an incentive to report the true service environment. However, when the true service environment indicates a lower cost of resources than the brand’s belief, the service provider will be better off not to report since doing so will only reduce its expected payment from the brand. In the latter case, the brand is essentially overcharged by the provider due to partial observability.

The brand, however, would anticipate the provider’s asymmetric reporting behavior and thus would adjust its belief accordingly. In particular, if the provider does not report the true market environment, the brand knows that the cost of resources is likely to be lower than its current belief—though it does not know how much lower the true cost is. That is, the brand partially learns the environment (that it is less challenging) even if the provider does not report. Therefore, the provider’s reporting strategy and the brand’s belief updating process are intimately intertwined. We shall analyze the equilibrium reporting and belief updating process in §5.1.
3.5. Monitoring Efforts
To compensate for the negative effects of partial observability, the brand could also exert costly monitoring efforts to improve its understanding of the service environment. Monitoring efforts help the brand better align its belief with the true environment. Let \( y_i \in \{0, 1\} \) denote the brand’s monitoring decision in period \( i \) with \( y_i = 1 \) denoting monitoring and \( y_i = 0 \) denoting not monitoring. Monitoring efforts include site visits, audits of provider processes and documents, and surveys of customers to better understand resource requirements and customer needs. We assume that the cost of the brand’s monitoring efforts does not depend on its current belief about the environment, and hence each time the brand exerts monitoring efforts, it incurs a fixed cost \( M \). Further, we assume that if the brand conducts monitoring efforts in period \( i \), then \( d_i = d_i \); that is, its belief is aligned with the underlying service environment. If there is still a residual belief gap after monitoring effort, then \( d_i \geq d_i \). All else being equal, this will reduce the benefit of monitoring. Since such a negative impact reduces the attractiveness of outsourcing in a monotone fashion, we do not analyze the residual belief gap case but recognize that ignoring such residual gap attains an upper bound on the potential value of outsourcing product service.

3.6. Problem Formulation and Basic Properties
In each period, the brand needs to set a target customer satisfaction level \( \theta(k) \) (and hence the implied resource level \( k \)) to maximize its expected profit. A higher level of customer satisfaction benefits the brand from multiple aspects, but it also requires more investments either by the brand if it provides the service directly in-house or by a third-party provider if the service is outsourced. We focus on the setting where, all else being equal, the third-party provider is more efficient in service provision than the brand (as otherwise outsourcing would not be attractive). In addition, we assume that transitioning product service from in-house operation to a third-party service provider incurs a fixed cost \( \tau \) (e.g., setup cost, re-allocating in-house resources).

3.6.1. Direct Service (DS). In a direct service setting, the brand provides product services in-house. Note that a provider might still be involved in product service, such as providing raw materials or component inputs, but the provider is invisible to the end customers. With DS, the brand understands the service environment since it is directly involved with product service delivery. Let \( 0 \leq \gamma \leq 1 \) be the discount factor, and define \( G(k_i, d_i) = R(k_i) - c(k_i, d_i) \). The brand’s value function can be defined as

\[
V^{DS}_i(d_i) = \max_{k_i \geq 0} \{G(k_i, d_i) + \gamma E_{i+1} V^{DS}_{i+1}(d_{i+1})\}, \quad i = 1, \ldots, T.
\]

For the finite horizon case, we set \( V^{DS}_{T+1}(d_{T+1}) = 0 \). Note that the cost of maintaining resource level \( k \) (i.e., \( c(k_i, d_i) \)) is an ongoing cost for each period. That is, even if the brand maintains a steady
resource level $k_i$, it will incur the same cost in each period. Similarly, if the resource level varies from period to period, then the cost will vary proportionally. We do not consider the fixed cost of varying resource levels, as the most significant cost driver for service providers is the variable labor cost of knowledgeable employees.

**Lemma 1 (Basic Property on Direct Service (DS))**

(a) $G(k_i, d_i)$ is concave in $k_i$, and the optimal $k^*_i$ satisfies $\frac{\partial c(k_i, d_i)}{\partial k_i} = \pi \frac{d\theta(k_i)}{dk_i} \bigg|_{k_i=k^*_i}$ if $d_i \leq \frac{1}{\beta} (\lambda \pi C - 1)$ and $k^*_i = 0$ otherwise. (b) All else being equal, $k^*_i$ decreases in $d_i$.

Lemma 1 characterizes the optimal service attained with DS. The optimal (interior) resource level is such that $\frac{\partial c(k, d_i)}{\partial k} = \pi d\theta(k)$; that is, the marginal cost of provisioning the resource equals the marginal expected savings in product services (i.e., the cost saved by avoiding expected penalty cost $\pi$). In addition, the optimal resource level decreases in the cost of resources, which is more likely to happen when service environment uncertainty increases since environment uncertainty makes it more challenging to maintain the same level of customer satisfaction.

This echoes our earlier discussions that, as the service environment becomes more challenging due to uncertainty, brands will face significant difficulties in maintaining a high level of product service in-house. Thus, as the service environment uncertainty increases, brands will be more likely to encounter a challenging service environment and hence have a stronger incentive to outsource the product service function to a third-party provider that enjoys more efficient cost structures.

To explore a brand’s incentive to outsource, we next analyze the model and basic properties of the third-party service (TS).

**3.6.2. Third-party Service (TS).** With service outsourcing, the brand contracts product services to a third-party provider. Let $c^S(k, d)$ denote the provider’s cost function, and consider the scenario where the provider is more efficient in service provision. To simplify the analysis while maintaining the key tradeoff, we assume that $c^S(k, d) = \alpha c(k, d)$ where $0 < \alpha \leq 1$ parameterizes the third-party provider’s cost efficiency. A lower value of $\alpha$ indicates a more efficient third-party provider. It follows immediately that (a) $c^S(k, d) \leq c(k, d)$ and (b) $\partial c^S(k, d)/\partial k \leq \partial c(k, d)/\partial k$, $\partial c^S(k, d)/\partial d \leq \partial c(k, d)/\partial d$, and (c) $\partial^2 c^S(k, d)/(\partial k\partial d) \leq \partial^2 c(k, d)/(\partial k\partial d)$. These conditions reflect settings where the provider possesses specialized expertise in service provision; that is, it is (a) more cost-efficient and (b) can scale its operations at a lower marginal cost.

Let $v^{\omega} \geq v$ denote the improved product value due to outsourcing as the brand can focus on its core business to improve product value. Note that the product value may also decline as the brand loses touch with its customer base, so the net effect can be negative over the long run. We maintain the above assumption for several reasons: (a) it provides an upper bound on the value of outsourcing, (b) the directional impact of declining $v^{\omega}$ is apparent, and (c) products may be
valued more heavily by technical advances as opposed to customers’ feedback. For the outsourcing contract, we adopt the direct contracting approach commonly used in practice, where the brand specifies the customer satisfaction level that the provider should provide. The brand pays the provider in each period for a target customer satisfaction level $\theta(\xi|_{ki})$. With partial observability, the brand’s payment is based on (and increases in) its belief instead of the true cost of services. As such, the provider may have an incentive to report the true market environment, and the brand anticipates the provider’s reporting strategy (see §3.4). To formalize such a strategic interaction, let $R(d_i)$ denote the provider’s reporting strategy and $d_i^{bs} = \Gamma(d_i|y)$ denote the brand’s updated belief given the provider’s report $R(d_i) = r$, where $\Gamma(\cdot)$ denote the brand’s belief updating function (which will be characterized later). We note that $r = \emptyset$ if the provider does not report.

Define $G_i^o(k_i, w_i) = R_i^o(k_i) - w_i$ as the brand’s profit in period $i$ under outsourcing, where $R_i^o(k_i) = v^o - \pi(1 - \theta(\xi|_{ki}))$. The brand’s problem can be defined as

$$V_i^{TS}(d_i^b, s_i) = \max_{w_i, k_i, y_i} \left\{ G_i^o(k_i, w_i) - y_i M + \gamma E_{c_{i+1}, (1-y_i) d_i^{bs} + y_i d_i} V_{i+1}^{TS}(d_i^{b}, s_i) \right\}, \quad i = 1, \ldots, T, \quad (3)$$

s.t. $d_i^{bs} = \Gamma(d_i|y_i)$, \quad (4)

$$r_i = \arg \max_{R(d_i)} \Gamma(d_i|y_i), \quad (5)$$

$$y_i c^S(k_i, d_i^{bs}) + y_i c^S(k_i, d_i) \leq w_i, \quad (6)$$

$$w_i \geq 0, \quad k_i \geq 0, \quad y_i \in \{0, 1\}. \quad (7)$$

For the boundary condition, we set $V_T^{TS}(d_{T+1}) = 0$. We note that constraints (4) and (5) implicitly depend on $s_i$ as well. The key differences between the third-party service and direct service are (a) the current period expected revenue function is given by $G_i^o(k_i, w_i)$, (b) the brand only partially observes service environment with its belief jointly updated by (4) and (5), (c) the brand’s payment is contingent on its learning and monitoring effort as specified in (6). In particular, the payment is based on partially updated belief $d_i^{bs}$ if the brand does not exert monitoring effort (i.e., when $y_i = 0$), whereas it is based on true service environment $d_i$ if the brand does exert monitoring effort (i.e., when $y_i = 1$). Note that in the former case, the payment covers the provider’s service cost ex-ante in expectation: $E_{\xi}^S(k_i, d_i^{bs} \xi_i^{D}) = c^S(k_i, E_{\xi}^S(d_i^{bs} \xi_i^{D})) = c^S(k_i, d_i^{bs} E_{\xi}^S(\xi_i^{D})) = c^S(k_i, d_i^{bs})$, where the equality holds because the payment contract $c^S$ is linear in service environment. Given that monitoring is costly (incurring a fixed cost $M$ each time), the brand may either actively monitor the environment at a cost or passively update its belief based on the provider’s reporting behavior. For notational ease, at times we simply use $d_i^{bs}$ to denote the brand’s updated belief, recognizing that

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5 Feng et al. (2019), for example, note that direct contracts are “straightforward to specify, easy to communicate, and relatively simple to implement” and “probably the most widely used form of contracts in service outsourcing” (p.689). Further, Krishnan and Mani (2020) empirically show that direct, cost-plus contracts are more prevalent when service environment is uncertain and/or ambiguous.
$d_{i}^{bs} = d_{i}$ if the brand exerts monitoring effort. We shall analyze both equilibrium belief updating and monitoring subsequently.

The structure of $V_{i}^{TS}(d_{i}^{b}, s_{i})$ is similar to $V_{i}^{DS}(d_{i})$, and the following is analogous to Lemma 1.

**Lemma 2 (Basic Property on Third-Party Service (TS))**

(a) $G^{o}(k_{i},w_{i})$ is concave in $k_{i}$, and the optimal $k_{i}^{*}$ satisfies $\frac{\partial c^{S}(k_{i},d_{i}^{b})}{\partial k_{i}} = \pi_{i}^{d_{i}^{b}}(\lambda \pi_{i}^{a} - 1)$ and $k_{i}^{*} = 0$ otherwise. (b) All else being equal, $k_{i}^{*}$ decreases in $d_{i}^{b}$. 

We will leverage Lemmas 1 and 2 in the next section to analyze how service environment uncertainty affects a brand’s preference toward these two strategies. In closing, we note that two key benefits of outsourcing are (a) improved product value due to focus on core business (i.e., improve $v$ to $v^{o}$) and (b) reduced service provision cost due to service provider’s cost efficiency (i.e., $c^{S}(k,d) < c(k,d)$). However, these benefits must be measured against the negative impact of partial observability of service environment and the fixed cost $\tau$ of switching from in-house operation to a third-party provider. A key question we explore in the next section (§4) is whether a brand benefits from service outsourcing, and if so, under what conditions. We then further explore the impact of partial observability and monitoring efforts on outsourced product services in §5.

### 4. Outsourcing Product Service: Value and Intention

In this section, we focus on the performance bound on the outsourced product service and hence key drivers toward a brand’s intention to outsource. According to the analyses in §3.6, regardless of whether the product service is performed in-house or contracted to a third party, the service performance declines with increased cost of resources due to environment uncertainty. Nevertheless, the brand enjoys a lower cost and increased product value by contracting its product service to a third-party provider. The following proposition formalizes the above observation.

**Proposition 1 (Upper Bound on Improved Service with TS)**

All else being equal, (a) $V_{i}^{TS}(d_{i}^{b}, s_{i})$ is upper bounded by $V_{i}^{TS}(d_{i}^{b}, 0)$ and (b) at this upper bound $k_{i}^{*} < k_{i}^{o}$ and $V_{i}^{DS}(d_{i}) < V_{i}^{TS}(d_{i}^{b}, 0)$.

The above proposition confirms that, under the best-case scenario (i.e., under full observability with outsourcing such that $s_{i} = 0$), (a) the service level $\theta(\cdot)$ is higher and (b) the brand enjoys a higher expected profit. Switching from in-house operation to a third-party provider, however, incurs a fixed cost $\tau$. Therefore, outsourcing is more attractive only if the expected benefits within the planning horizon exceed the switching cost. The following proposition partially characterizes the brand’s intention to outsource product services.

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6 The planning horizon is the brand’s contract review cycle, at the end of which the brand may choose a different provider or bring operations in-house (both at the same switching cost $\tau$) depending on past performance and service environment.
Proposition 2 (Switching to TS)

Define $L(x) = x(1 - \ln(x))$ and $A_i = C(1 + \beta d_i)/(\lambda \pi)$. (a) If the service environment is stable (i.e., $d_i = d$ for all $i$), then there exists a finite threshold $\overline{T}$ such that the brand benefits from service outsourcing whenever $T \geq \overline{T}$, where $\overline{T}$ is given by

$$\overline{T} = 1 + \log \left( 1 - \frac{(1 - \gamma)\tau}{\nu^o - \nu + \pi(L(A_i) - L(\alpha A_i))} \right) / \log (\gamma). \quad (8)$$

(b) Furthermore, $T_i$ (i) is convex in $d_i$ (initially decreasing and eventually increasing), and (ii) increases if the environment is uncertain (i.e., $d_i$’s form MMFE).

When the service environment is stable, the brand is more likely to benefit from service outsourcing even with significant switching costs. Part (a) implies that, in a stable environment, the optimal switching decision is nested: if it is optimal to switch with planning horizon $T$, then it is also optimal to switch with any planning horizon longer than $T$. In fact, the brand always benefits from outsourcing with an infinite planning horizon (i.e., $T \to \infty$). Conversely, if it is not optimal to switch with a planning horizon $T$, then the brand should never switch with any planning horizon shorter than $T$. Additionally, part (b)(i) suggests that a more challenging environment makes outsourcing more attractive (since a shorter contract length is needed to offset the switching cost). However, if the environment becomes extremely challenging, then the benefit of outsourcing diminishes because the performance of direct service and third-party service converges (since neither approach can maintain sufficient customer satisfaction). We shall show subsequently that if a target service level must be maintained, then the more challenging the environment, the more attractive service outsourcing is.

Part (b)(ii) suggests that environment uncertainty weakens the expected benefit of outsourcing, such that a longer planning horizon (relative to the stable environment case) is needed to break even the switching costs. Therefore, the brand can be worse off with outsourcing if it sets a planning horizon assuming a stable environment. Part of the intuition is that the benefit of outsourcing is a concave function of environment uncertainty, and thus increased environment uncertainty reduces (due to Jensen’s inequality) the expected benefit of outsourcing. A direct implication is that environment uncertainty (which is more likely to result in more extreme cost of resources), while incentivizes brands to outsource, nevertheless dampens the brand’s expected benefit of product service outsourcing. From a product life cycle perspective, an important implication is that the brand is more likely to benefit from outsourcing if the product has a long-life cycle such that the outsourcing contract can be planned for a longer planning horizon. In contrast, for a short life cycle product, all else being equal, it is less likely to benefit from outsourcing due to fixed switching costs.
The above observation is based on the brand setting the optimal resource level contingent on service environment in each period. In practice, brands typically need to specify and maintain a fixed target customer satisfaction level $\overline{\theta}(\cdot)$ in an SLA (service level agreement), which can be influenced by many factors, including industry norms (Overby et al. 2017, SQM 2023) or pressures from customers (Buesing et al. 2018, Businesswire 2022). We therefore examine the brand’s performance when a fixed target service level must be maintained in all our subsequent analyses. This is equivalent to modify constraint (7) such that $k_i = \overline{k}$, which denotes the resource level required to maintain a fixed target customer satisfaction level $\overline{\theta}(\cdot)$.

**Proposition 3 (Bound on Planning Horizon)**

The brand cannot benefit from outsourcing product service with a planning horizon $T$ for any

$$T \leq 1 + \log \left( 1 - \frac{(1 - \gamma)\tau}{v^\rho - v + (1 - \alpha)\overline{k}(1 + \beta d_i)} \right) / \log (\gamma).$$

A target service level requirement fundamentally changes the impact of environment uncertainty on the brand’s incentive to outsource. In contrast to Proposition 2, here $T$ monotonically decreases in $d_i$, suggesting that the more challenging the environment is, the more attractive outsourcing is (i.e., $T$ shrinks in $d_i$). The reason is that the benefit of outsourcing under a fixed target service level relates only to the cost savings of service provision by the third-party provider. These cost savings are proportional to the expected cost of resources as influenced by the service environment. Therefore, as long as the expected environment remains the same (i.e., the brand does not have additional information that foresees an upward or downward trend in the service environment), the expected cumulative discounted benefit is the same as if the current environment will persist into the future. Further, the brand’s incentive to outsource is increasing in the required resource level $\overline{k}$ and in the third-party provider’s cost efficiency (i.e., decreasing $\alpha$). Thus, when the industry expectation on service level is high, the brand is more likely to outsource the product service to a third-party provider. This observation also echoes our earlier discussions that brands that must meet high service levels will have a stronger incentive to outsource the product service function.

The above analysis establishes that brands have a stronger incentive to outsource when the environment becomes more challenging due to high uncertainty, and all else being equal there is an upper bound on the brand’s product outsourcing benefit. Note that a resource budget constraint on the brand can reduce the benefit of outsourcing, and we analyze the impact of budget constraints in Appendix A1. In the next section, we explore how partial observability affects the outsourcing benefit and whether the learning and monitoring efforts can help the brand to better retain the product outsourcing benefit.
5. Impact of Partial Observability, Learning, and Monitoring

To develop insights, we first consider a benchmark case where there is no reporting and learning between the service provider and the brand nor any monitoring efforts by the brand.

Lemma 3 (Service Environment Belief under Partial Observability)

(a) The brand’s belief about the service environment is unbiased, that is, \( E[d_i] = E[d_i] \).

(b) The probability that the brand’s belief exceeds the true service environment by an arbitrary value \( X \) is given by

\[
P(d_b^i - d_i \geq X | s_i) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{\ln(1 - X/d_b^i) + s_i^2/2}{\sqrt{2} s_i} \right) \right),
\]

where \( \text{erf}(\cdot) \) is the standard error function. (c) \( \partial P(d_b^i - d_i \geq X | s_i) / \partial s_i \geq 0 \).

While outsourcing can lead to limited visibility, the brand’s belief about the service environment is unbiased. The key reason is that the service environment is expected to remain the same given the MMFE property. If there is a positive or negative trend in the service environment, then the trend can be incorporated into the brand’s belief such that the expected service environment is unbiased.

Nevertheless, the probability that the brand’s belief diverges from the true service environment by more than \( X \) can be quite significant. For example, if \( s_i^2 = 1 \), then by (10) the probability that the brand’s belief diverges by more than twice the true service environment (i.e., \( d_b^i/d_i \geq 2 \)) is around 0.5, and this probability rapidly approaches to 1 as \( s_i^2 \) increases. This suggests that, while the brand holds an unbiased belief, there is a high probability that the actual service environment can diverge significantly from the belief. This is more likely to happen when (a) environment uncertainty is high and/or (b) the outsourcing relationship \( T \) is long. That latter implies that partial observability is more likely to occur for long-life cycle products.

A diverging belief in the service environment directly influences the value of outsourcing. First consider the case when \( d_b^i \geq d_i \); that is, when the cost of resources under the actual service environment is less than that under the brand’s belief. This can happen when the service provider obtains more efficient resources (e.g., high-quality workers from lower-cost regions, better training practices, new technologies), accumulates knowledge on how to provide product services more efficiently, or when market demographics shift service requests to be more congruent to service provider’s resources. In such cases, the brand could have realized lower costs and higher service levels if it knew the true service environment, so the brand incurs an opportunity cost for potential cost savings. The service provider’s participation constraint is non-binding because \( c^S(\overline{k}, d_i) \leq c^S(\overline{k}, d_b^i) \) where recall that \( \overline{k} \) is the target resource level specified by the brand. The brand’s value loss from outsourcing due to partial observability is therefore given by \( c^S(\overline{k}, d_b^i) - c^S(\overline{k}, d_i) \geq 0 \).
In contrast, next consider the case when \( d^b_i < d_i \); that is, when the actual service environment is more challenging than the brand’s belief. In such a case, the service provider would have an incentive to report the true service environment such that the brand increases payment from \( c^S(\bar{k}, d^b_i) \) to \( c^S(\bar{k}, d_i) \). Although the brand increases its service payment to the provider here, the value of outsourcing, in this case, is not impacted by partial observability. The reason is that, even with full observability, the brand would have set the same payment to the provider given the observed service environment \( d_i \). Therefore, partial observability affects the value of outsourcing only when the brand’s belief drifts above the actual cost of service provision.

**Proposition 4 (Value of TS under Partial Observability)**

Denote \( \Delta_i = V^{TS}_i(d^b_i, s_i) - V^{DS}_i(d_i) \) as the value of product service outsourcing. Then the value \( \Delta_i \) (a) is affected by partial observability only when the belief drifts above the actual cost of service provision, (b) strictly declines in the variance of partial observability \( s^2_i \), and (c) can be strictly negative under partial observability.

Partial observability can therefore negate the value of product service outsourcing, despite the fact that the third-party provider is more cost-efficient and that the brand gains product value by focusing on its core operations. The above result holds even when there is no switching cost involved from in-house service to third-party service. This is in contrast to the case of full observability where, in absence of switching cost, outsourcing is always beneficial as outlined in Proposition A1. Since partial observability is cumulative over time, the above result also suggests that the longer the outsourcing relationship (e.g., for longer life cycle products), the more likely that the brand’s value gained from outsourcing is wiped out by partial observability. This is somewhat consistent with Austin (2011)’s observation that the typical outsourcing relationship is shrinking over time and that agrees with the dilemma facing airlines in our motivating example. We next examine whether brands can improve product service outsourcing performance through dynamic learning.

### 5.1. Equilibrium Learning Dynamics

Recall that under partial observability, the service provider is paid based on the brand’s belief about the expected cost of resources, not the actual cost incurred under the true service environment. Since the provider’s pay is increasing in the brand’s belief about the expected cost of resources, the provider is interested in maximizing the brand’s belief in each period through its reporting strategy. Naturally, this implies that the provider may voluntarily report to the brand the true service environment when doing so would raise the brand’s belief in the cost of resources, but the provider would be unlikely to do so if such reporting may lower the brand’s belief. We assume that the service provider is truthful if it chooses to report service environment; that is, it may hold private information (i.e., choose not to report) but it does not distort information when it chooses to report
to the brand. This assumption is reasonable because the brand can end the relationship for any fraudulent behavior, and the mere possibility of termination could encourage truthful reporting. The truthful reporting assumption does not imply that the provider never overcharges the brand. When the provider chooses not to report the service environment, it is in effect overcharging the brand. This echoes the survey results in BPG (2022) discussed in the introduction.\footnote{The provider may also falsify the information it reports (i.e., exaggerate the cost of resources). In such a case, the brand may either ignore or discount the provider’s report. In the former case, the brand will only use its own belief as a basis for contract payment (i.e., a special case of our model where no learning occurs). In the latter case, the brand anticipates and discounts the provider’s report and the provider knows this, which likely unravels the equilibrium such that the report becomes cheap talk.}

Such an asymmetric behavior by the service provider has significant implications on the brand’s belief and, consequently, the benefit of outsourcing. The brand’s belief is influenced by the service provider’s reporting strategy. If the provider reports \( d_i \), then the brand’s belief is updated to \( d_{i}^{bs} = d_i \) (since the provider does not falsify information about the service environment). However, if the provider does not report, the brand’s belief is also updated since the true environment must be more favorable to the provider (i.e., resource is less costly than the brand’s current belief). As such, even if the provider does not report \( d_i \), the brand’s belief is still partially updated closer to the true service environment. Because disclosing the true environment information can be costly for the provider (e.g., preparing and communicating detailed operational and financial data or sharing proprietary information (Verrecchia 1983)), we let \( c^R \geq 0 \) denote the provider’s fixed cost of reporting the true service environment to the brand.

To derive the provider’s optimal disclosure strategy, we define a perfect Bayesian equilibrium (PBE) consisting of the provider’s reporting strategy \( R(d_i) \) and the brand’s conditional belief \( d_{i}^{bs} = \Gamma(d_i^b \mid R(d_i)) \). The PBE satisfies two axiomatic conditions: (a) the reporting strategy \( R(d_i) \) maximizes the provider’s payoff (i.e., maximizes the brand’s conditional belief) such that for any realized \( d_i, r = \text{arg} \max_{R(d_i)} \Gamma(d_i^b \mid R(d_i)) \) and (b) the beliefs are consistent such that \( \Gamma(d_i^b \mid R(d_i)) \) is consistent with the provider’s report \( R(d_i) \). Since the provider is truthful (and to make the analysis tractable), we also assume that the provider’s reporting strategy is not contingent on the brand’s future monitoring activities. The following proposition characterizes the provider’s optimal PBE reporting strategy and the brand’s equilibrium belief updates.

**Proposition 5 (Equilibrium Reporting and Learning)**

(a) The service provider’s PBE reporting strategy \( R(d_i) \) is a threshold type such that

\[
R(d_i) = \begin{cases} 
    d_i, & d_i \geq d_i^R; \\
    \emptyset, & d_i < d_i^R,
\end{cases}
\]

where \( \emptyset \) denotes non-disclosure and \( d_i^R \) is a threshold environment that uniquely satisfies

\[
d_i^R = d_i^b F_i^{-1} \left( \frac{d_i^R}{d_i^b} \right) \int_{\xi_i^b \leq d_i^R/d_i^b} \xi_i^b F_i(\xi_i^b) + \frac{c^R}{\alpha C \beta k},
\]

where \( \emptyset \) denotes non-disclosure and \( d_i^R \) is a threshold environment that uniquely satisfies
where $F_i(\cdot)$ denotes the distribution function of $\xi^b_i$.

(b) The brand’s PBE belief in the service environment is updated by the following rules.

$$d_i^* = \Gamma(d_i^R | R(d_i)) = \begin{cases} d_i, & R(d_i) = d_i \text{ (provider reports)}; \\ d_i^R - c^R/(\alpha C \beta \bar{K}), & R(d_i) = \emptyset \text{ (provider does not report)}. \end{cases}$$

(13)

The brand receives asymmetric reporting of the true service environment by the provider: there exists a threshold level of service environment where the service provider will report the true environment only when the realized service environment is more challenging than the threshold condition but otherwise will not disclose. The brand is therefore partially informed about the true service environment: it is perfectly informed when the environment becomes more challenging but only partially informed when the service environment becomes less challenging. The intuition is that when the service provider does not disclose the service environment, the brand partially updates its belief downwards by recognizing that the true environment must be below the threshold level. Thus, in equilibrium the brand learns more accurately about the service environment when it becomes more challenging but not so when it becomes less challenging.

We next examine how the reporting cost $c^R$ and environment volatility impact the service provider’s threshold reporting strategy. The environment volatility can be captured by the variance parameter $s_i$ associated with the distribution function of $\xi^b_i$, that is, the parameter $s_i$ associated with $F_i(\cdot)$. Note, however, that $F_i(\cdot)$ may not be lognormal if the provider has not reported true service environment in prior periods, because the brand’s posterior distribution of $\xi^b_i$ is truncated due to partial belief updating. To facilitate subsequent analysis, we adopt a lognormal moment matching approximation to $F_i(\cdot)$. Specifically, in any period when the provider does not report, let $\tilde{F}_i(\cdot)$ denote the brand’s posterior (truncated) distribution and $\tilde{F}_i(\cdot)$ denote the matching lognormal distribution such that $d_i \cdot \int xd\tilde{F}_i(x) = d_i^* \cdot \int xd\tilde{F}_i(x) = d_i^*$ and $\int x^2d\tilde{F}_i(x) - (\int xd\tilde{F}_i(x))^2 = \int x^2d\tilde{F}_i(x) - (\int x^2d\tilde{F}_i(x))^2 = \int x^2d\tilde{F}_i(x) - 1$. Thus, the matched distribution $\tilde{F}_i(\cdot)$ is lognormally distributed with mean equal to 1 and has the same variance as the posterior truncated distribution $\tilde{F}_i(\cdot)$. If the brand regards the underlying environment uncertainty as lognormally distributed with uncertain parameters, then the moment matching approach has an intuitive interpretation: whenever the provider does not report, the brand shifts its beliefs downwards with reduced variance, that is, the brand is more confident that the service environment is less challenging (while maintaining a lognormal belief). In the subsequent analysis, we use the matched lognormal approximation approach at places when explicit expressions are desired. Specifically, this approximation is used in Corollaries 1-3 and Propositions 7-9.

A numerical simulation of 86,905 scenarios with varying reporting cost and environment uncertainty show that, with the matched approach, the mean difference in reporting threshold is 1.85%, with a maximum of 37.90% and minimum of -9.99% (but less than 6% of scenarios has a difference greater than 15%).
Corollary 1 (Comparative Statics on Reporting Threshold)

All else being equal, the threshold reporting level \( d_i^R \) (a) increases in the fixed reporting cost \( c_i^R \) and (b) decreases in the environment volatility \( s_i^2 \).

Part (a) is intuitive since the service provider is less likely to report the true environment when its fixed reporting cost increases. The reporting cost can also include the potential negative impact of revealing confidential information, such as operating strategies and detailed financial data that may be leaked to third parties (thus negatively affecting the service provider). Therefore, the brand would obtain better information if it can assure the service provider’s data security concerns, or if it can help the provider to ease the reporting burden and hence reduce the reporting cost.

Part (b) suggests the provider’s reporting threshold decreases over time; that is, the provider is more likely to report true environment as time evolves (given that the provider has not reported since the last disclosure). The intuition is that the brand will become increasingly suspicious that its belief about the cost of resources is higher than the actual cost under the true service environment. The service provider recognizes the brand’s suspicion and must lower its reporting threshold over time to keep the brand’s belief consistent with its reporting strategy. Therefore, the brand partially calibrates its belief periodically even when the service provider does not disclose true service environment and hence can more accurately track the service environment over time.

Nevertheless, the brand’s belief is only partially calibrated and becomes more accurate only with the accumulation of time. Given that the provider has no incentive to report a more favorable (less costly) service environment (c.f. Lemmas 1 and 2), the brand overpays the provider whenever the true environment is more favorable (less costly) than what the brand believes. As a result, partial observability, coupled with asymmetric reporting by the provider, can reduce and may even completely negate the value of outsourcing. The following proposition partially characterizes conditions under which outsourcing is indeed not attractive due to partial observability.

Proposition 6 (Value Loss under Partial Observability and Learning)

(a) The brand incurs a higher cost with outsourcing in any period whenever

\[
\int_{\xi_i^b \geq 1} \xi_i^b dF_i(\xi_i^b) \geq \frac{1}{\alpha},
\]

where \( F_i(\cdot) \) denotes the distribution function of \( \xi_i^b \). (b) There exists a finite upper bound on the number of periods that (14) holds after the service provider’s last disclosure.

In part (a), condition (14) is a sufficient condition that weighs the negative impact of partial observability (as captured by distribution \( F_i \)) and the positive impact of cost efficiency attained by the third-party provider (as captured by \( 1/\alpha \)). The brand is therefore less likely to benefit from
service outsourcing when visibility is limited (i.e., \( \int_{\xi_i \geq 1} \xi_i dF_i(\xi_i) \) is large). This occurs when the service environment is evolving (e.g., external events influencing customer needs, resource mixes, or cost of labor) and the outsourcing relationship is long (e.g., for long life cycle products). An evolving environment implies that a brand is both likely to see a more challenging environment (high \( d_i \)) and a more accommodating environment (low \( d_i \)). The more challenging environment creates a strong incentive for brands to outsource (see Lemma 1 and discussions thereafter), but the brand will reap less cost savings under a more accommodating service environment under partial observability. Therefore, an evolving service environment makes it more likely for brands to outsource, and yet this condition can also cause brands to suffer from partial observability, potentially negating the benefit of outsourcing in the first place. It is therefore important for brands to assess the volatility of the service environment before deciding whether to outsource product services.

Part (b) suggests, however, that there is a limit in the number of periods that the brand suffers from partial observability in each cycle of the provider’s non-disclosure decisions. That is, as the service provider continues to not disclose, the brand suffers only up to a certain time period limit, beyond which the brand may benefit from the outsourcing arrangement. This result applies to each cycle starting from the last disclosure until the service provider reports the true service environment again. Note that while there is a limit in the time periods when the brand suffers from partial observability, this does not imply that the negative impact of limited visibility can be ignored. Quite on the contrary, the costs incurred during the limited time periods can still be significant enough to outweigh the benefit of initial cost savings.

From a practical perspective, partial observability with asymmetric partial belief updates suggests that a reasonable policy for the brand to manage the outsourcing relationship, especially for long life cycle products, is to gradually reduce the payment to the service provider unless the service provider reports convincing evidence that the service environment becomes more challenging. When the provider does report the true environment, the brand resets the payment terms and then gradually reduces the payment from this point on until the service provider reports again. This policy coincides with casual observations that brands often “squeeze” service providers over time. For example, in the airline example discussed in the Introduction, The Economist (2017) observed that “Airbus and Boeing are adopting a more aggressive stance towards the suppliers [service providers]. This means trying to push them into offering much lower prices today, in return for future contracts” (para. 4). This is also a key underlying reason that suppliers, such as UTC and Collins, decided to merge to save costs (nudging lower \( d_i \) in our context). In practice, there are multiple reasons why brands may squeeze service providers and why providers may decide to merge, but our findings do suggest that there exist systematic forces explaining why such a phenomenon can happen in many outsourcing relationships. We next explore whether the brand can improve its outsourcing performance by additionally exerting monitoring efforts.
5.2. Monitoring

Recall that the service provider will provide asymmetric updates whenever \( d_i \geq d_i^R \); that is, whenever the underlying environment is more challenging than a threshold level. As such, the brand’s monitoring decision is made after observing whether the provider reports the true service environment. Note that if the provider reports in period \( i \), then the brand’s belief is updated to true service environment \( d_i \) with no monitoring effort \((y_i = 0)\).

**Proposition 7 (Threshold Monitoring Policy)**

*The optimal policy for monitoring efforts is of a contingent threshold type, such that \( y_i^* = 1 \) if and only if (a) the provider does not report in period \( i \) and (b) \( d_i^R \geq \bar{d}_i(s_i) \). Further, \( \partial \bar{d}_i(s_i)/\partial s_i \leq 0 \).*

It is optimal to exert monitoring effort only when the brand’s belief about the environment is above a certain threshold, and this threshold is decreasing in the volatility of random events affecting the environment. Therefore, the more turbulent the environment, the lower the threshold for exerting monitoring effort. This agrees with the intuition that the brand should be more vigilant with monitoring efforts when the service environment is turbulent; for example, when customer service needs shift more often and/or when technologies advance more rapidly on how service can be rendered by the service provider.

We note that a more turbulent service environment also lowers the threshold for the service provider to report the true environment (see Corollary 1(b)). This suggests that both the monitoring threshold and reporting threshold decrease over time. As a result, the brand should not exert monitoring effort when the service provider’s reporting threshold becomes sufficiently low (see Corollary 6(b)). From a managerial perspective, if the service provider has not reported for many periods, then instead of exerting monitoring efforts, the brand should follow a *passive monitoring strategy* and gradually lower the payments based on its updated beliefs of the service environment. The reason is that in such instances the service provider will either accept the lower payments (because the cost of resources becomes more favorable) or choose to disclose the true environment (because the cost of resources is higher than the brand’s beliefs) to calibrate payments. Either way, the brand is better off with passive monitoring since it saves monitoring costs without incurring large, misaligned payments. Similarly, the brand is less likely to benefit from monitoring efforts in periods immediately after the provider’s true environment disclosure. In general, the brand is more likely to benefit from monitoring after some moderate periods have passed since the last disclosure but not to wait too long, after which the brand is more likely to benefit from gradually reducing payments. Next, we examine how the cost and service characteristics influence the brand’s monitoring strategy.
Corollary 2 (Sensitivity of the Monitoring Threshold)
The threshold \( d_i(s_i) \) satisfies the following property: (a) \( \partial d_i(s_i)/\partial \alpha \leq 0 \), (b) \( \partial d_i(s_i)/\partial \bar{k} \leq 0 \), and (c) \( \partial d_i(s_i)/\partial M \geq 0 \).

All else being equal, the brand is less likely to conduct monitoring efforts when (a) the provider offers a lower cost (relative to the brand’s internal operation), (b) the brand maintains a lower service level requirement, and (c) the brand incurs a higher cost to conduct monitoring efforts. These three observations suggest that if the brand outsources to a low-cost service provider, then it is less inclined to monitor the provider. On the other hand, if the brand needs to maintain a high service level, then it is more inclined to monitor. In addition, the brand will never monitor when \( M \) is sufficiently large.

The above results have important practical implications for the brand’s outsourcing and monitoring efforts. If the brand outsources product service to a low-cost, far-away provider, then it is less likely to conduct monitoring efforts since these are settings where the threshold level for monitoring is very high. Thus, the brand is least likely to monitor precisely under the most vulnerable conditions for the product service to go awry. Such service risks, however, can be partially mitigated by the passive monitoring strategy discussed earlier. That is, the brand can gradually lower the payments whenever the service provider chooses not to disclose the true environment. On the other hand, when a brand exerts monitoring efforts, then its efforts are reinforcing with the provider’s reporting frequency, allowing the brand to track the service environment more accurately.

Corollary 3 (Reinforcing Monitoring and Reporting)
All else being equal, the brand’s monitoring and the provider’s reporting decisions are reinforcing: the more frequently the brand monitors, the more frequently the provider reports, and vice versa.

Therefore, the more efficient (less costly) a brand is with its monitoring process, the more likely the service provider will report the true service environment. From a practical point of view, the brand may improve its monitoring process by implementing supply chain digitization and multi-channel communication with the service provider. Such efforts can lead to a virtuous cycle where there is frequent monitoring by the brand and frequent reporting by the service provider. Nevertheless, oftentimes monitoring is costly since the outsourced service is not considered a core business and partial observability has the side effect of rendering the brand not being able to properly verify the service provider’s work. As a result, the brand could find it costly to conduct frequent monitoring activities. A natural question arises as to whether there are instances where active monitoring efforts are not critical for the brand to arrest the negative impact of partial observability. In the next proposition, we show that under certain conditions following the passive monitoring strategy is sufficient for the brand to contain the negative impact of partial observability such that the brand is still better off with outsourcing the service.
Proposition 8 (Cost Bound with Passive Monitoring)
There exists a threshold level reporting level $\bar{d}^R$ such that: if $d^R_i \leq \bar{d}^R$ almost surely in any given time period $i$, then the brand’s outsourcing cost is guaranteed to be lower than the cost with direct service provision. Furthermore, $\bar{d}^R$ increases when the service provider’s cost efficiency improves.

If the brand expects that the service provider’s report threshold be below a certain threshold almost surely, then passive monitoring based on the provider’s self-reporting is sufficient to ensure that it is more attractive to outsource than provide the service directly. In addition, when the brand works with a more cost-efficient provider, passive monitoring is more likely to be sufficient for the brand to contain the cost risk associated with partial observability. Note that this does not imply that monitoring is not useful, rather, monitoring (if not too costly) provides further improvement over the outsourcing performance beyond the benchmark of direct service provision.

5.3. Heuristic Policy
The optimal monitoring decision is influenced by the variance of the cumulative information that the brand does not learn, which poses a practical challenge for brands to implement the optimal policy. It is therefore of value to explore a policy form that does not depend on the knowledge of information that the brand does not know. Toward this goal, we simplify two effects that influence the optimal monitoring decision. First, we decouple the marginal benefit of monitoring in the current period from the future period and consider only the benefit in the current period. This obtains a myopic policy that is easy to compute. Second, we inflate the variance of the cumulative information that the brand has not learned up to the beginning of the current period. This yields a policy that performs well under worst-case scenarios (where partial observability is significant). Based on the above two simplifications, the heuristic policy takes a simple form as described below.

Proposition 9 (Heuristic Policy)
Consider the myopic heuristic policy for monitoring efforts as $\hat{g}_i = 1$ if and only if (a) the provider does not report in period $i$ and (b) $d^b_i \geq \bar{d}^b_i := M(\alpha C \bar{k} \beta)^{-1}$. As the variance of partial observability $s^2_i$ increases, the above policy is asymptotically optimal among all possible myopic policies. Furthermore, the monitoring threshold satisfies $\bar{d}^b_i \geq \bar{d}^b_i$ and, as $s^2_i$ increases, the maximum threshold gap $\bar{d}^b_i - \bar{d}^b_i \leq [1/(1 + r)]\bar{d}^b_i$.

The above heuristic policy takes a simple form, which does not depend on the variance of cumulative information that the brand does not learn up to the decision period. Under this policy, the brand conducts monitoring efforts whenever its belief about the environment drifts above a fixed threshold. Therefore, the brand conducts more frequent monitoring when the monitoring cost is low (i.e., $M$ is small), the provider’s cost efficiency is low (i.e., $\alpha$ is large), the desired service level is high ($\bar{k}$ is large), or when the environment has a larger impact on resource cost (i.e., $\beta$ is high).
Note that as the monitoring cost $M \to 0$, the heuristic policy coincides with the global optimal policy, since in both cases the optimal policy is to monitor in every period except for when the provider reports the service environment. Interestingly, the above results suggest that brands are less likely to conduct monitoring efforts when the provider offers large cost reductions ($\alpha$ is small). This implies that when the brand outsources product service to a very low-cost third-party provider, it will have little incentive to conduct monitoring efforts. As a result, the brand is likely to lose significant visibility over time, representing missed opportunities for potential cost savings by monitoring more often. Conversely, when the monitoring cost $M \to \infty$, then the heuristic policy also coincides with the global optimal policy, since in both cases the optimal policy is to never monitor in any period. This suggests that if the third-party provider is in a foreign country such that monitoring cost is prohibitive, then the brand suffers the most from partial observability, and hence the anticipated benefit of outsourcing product service may not materialize.

The threshold gap between the heuristic policy and the global optimal policy can be bounded within a fraction of $\frac{b}{d}$. This bound is tighter under low monitoring cost but high target service level $k$, precisely the setting when the heuristic policy is more likely to conduct monitoring effort. On the other hand, when the above setting is reversed, then the threshold gap increases, suggesting that the heuristic policy is less likely to monitor as compared with a global optimal policy. Note that a numerical study that sheds further insights into the magnitude of performance gaps caused by partial observability, learning, and monitoring is available upon request.

6. Discussion and Conclusion
Product service plays a pivotal role in retaining customers and spurring revenue growth. It is therefore critical to manage product service effectively, especially when product service is outsourced. A critical factor that influences a brand’s ability to provide high-quality service is environment uncertainty, that is, when the customer needs and service provision efficiency change over time. We find that a more challenging environment creates strong incentives for brands to switch the product service process from in-house to a third-party provider who offers a marginal cost advantage in provisioning the service. Therefore, a brand is more likely to outsource under environment uncertainty (which makes it more likely for brands to encounter a more challenging environment). The brand typically benefits from outsourcing, at least initially, when the third-party provider offers significant cost advantages. Yet, such benefits can dissipate rapidly over time due to partial observability, making it difficult for the brand to set accurate payments for service outcomes and therefore incur increased costs to the provider over time. If left unattended, the brand can be worse off with outsourcing for long-life cycle products over the long run.

On the other hand, when monitoring cost is low or the provider’s reporting cost is low, then the brand may capture the most outsourcing benefit. Therefore, from a brand’s perspective, an
important consideration in product outsourcing is to weigh whether it can monitor the service environment efficiently or whether it can nudge the service provider to self-report the cost of resources frequently by reducing the provider’s reporting cost. Typically, both monitoring cost and reporting costs are lower when the provider is closer to the brand, where regular interaction is expected and the cost of such interactions is low. In contrast, when the provider is in distant locations, then regular interactions, and hence monitoring and reporting, can be challenging and the anticipated outsourcing benefits may be jeopardized. In the airline example discussed in the introduction, maintenance providers are often scattered in far-flung regions with sophisticated service provision processes, making regular monitoring challenging and expensive. This leads to limited visibility such that the airplane makers incurred a higher than anticipated cost (The Economist 2017) and yet they were seeing reduced levels of customer satisfaction by UTC and Collins (Shay 2020). Thus, product service outsourcing in itself does not imply that the service outcome will improve or worsen. The key determining factor is whether the brand manages the outsourcing process with service environment clarity at a minimal cost. Our findings suggest that brands have to engage service providers with either increased monitoring efficiency or improved self-reporting to mitigate the negative impact of partial observability in outsourced product services.

There are a number of interesting research directions that ensue. First, a logical next step is to explicitly link a brand’s core business (of manufacturing products) with the non-core business (of providing after-sales product service), and examine how the non-core business performance directly impacts the core business. Second, it is useful to distinguish desirable and non-desirable customers in terms of their service requests and impacts on the brands’ bottom line. When a large fraction of service failures is generated by a small fraction of customers, outsourcing the product service may not entail rigorous monitoring efforts from the brand’s perspective. Instead, one possible strategy can be a tiered service provision that devotes different resource levels to different segments of customers. Third, it is of value to investigate how to leverage the expertise of the third-party provider: the feedback loop may not always be negative, and proper incentives could be designed to motivate the provider to voluntarily tailor resource levels to meet customer needs while keeping the brand informed. We hope that future research by us and others will further our understanding of this important area of research.

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A1. Limited Resource Budget

In the following analysis, we consider a product service outsourcing setting with a budget constraint. In such a setting, because the service environment is uncertain, a brand can only maintain the required service level in a probabilistic sense. That is, the brand will maintain resource level $k$ when the corresponding cost $c(k, d_i)$ is less than the budget $\bar{w}$ but maintains a reduced level $\hat{k} < k$ such that $c(\hat{k}, d_i) = \bar{w}$. With fixed budget $\bar{w}$, the benefit of outsourcing (besides product value gain $v^o - v$) can be partitioned into three regions depending on the service environment $d_i$. The following proposition illustrates the drivers of outsourcing benefits under different levels of environment uncertainty.

Proposition A1 (Value Drivers of TS)

Let $H(x) = \frac{1}{\beta} (\bar{w}/x - 1)$. Define three regions of service environment as

$$
\Omega_1 = \{d_i: d_i \leq H(\bar{k})\}, \quad \Omega_2 = \{d_i: H(\bar{k}) < d_i \leq H(\alpha\bar{k})\}, \quad \Omega_3 = \{d_i: d_i > H(\alpha\bar{k})\}
$$

(a) If $d_i \in \Omega_1$, then the outsourcing benefit is driven entirely by cost savings, which is linearly increasing in $d_i$. (b) If $d_i \in \Omega_2$, then the outsourcing benefit is driven by both higher service levels and cost savings. In this region, the outsourcing benefit is decreasing in $d_i$ if $\alpha > \pi e^{-1}/\bar{k}$. (c) If $d_i \in \Omega_3$, then the outsourcing benefit is driven purely by improved service levels. In this region, the outsourcing benefit is unimodal in $d_i$.

Part (a) captures settings where the environment is favorable. In this region, outsourcing benefits the brand through cost savings (besides product value gain $(v^o - v)$), and the benefit is increasing even as the environment becomes less favorable. However, as the environment becomes more challenging, part (b) suggests that the benefit is driven by higher service levels and lower costs. While this appears to increase the value of outsourcing, it turns out that the benefit can be decreasing in this region. Part of the intuition is that the higher service level may not offset the increased cost (due to a more challenging environment), resulting in a lower outsourcing benefit. As the environment becomes even more challenging, part (c) suggests that the benefit is unimodal and eventually approaches zero.

With a budget constraint, the brand can achieve its target service level (i.e., ensuring $\bar{k}$ amount of resources) in case (a) only if the product service is carried out in-house, but can achieve in both cases (a) and (b) if outsourced. However, when the environment is highly challenging (i.e., case (c)), the target service level cannot be attained even with outsourcing. Thus, budget constraints
hurt the benefit of outsourcing under moderately to highly challenging environments. As a result, the expected cumulative outsourcing benefit is lower than that without a budget constraint (or with a larger budget). The following corollary formalizes the above observation.

**Corollary A1 (Impact of Fixed Budget)**

Suppose that a minimum service level \( \bar{\theta} \) (equivalently \( \bar{k} \)) is desired with a fixed budget \( \bar{w} \). The benefit of outsourcing declines in the available budget. Consequently, the brand needs a longer planning horizon to benefit from outsourcing.

While budget constraint lowers the potential benefit of outsourcing, it does not eliminate such potential benefit. If the product value gain from outsourcing is significant enough such that \( v^o - v \approx \tau \), then the brand is always better off switching to outsourcing regardless of budget limitations.

**Proof of Proposition A1.**

**Part (a).** In the first region when the environment uncertainty is low, then the benefit comes from cost savings in maintaining resource level \( k \), which is given by \( B_1(d_i) = (1 - \alpha)C\bar{k}(1 + \beta d_i) \).

In this region, the benefit is linearly increasing in the service environment.

**Part (b).** In the second region when the environment uncertainty is medium, the benefit comes from a combination of higher service level and lower cost. In this region, the in-house operation maintains a lower service level with resource \( b_k \) at a cost of \( \bar{w} \), whereas the third-party provider still maintains \( \bar{k} \) at a cost lower than \( \bar{w} \). The benefit is therefore given by (ignoring the product value gain \( v^o - v \)) \( B_2(d_i) = \pi(\theta(\bar{k}) - \theta(\hat{k})) + \bar{w} - \alpha c(\bar{k}, d_i) \), where \( \hat{k} = \bar{w}/(C(1 + \beta d_i)) \). For notational ease, we at times use superscript ‘ and ” to denote first order and second derivatives. It follows that

\[
B_2'(d_i) = -\pi\theta'(\hat{k}) \frac{\partial \hat{k}}{\partial d_i} - \alpha \frac{\partial c(\bar{k}, d_i)}{\partial d_i}, \quad B_2''(d_i) = -\pi\theta''(\hat{k}) \left( \frac{\partial \hat{k}}{\partial d_i} \right)^2 - \pi\theta'(\hat{k}) \frac{\partial^2 \hat{k}}{\partial d_i^2}.
\]

Note that \( \theta'(\hat{k}) = \lambda e^{-\lambda \hat{k}} \), \( \theta''(\hat{k}) = -\lambda^2 e^{-\lambda \hat{k}} \), \( \frac{\partial \hat{k}}{\partial d_i} = \frac{-\pi \beta}{c(1 + \beta d_i)^2} \), and \( \frac{\partial^2 \hat{k}}{\partial d_i^2} = \frac{2 \pi \beta^2}{c(1 + \beta d_i)^3} \). Substituting these terms into \( B_2''(d_i) \), we have

\[
B_2''(d_i) = \pi \lambda \hat{k} e^{-\lambda \hat{k}} \left( \frac{\beta}{1 + \beta d_i} \right)^2 \left\{ \frac{\lambda \bar{w}}{C(1 + \beta d_i)} - 2 \right\}
\]

It follows that when \( \bar{w} \) is large (i.e., \( \bar{w} \geq \frac{2}{\lambda} C(1 + \beta d_i) \)), then the benefit is convex in \( d_i \). Conversely, when \( \bar{w} \) is small, then the benefit is more likely to be concave in \( d_i \). Notice that as the environment becomes more challenging (\( d_i \) increases), then the benefit is more likely to be concave. Furthermore, note that

\[
B_1'(d_i) = \pi \lambda e^{-\lambda \hat{k}} \frac{\bar{w} \beta}{C(1 + \beta d_i)^2} - \alpha C/\beta \bar{k} = \pi \lambda \hat{k} e^{-\lambda \hat{k}} \frac{\beta}{1 + \beta d_i} - \alpha C/\beta \bar{k}.
\]
Note that $\lambda \hat{k} e^{-\lambda \hat{k}} \leq e^{-1}$, and hence we have $B'_2(d_i) < \pi e^{-1} \frac{\beta}{1 + \beta d_i} - \alpha C \beta \overline{K}$. It follows that if $\alpha > \pi e^{-1} / (C \beta \overline{K})$, then the benefit of outsourcing is strictly decreasing in $d_i$ in this region. That is, unless the third-party provider is extremely efficient (very small $\alpha$), the benefit of outsourcing in this region is likely to decrease as the service environment becomes more challenging.

**Part (c).** Finally, in the third region when the environment is highly challenging (i.e., high $d_i$), the benefit is driven by differences in service level provided as both in-house operation and third-party provider incur the same cost $\overline{w}$. The benefit is therefore given by $B_3(d_i) = \pi \left( \theta(\hat{k}^o) - \theta(\tilde{k}) \right)$, where $\hat{k}$ and $\tilde{k}^o$ are bounded by the budget $\overline{w}$. Note that $\hat{k} = \alpha \tilde{k}^o$, hence

$$B'_3(d_i) = \pi \left( -\alpha \lambda e^{-\alpha \lambda \tilde{k}^o} + \lambda e^{-\lambda \tilde{k}^o} \right) \frac{\partial \tilde{k}^o}{\partial d_i},$$

$$B''_3(d_i) = \pi \left( \alpha^2 \lambda^2 e^{-\alpha \lambda \tilde{k}^o} - \lambda^2 e^{-\lambda \tilde{k}^o} \right) \left( \frac{\partial \tilde{k}^o}{\partial d_i} \right)^2 + \pi \left( -\alpha \lambda e^{-\alpha \lambda \tilde{k}^o} + \lambda e^{-\lambda \tilde{k}^o} \right) \frac{\partial^2 \tilde{k}^o}{\partial d_i^2}.$$ 

When $B'_3(d_i) = 0$, we have $\alpha \lambda e^{-\alpha \lambda \tilde{k}^o} = \lambda e^{-\lambda \tilde{k}^o}$. Substituting this expression into $B''_3(d_i)$, we have

$$B''_3(d_i) = \pi \alpha \lambda^2 e^{-\alpha \lambda \tilde{k}^o} (\alpha - 1) \left( \frac{\partial \tilde{k}^o}{\partial d_i} \right)^2 < 0.$$ 

It follows that the benefit in the third region is unimodal in the service environment $d_i$. □

**Proof of Corollary A1.** Observe that the maximum gain (besides those in regions 1 and 2) is given by $(1 - \alpha) \pi e^{-\lambda \hat{k}^o}$, where $\hat{k}^o = \frac{\overline{w}}{\alpha \lambda (1 + \beta d_i)}$. It follows that the maximum benefit is bounded by $(1 - \alpha) \pi$. In contrast, with no budget limit, the benefit is given by $(1 - \alpha) C \overline{K}(1 + \beta d_i)$, which grows in $d_i$ without bound. It follows that if $d_i > \frac{1}{\beta} \left( \frac{\overline{w}}{C \overline{K}} - 1 \right)$, then the benefit of no budget constraint exceeds that with budget constraint. By definition, in $\Omega_3$ we have $d_i > \frac{1}{\beta} \left( \frac{\overline{w}}{\alpha C \overline{K}} - 1 \right)$. Thus, the condition $d_i > \frac{1}{\beta} \left( \frac{\overline{w}}{C \overline{K}} - 1 \right)$ is always satisfied in region 3. As such, all else being equal, the budget constraint reduces the value of outsourcing. It then follows that the brand needs a longer planning horizon $T$ for the accrued benefit of outsourcing (if attractive) to outweigh the fixed switching cost $\tau$. □

A2. Proofs

Note that for notational ease, we at times use superscript ‘$\prime$’ and ‘$\prime\prime$’ to denote first order and second derivatives when there is no ambiguity. Otherwise, we use $\partial$ to denote derivatives.

**Proof of Lemma 1.**

**Part (a).** Note that $G'_k(k|v, d) = \pi \theta'(k) - c'_k(k, d)$ and $G''_k(k|v, d) = \pi \theta''(k) - c''_k(k, d)$. Because $\theta''(k) \leq 0$ (due to concavity of $\theta(\cdot)$ function) and $c''_k(k, d) = 0$ (due to the linearity of $c(k, d)$ function), it follows that $G''_k(k|v, d) \leq 0$.

**Part (b).** Note that $G'_{k, d}(k|v, d) \geq 0$ (due to supermodular property of $c(k, d)$), it follows that $G'_k(k|v, d)$ increases in $d$. Substituting this into $\pi \theta'(k) = c'_k(k, d_i)$, we have the RHS increasing in
$d_i$. To maintain the equality, it is necessary to reduce $k$. Note that the above first-order condition does not depend on future periods’ value function because resource maintenance cost is a variable cost incurred in every period. □

**Proof of Lemma 2.** The proof follows analogously to the proof of Lemma 1. Specifically, notice that $G_k^i(k|v,d) = \pi\theta'(k) - c_k^G(k,d)$ and $G_k^{G}(k|v,d) = \pi\theta''(k) - c_k^{G''}(k,d)$. Because $\theta''(k) \leq 0$ (due to concavity of $\theta(\cdot)$ function) and $c_k^{G''}(k,d) \geq 0$ (due to convexity of $c^G(k,d)$ function), it follows that $G_k^{G}(k|v,d) \leq 0$. In addition, observe that $G_{k,d}(k|v,d) \geq 0$ (due to supermodular property of $c^G(k,d)$), it follows that $G_k^i(k|v,d)$ increases in $d$. Substituting this into $\pi\theta'(k) = c_k^G(k,d_{i*})$, we have the RHS increasing in $d_{i*}$. To maintain the equality, it is necessary to reduce $k$. The lemma statement then follows. □

**Proof of Proposition 1.**

**Part (a).** Note that $s_i = 0$ is equivalent to the brand having full observability into the true market environment, and therefore the brand observes $d_i$ even with the third-party service. Given that full observability is a relaxation of the original problem with partial observability, it follows that the value function under $s_i = 0$ must dominate that under $s_i > 0$. Therefore, the value function under $s_i = 0$ must be an upper bound on any achievable performance under $s_i > 0$.

**Part (b).** By Lemma 1, with direct service, the optimal resource level in an arbitrary period $i$, $k_i^*$, satisfies $\pi\theta'(k_i^*) = c^G_k(k_i^*,d_i)$ (denote as E1). Similarly, by Lemma 2, with third-party service, the optimal resource level in period $i$ satisfies $\pi\theta'(k_i^*) = c^{G'}_k(k_i^*,d_{i*})$ (denote as E2). Since $s_i = 0$, we have $d_{i*} = d_i$. Recall that all else equal $c^G_k(k,d) \leq c^G_k(k,d)$ for any arbitrary $k$ and $d$ (see §3.6.2). Additionally, because the left-hand side of E1 and E2 are identical (for the same resource level), it follows that $k_i^* \geq k_i^*$. In addition, recall that under direct service the brand’s value function is $V_i^{DS}(d_i) = \max_k \{G(k_i,d_i) + \gamma E_{v,i+1} V_i^{DS}(d_{i+1})\}, \; i = 1,\ldots,T$, whereas under the third-party service (with $s_i = 0$), the brand’s value function is simplified to $V_i^{TS}(d_i) = \max_k \{G^o(k_i,d_i) + \gamma E_{v,i+1} V_i^{TS}(d_{i+1})\}, \; i = 1,\ldots,T$. Now, suppose the brand endorses a resource level in each period that is optimal under the direct service and imposes such series of resource levels for the third-party service, then we must have $G(k_i^*,d_i) \leq G^o(k_i^*,d_i)$ because $c^G(k,d) \leq c(k,d)$ for any arbitrary $k$ and $d$. If follows that $V_i^{DS}(d_i) \leq V_i^{TS}(d_i)|k_i^* = k_i^*$, that is, when the resources levels are set identically based on the direct service setting. However, the brand could optimally choose $k_i^*$ based on the third-party service setting and this can only improve its value function; that is, $V_i^{DS}(d_i)|k_i^* = k_i^* \leq V_i^{TS}(d_i)|k_i^*$ when $k_i^*$ is optimally chosen. The proposition statement then follows. □

**Proof of Proposition 2.**
Part (a). When the service environment is stable, then the expected value of switching to the third-party service provider is the sum of discounted benefits over the \( T \) time periods. Leveraging Lemmas 1 and 2, we have

\[
G(k^*_i) = v - \frac{C(1 + \beta d_i)}{\lambda} \left( 1 - \ln \left( \frac{C(1 + \beta d_i)}{\lambda \pi} \right) \right),
\]

\[
G(k^o_i) = v^o - \frac{\alpha C(1 + \beta d_i)}{\lambda} \left( 1 - \ln \left( \frac{\alpha C(1 + \beta d_i)}{\lambda \pi} \right) \right).
\]

The benefit of switching to a third-party service provider is therefore given by

\[
G(k^o_i) - G(k^*_i) = v^o - v + \frac{C(1 + \beta d_i)}{\lambda} \left( 1 - \ln \left( \frac{C(1 + \beta d_i)}{\lambda \pi} \right) \right) - \frac{\alpha C(1 + \beta d_i)}{\lambda} \left( 1 - \ln \left( \frac{\alpha C(1 + \beta d_i)}{\lambda \pi} \right) \right).
\]

Using the definition of \( L(x) = x(1 - \ln(x)) \) and \( A_i = C(1 + \beta d_i)/(\lambda \pi) \), the above expression can be simplified to \( G(k^o_i) - G(k^*_i) = v^o - v + \pi(L(A_i) - L(\alpha A_i)) > 0 \), where the inequality follows from the fact that \( v^o > v \) and \( L(x) \) is an increasing function of \( x \) for any \( x \leq 1 \). Given \( d_i = d \) for all \( i \), it follows that \( G(k^o_i) - G(k^*_i) \) is a constant in each period. The sum of discounted values of \( G(k^o_i) - G(k^*_i) \) over \( T \) periods is therefore given by \( B(T) = \frac{1 - \gamma^{T-1}}{1 - \gamma} (G(k^o_i) - G(k^*_i)) \). The brand, therefore, benefits from outsourcing product service as long as the \( B(T) \geq \tau \). Therefore, the boundary planning horizon \( T \) is obtained by considering the indifference point \( B(T) = \tau \). The exact expression follows from rearranging the terms in \( B(T) - \tau = 0 \) and substituting \( G(k^o_i) \) and \( G(k^*_i) \) into \( B(T) \).

Part (b)(i). Observe that \( T \) satisfies \( B(T) - \tau = 0 \), which is equivalent to

\[
[v^o - v + \pi(L(A_i) - L(\alpha A_i))] \frac{1 - \gamma^{T-1}}{1 - \gamma} - \tau = 0.
\]

Treating the terms in the square bracket as a function of \( d_i \), the above expression can be re-written as \( J(d_i) \frac{1 - \gamma^{T-1}}{1 - \gamma} - \tau = 0 \). Notice that if \( J(d_i) \) increases, then \( T \) must decrease to maintain the equality, and vice versa. In other words, the directional change in \( T \) is the opposite of the directional change in \( J(d_i) \). Now, observe that \( \partial J(d_i)/\partial d_i \sim \alpha \ln(\alpha d_i) - \ln(d_i) \). Furthermore, \( \partial^2 J(d_i)/\partial d_i^2 \sim (\alpha - 1)/d_i < 0 \). Therefore, \( J(d_i) \) is concave in \( d_i \), suggesting that \( J(d_i) \) initially increases and eventually decreases in \( d_i \). As such, this means that \( T \) must be convex: initially decrease and then increase in \( d_i \).

Part (b)(ii). First consider the case when the brand has full observability of the service environment. Because \( L(x) \) is concave in \( x \), by Jensen’s inequality, we have \( E_x L(x) \leq L(E_x x) \). It follows that, starting from period \( i \), the expected benefit in any subsequent period \( i < j \leq T \) must be such that \( E_{d_j} L(A_j) \leq L(E_{d_j} A_j) \). Note that \( L(x) - L(\alpha x) \) is also concave in \( x \) because \( L''(x) - L''(\alpha x) = -(1 - \alpha)/x < 0 \). Therefore, we have \( E_{d_j} (G(k^o_j) - G(k^*_j)) \leq G(E_{d_j} k^o_j) - G(E_{d_j} k^*_j) = G(k^o_i) - G(k^*_i) = v^o - v + \pi(L(A_i) - L(\alpha A_i)) \), where the first equality follows from the fact that \( E[d_j d_i] = d_i \) by the property of MMFE. It then follows that under uncertain \( d_i \), the sum of discounted expected
benefit is strictly less than that attained under constant service environment \( d_i \). This implies that \( T \) must be increased to compensate for the reduced benefit of service outsourcing; that is, \( T \) must be increased to render outsourcing attractive. Now, by Proposition 1(a), the brand’s value function is upper bounded by the full observability case, and therefore for any \( s_i > 0 \), we have \( V_i^{TS}(d_i, s_i) \leq V_i^{TS}(d_i, 0) \) such that the benefit of outsourcing is strictly lower than that under the full observability case. Therefore, \( T \) must be further increased beyond the full observability case to offset the switching cost \( \tau \). □

**Proof of Proposition 3.** With a minimum service requirement, the benefit of outsourcing in the current period is given by \( B_i = R^o(\bar{k}) - c^s(\bar{k}, d_i) - (R(\bar{k}) - c(\bar{k}, d_i)) = v^o - v + (1 - \alpha)C\bar{k}(1 + \beta d_i) \). At the beginning of a planning horizon of \( T \), the expected sum of discounted benefit is given by \( B = \sum_{t=1}^{T} \gamma^{t-1} E[B_t] = \sum_{i=1}^{T} \gamma^{t-1} E[v^o - v + (1 - \alpha)C\bar{k}(1 + \beta d_i)] = \sum_{t=1}^{T} \gamma^{t-1} (v^o - v + (1 - \alpha)C\bar{k}(1 + \beta E[d_i])) = \sum_{t=1}^{T} \gamma^{t-1} (v^o - v + (1 - \alpha)C\bar{k}(1 + \beta d_i)), \) where the last equality follows from the martingale property. It follows that \( B = (v^o - v + (1 - \alpha)C\bar{k}(1 + \beta d_i)) \frac{1 - \gamma^{T-1}}{1 - \gamma}. \) The brand benefits from outsourcing if \( B \geq \tau \). The proposition statement follows by re-arranging the above inequality. □

**Proof of Lemma 3.** First note that without learning and monitoring effort, the brand’s belief is simply \( d_i^0 \) (as opposed to the partially learning updated belief \( d_i^{*m} \)). As such, we only need to consider the brand’s belief \( d_i^0 \). Part (a). By definition, we have \( \xi_i^b = \Pi_{k=0}^{s_i} e_k / \epsilon_k = \Pi_{k=m}^{s_i} \Pi_{m \in |I_i \setminus I_i^b|} \xi_{k,m} \), where the notation \( m \in |I_i \setminus I_i^b| \) refers to those random events that the brand does not observe due to outsourcing. Because \( \xi_{k,m} \)’s are independent across \( k \) and \( m \), and lognormally distributed with mean equal to 1, it follows that \( \xi_i^b \), as a multiplication of lognormal random variables, is also lognormally distributed with mean 1 and variance parameter \( s_i^2 \). The variance parameter captures the cumulative uncertainty captured by these random events. Therefore, we have \( E[d_i] = E[d_i^0 \xi_i^b] = E[d_i^0] E[\xi_i^b] = E[d_i^0] \), where the second equality follows from the fact that \( d_i^0 \) and \( \xi_i^b \) are independent. Part (b). Observe that \( P(d_i^0 - d_i \geq X|s_i) = P(d_i^0 - \xi_i^b d_i^0 \geq X|s_i) = P(d_i^0 (1 - \xi_i^b) \geq X|s_i) = P(1 - \xi_i^b \geq X/d_i^0|s_i) = P(\xi_i^b \leq 1 - X/d_i^0|s_i) \). The lemma statement follows from the fact that \( \xi_i^b \) is lognormally distributed with mean equal to 1 and arbitrary variance parameter \( s_i^2 \). Finally, let \( \Gamma(s) = (A + s_i^2/2)/(\sqrt{2}s_i) \), we have \( \Gamma'(s_i) = \frac{1}{\sqrt{2}}(1 - A/s_i^2) \). Substituting \( A = \ln(1 - X/d_i^0) \leq 0 \) into \( \Gamma'(s_i) \), we have \( \Gamma'(s_i) \geq 0 \). □

**Proof of Proposition 4.**

Part (a). Consider any period \( i > 0 \). Suppose that partial observability has resulted in the brand holding a belief \( d_i^0 \) with the variance parameter of the information gap as \( s_i^2 \). The brand sets a target resource level \( \bar{k} \) with payment \( w_i = c^s(\bar{k}, d_i^0) \) to satisfy the provider’s participation constraint. If \( d_i^0 \geq d_i \), then the provider’s participation constraint is satisfied (but not binding). In this case, the value of outsourcing is reduced by \( c^s(\bar{k}, d_i^0) - c^s(\bar{k}, d_i) \geq 0 \), where the inequality follows from
the fact that $d_i^b \geq d_i$. On the other hand, if $d_i^b < d_i$, then the provider’s participation constraint is violated (because $c^S(\bar{k}, d_i) > c^S(\bar{k}, d_i^b)$). This forces the brand to adjust its payment to $c^S(\bar{k}, d_i)$ based on actual environment $d_i$. The gain from outsourcing is therefore exactly the same as if the brand observes the service environment $d_i$. In the above payment structure, the brand acts on the point estimate of $d_i^b$, although the brand is aware that $d_i^b$ is not perfect. Nevertheless, the payment structure is valid even if the brand incorporates the uncertainties surrounding $d_i^b$. To see this, recall that the functional form of $c^S(\bar{k}, d_i^b)$ is given by $c^S(\bar{k}, d_i^b) = \alpha C \bar{k}(1 + \beta d_i^b)$. Suppose that the brand recognizes that the true environment is random which we denote as $\tilde{d}_i = d_i^b \xi_i^b$ with some estimated distribution function of $\xi_i^b$ as $F_i(\xi_i^b)$. Then for any given $\bar{k}$ the expected service provision cost is given by $E[c^S(\bar{k}, \tilde{d}_i)] = \int_{\xi_i^b} c^S(\bar{k}, \tilde{d}_i) dF_i(\xi_i^b) = \int_{\xi_i^b} \alpha C \bar{k}(1 + \beta \tilde{d}_i) dF_i(\xi_i^b) = \alpha C \bar{k}(1 + \beta d_i^b) E[\xi_i^b] = \alpha C \bar{k}(1 + \beta d_i^b) = c^S(\bar{k}, d_i^b)$, where the second last equality follows from the MMFE property (as $d_i^b$ is an unbiased estimate of $\tilde{d}_i$). Such a contract is similar to the specified capacity and quality (SCQ) contract studied in Feng et al. (2019). Note that the diminishing marginal effect is captured in the customer’s satisfaction function $\theta(\bar{k})$ so the functional form of $c^S(\bar{k}, d_i^b)$ still allows the non-linear effect of resource level and customer satisfaction.

Part (b). The proposition statement follows from part (c) of Lemma 3.

Part (c). Note that the expected value loss from the belief drifting above the actual environment in an arbitrary period after outsourcing is given by

$$E\left[c^S(\bar{k}, d_i^b) - c^S(\bar{k}, d_i)|d_i \leq d_i^b\right] = \int_{\xi_i^b \leq 1} (c^S(\bar{k}, d_i^b) - c^S(\bar{k}, d_i^b \xi_i^b)) dF_i(\xi_i^b),$$

where $\xi_i^b$ is the cumulative loss of visibility by period $i$ and $F_i(\cdot)$ denote the lognormal distribution of $\xi_i^b$ with mean 1 and variance $s_i^2$. Substituting the lognormal distribution into the above expression and recognizing that $c^S(k, d) = \alpha C k(1 + \beta d)$, we obtain

$$E\left[c^S(\bar{k}, d_i^b) - c^S(\bar{k}, d_i)|d_i \leq d_i^b\right] = \alpha C \beta \bar{k} d_i^b \frac{1}{F_i(1)} \int_{\xi_i^b \leq 1} (1 - \xi_i^b)) dF_i(\xi_i^b) = \alpha C \beta \bar{k} d_i^b \left(1 - \frac{1 - \text{erf} \left(\frac{s_i}{\sqrt{2}}\right)}{1 - \text{erf} \left(-\frac{s_i}{\sqrt{2}}\right)}\right).$$

As $s_i$ increases (visibility worsens), $\lim_{s_i \to \infty} E\left[c^S(\bar{k}, d_i^b) - c^S(\bar{k}, d_i)|d_i \leq d_i^b\right] \to \alpha C \beta \bar{k} d_i^b$. Therefore, there exists a threshold level of visibility $\bar{s}_i$ such that beyond which the loss in value due to partial observability satisfies $E\left[c^S(\bar{k}, d_i^b) - c^S(\bar{k}, d_i)|d_i \leq d_i^b\right] \geq \Delta_i$, where $\Delta_i$ represents the expected value gain necessary in period $i$ as implied by $V_0^{DS}(d_0) + \tau \leq V_0^{TS}(d_0, s_0)$ at the start of the planning horizon. Furthermore, note that the partial observability variance is cumulative over the planning horizon, so that $s_i^2$ is never decreasing. Therefore, if $s_i > \bar{s}_i$ at some period $i$, then the condition holds for all of the remaining periods. It then follows that there exists a $\bar{s}_i$ such that if $s_i > \bar{s}_i$ occurs within the planning horizon, then the value of outsourcing is strictly negative. \(\square\)
Proof of Proposition 5.

Part (a). We first show that the optimal PBE reporting strategy must be of a threshold type. Observe that the service provider’s profit in period $i$ is given by $u = w_i - c^S(\bar{k}, d_i) - c_R$, where $w_i$ satisfies $w_i = c^S(\bar{k}, d_i^*)$. All else being equal, the provider’s payoff $w_i$ is therefore increasing in the brand’s belief $d_i^*$. It follows that any incentive compatible reporting strategy $R(d_i)$ must satisfy the conditions that (a) $R(d_i) = d_i$ if $c^S(\bar{k}, d_i) \geq E_{\xi_i}(c^S(\bar{k}, d_i^*)|\emptyset) + c_R \Rightarrow d_i \geq d_i^* E(\xi_i|\emptyset) + c_R/\alpha C\beta \bar{k}$ and (b) $R(d_i) = \emptyset$ if $c^S(\bar{k}, d_i) < E_{\xi_i}(c^S(\bar{k}, d_i^*)|\emptyset) + c_R \Rightarrow d_i < d_i^* E(\xi_i|\emptyset) + c_R/\alpha C\beta \bar{k}$. The two conditions (a) and (b) imply that any PBE optimal strategy must be of a threshold type and hence (11) must hold.

It remains to be shown that the optimal threshold satisfies condition (12). Notice that if the provider discloses true environment information $d_i$, then its expected payoff is $w_i - c_R = E[c^S(\bar{k}, d_i^*)|d_i] - c_R = c^S(\bar{k}, d_i) - c_R$. Because any optimal disclosure strategy is of a threshold type, there must exist a threshold $d_i^R$ such that the provider does not disclose the true environment when $d_i < d_i^R$ but reports the true environment otherwise. For any given $d_i^R$, if the provider does not disclose, then the brand’s (adjusted) belief about the service environment is given by

$$E[d_i^b\xi_i|\emptyset] = E[d_i^b\xi_i|d_i^b < d_i^R] = d_i^R E[\xi_i|\xi_i < d_i^R/d_i^b].$$

(A-1)

At the boundary point of $d_i^R$, the provider is indifferent between disclosure and non-disclosure. Thus, we have

$$c^S(\bar{k}, d_i^R) - c_R = E[c^S(\bar{k}, d_i^b\xi_i)|d_i^b < d_i^R] = c^S(\bar{k}, d_i^b E[\xi_i|\xi_i < d_i^R/d_i^b]).$$

(A-2)

Recall that $c^S(k, d) = \alpha C \cdot k (1 + \beta d)$, therefore (A-2) can be expressed as

$$\alpha C \cdot \bar{k} (1 + \beta d_i^R) - c_R = \alpha C \cdot \bar{k} (1 + \beta d_i^b E[\xi_i|\xi_i < d_i^R/d_i^b])$$

$$\Leftrightarrow 1 + \beta d_i^R - \frac{c_R}{\alpha C \cdot \bar{k}} = 1 + \beta d_i^b E[\xi_i|\xi_i < d_i^R/d_i^b]$$

$$\Leftrightarrow \beta d_i^R = \beta d_i^b E[\xi_i|\xi_i < d_i^R/d_i^b] + \frac{c_R}{\alpha C \cdot \bar{k}}$$

$$\Leftrightarrow d_i^R = d_i^b E[\xi_i|\xi_i < d_i^R/d_i^b] + \frac{c_R}{\alpha C \cdot \bar{k}}.$$  

(A-3)

Let $F_i(\cdot)$ denote the distribution of $\xi_i$. We have

$$E[\xi_i|\xi_i \leq \tau] = F_i(\tau)^{-1} \int_{x \leq \tau} xdF_i(x).$$

(A-4)

Letting $\tau = d_i^R/d_i^b$ and substituting (A-4) into (A-3) yield the expression (12).

To show the uniqueness of $d_i^R$, it is sufficient to show that $c^S(\bar{k}, d_i^R) - c^S(\bar{k}, d_i^b E[\xi_i|\xi_i < d_i^R/d_i^b])$ is monotone in $d_i^R$. Note that $c^S(\bar{k}, d_i^R)$ is linearly increasing in $d_i^R$. In addition, we can evaluate
the directional impact of \( d^R_i \) by examining how \( d^R_i \) influences \( \mathbb{E}[\xi_i^b | \xi_i^b < d^R / d_i^b] \). For notational ease define \( G(k) = \mathbb{E}[\xi_i^b | \xi_i^b < k] \). It follows that \( \lim_{k \to \infty} G(k) = 1 \) and \( \lim_{k \to 0} G(k) = 0 \). Furthermore, it is clear that \( G(k) \) monotonically increasing in \( k \). By (A-2), since \( c^S(\bar{k}, \infty) - c^R > c^S(\bar{k}, d_i^b \mathbb{E}[\xi_i^b | \xi_i^b < (\infty / d_i^b)]) = c^S(\bar{k}, d_i^b) \) and \( c^S(\bar{k}, 0) - c^R < c^S(\bar{k}, d_i^b \mathbb{E}[\xi_i^b | \xi_i^b < (0 / d_i^b)]) = c^S(\bar{k}, 0) \), it follows that there must exist a unique \( d^*_i \) that satisfies (A-2).

Part (b). The proposition statement follows from the fact that (i) if the provider reports the true environment then the brand’s belief is updated to the true environment (given the assumption that the provider does not distort information) and (ii) the threshold reporting level \( d^R_i \) is exactly the indifference point where the provider chooses non-disclosure. Recall that by (A-1), the brand’s updated belief \( d^*_i \) is given by \( d^*_i = \mathbb{E}[d_i^b \xi_i^b | \emptyset] = d^*_i \mathbb{E}[\xi_i^b | \xi_i^b < d^R / d_i^b] \). By (A-3), we have

\[
d^R_i = d_i^b \mathbb{E}[\xi_i^b | \xi_i^b < d^R / d_i^b] + \frac{c^R}{\alpha C \beta k} \Leftrightarrow d^R_i = \mathbb{E}[d_i^b \xi_i^b | \emptyset] + \frac{c^R}{\alpha C \beta k} \Leftrightarrow \mathbb{E}[d_i^b \xi_i^b | \emptyset] = d^R_i - \frac{c^R}{\alpha C \beta k}.
\]

It follows that if the provider does not report, then the brand’s updated belief is given by \( d^*_i = \mathbb{E}[d_i^b \xi_i^b | \emptyset] = d^R_i - \frac{c^R}{\alpha C \beta k} \); that is, the brand’s belief about the service environment under non-disclosure is exactly the equilibrium threshold reporting point with a downward adjustment of the scaled reporting cost \( c^R / (\alpha C \beta k) \).

**Proof of Corollary 1.** Define

\[
\mathcal{L}(d^R_i) = d^R_i - d_i^b F_i^{-1}(d^R_i / d_i^b) \int_{\xi \leq d^R_i / d_i^b} \xi dF_i(\xi) - \frac{c^R}{\alpha C \beta k}.
\]

Part (a). We have \( \partial d^R_i / \partial c^R = -\partial \mathcal{L} / \partial c^R ) / (\partial \mathcal{L} / \partial d^R_i) \). By the proof of Proposition 5, we have \( \partial \mathcal{L} / \partial d^R_i \geq 0 \). Furthermore, note that \( \partial \mathcal{L} / \partial c^R = -1 / (\alpha C \beta k) < 0 \). It then follows that \( \partial d^R_i / \partial c^R \geq 0 \).

Part (b). We have \( \partial d^R_i / \partial s_i = -\partial \mathcal{L} / \partial s_i ) / (\partial \mathcal{L} / \partial d^R_i) \). Analogous to part (a), we have \( \partial \mathcal{L} / \partial d^R_i \geq 0 \). Furthermore, under the lognormal approximation we have (recognizing that \( \mathbb{E}[\xi_i^b] = 1 \))

\[
\mathbb{E}[\xi_i^b | \xi_i^b \leq \tau] = \Phi \left( \frac{\ln(\tau) - s_i^2 / 2}{s_i} \right) / \Phi \left( \frac{\ln(\tau) + s_i^2 / 2}{s_i} \right),
\]

where \( \tau = d^R_i / d_i^b \) and \( \Phi(\cdot) \) denote the standard normal distribution. Notice that \( \mathcal{L}(d^R_i) \) can be equivalently written as \( \mathcal{L}(d^R_i) = d^R_i - d_i^b \mathbb{E}[\xi_i^b | \xi_i^b \leq \tau] - \frac{c^R}{\alpha C \beta k} \). We have

\[
\frac{\partial \mathcal{L}}{\partial s_i} = -d_i^b \partial s_i \left( \frac{\Phi(\ln(d^R_i / d_i^b) / s_i - s_i / 2)}{\Phi(\ln(d^R_i / d_i^b) / s_i + s_i / 2)} \right).
\]

Observe that

\[
\lim_{s_i \to \infty} \frac{\Phi(\ln(d^R_i / d_i^b) / s_i - s_i / 2)}{\Phi(\ln(d^R_i / d_i^b) / s_i + s_i / 2)} = 0 \quad \text{and} \quad \lim_{s_i \to \infty} \frac{\Phi(\ln(d^R_i / d_i^b) / s_i - s_i / 2)}{\Phi(\ln(d^R_i / d_i^b) / s_i + s_i / 2)} = 1.
\]

Furthermore, let \( \Delta = \ln(d^R_i / d_i^b) \), then the following derivatives are equivalent:

\[
\partial s_i \left( \frac{\Phi(\ln(d^R_i / d_i^b) / s_i - s_i / 2)}{\Phi(\ln(d^R_i / d_i^b) / s_i + s_i / 2)} \right) \Leftrightarrow \partial s_i \left( \frac{\frac{\Delta - s_i}{\sqrt{2}}}{\frac{\Delta + s_i}{\sqrt{2}}} e^{-\frac{\Delta^2}{2}} dt \right).
\]
Taking the derivative on the right-hand side of the above expression, we have
\[
\partial s_i \left( \int_{-\infty}^{\Delta/s_i} e^{-ts^2/2} dt \right) = e^{-\Delta s_i^2/2} \left( -\Delta/s_i^2 - \frac{1}{2} e^{-t^2/2} dt \right) \int_{-\infty}^{\Delta/s_i^2} e^{-ts^2/2} dt - e^{-\Delta/s_i^2} \left( -\Delta/s_i^2 + \frac{1}{2} \right) \left( \int_{-\infty}^{\Delta/s_i^2} e^{-ts^2/2} dt \right)^2 .
\]

The sign of the above expression can be determined by evaluating the sign of
\[
D = e^{-\Delta/s_i^2} \left( -\Delta/s_i^2 - \frac{1}{2} \right) \left( \int_{-\infty}^{\Delta/s_i^2} e^{-ts^2/2} dt \right) - e^{-\Delta/s_i^2} \left( -\Delta/s_i^2 + \frac{1}{2} \right) \left( \int_{-\infty}^{\Delta/s_i^2} e^{-ts^2/2} dt \right)^2 .
\]
Notice that (a) \( \int_{-\infty}^{\Delta/s_i^2} e^{-ts^2/2} dt > \left( \int_{-\infty}^{\Delta/s_i^2} e^{-ts^2/2} dt \right)^2 \), (b) \( \frac{\Delta}{s_i^2} + \frac{1}{2} > \frac{\Delta}{s_i^2} - \frac{1}{2} \), and (c) \( e^{-\Delta/s_i^2} > e^{-\Delta/s_i^2} \). These three inequalities together imply that in the above expression \( D < 0 \). Recognizing the \(-d^R_i\) factor in \( \partial \mathcal{L}/\partial s_i \), we can conclude that \( \partial d^R_i / \partial s_i \leq 0 \). \( \square \)

**Proof of Proposition 6.**

**Part (a).** Consider any period where the brand’s partial observability is captured by \( \xi_i^b \). Note that \( \xi_i^b \) is unbiased and \( \mathbb{E}[\xi_i^b] = 1 \); that is, the service environment remains the same in expectation (by the MMFE property). Had the brand maintained product service in-house, its cost would be given by \( c(\bar{k}, d_i) \), where \( d_i \) is the true environment since the brand provides product service directly. In contrast, if the brand outsources the product service to a third-party provider, its payment is given by \( c^S(\bar{k}, d_i^b) \), where \( d_i^b \) is the brand’s belief about the service environment. The expected cost savings due to outsourcing in period \( i \) can be expressed as
\[
\mathbb{E} \left[ c(\bar{k}, d_i) - c^S(\bar{k}, d_i^b) \right] = \mathbb{E} \left[ c(\bar{k}, d_i^b \xi_i^b) - c^S(\bar{k}, d_i^b) \right] = \int_{\xi_i^b < 1} (c(\bar{k}, d_i^b \xi_i^b) - c^S(\bar{k}, d_i^b)) dF_i(\xi_i^b) + \int_{\xi_i^b \geq 1} (c(\bar{k}, d_i^b \xi_i^b) - c^S(\bar{k}, d_i^b)) dF_i(\xi_i^b).
\]
Substituting \( c(k, d) = Ck(1 + \beta d) \) and \( c^S(k, d) = \alpha c(k, d) \) into the above expression, we have
\[
\mathbb{E} \left[ C(\bar{k}, d_i) - c^S(\bar{k}, d_i^b) \right] = C\bar{k} \int_{\xi_i^b < 1} ((1 + \beta d^b\xi_i^b) - (1 + \alpha d^b)) dF_i(\xi_i^b)
+ C\bar{k} \int_{\xi_i^b \geq 1} ((1 + \beta d^b\xi_i^b) - (1 + \alpha d^b)) dF_i(\xi_i^b)
= C\bar{k} \beta d^b \left( 1 - \alpha F_i(1) - \int_{\xi_i^b \geq 1} \xi_i^b dF_i(\xi_i^b) \right) \leq C\bar{k} \beta d^b \left( 1 - \alpha \int_{\xi_i^b \geq 1} \xi_i^b dF_i(\xi_i^b) \right).
\]
The proposition follows by setting \( 1 - \alpha \int_{\xi_i^b \geq 1} \xi_i^b dF_i(\xi_i^b) \leq 0 \).

**Part (b).** By Proposition 5, in each period when the service provider does not report, the brand’s belief is adjusted downwards from \( d_i^b \) to \( d_i^R / (\alpha C\beta k_i^o) \). By the definition of PBE threshold \( d_i^R \), we have \( d_i^b > d_i^R \geq d_i \). Let \( \xi_i^b \) and \( \xi_i^\prime \) denote the cumulative uncertainty that has not been learned by the brand in period \( i \) just before and after the belief updating respectively (assuming the service
provider does not disclose true environment condition in period \( i \). By the martingale property, we have \( \mathbb{E}[d^b_i \xi^b_i] = \mathbb{E}[d^R_i \xi^b_i] = d_i \). It follows that \( \mathbb{E}[\xi^b_i] = d^b_i/d^R_i \mathbb{E}[\xi^b_i] > \mathbb{E}[\xi^b_i] \), implying that a subset of random events in \( \xi^b_i \) is learned by belief updating and that the learned events are biased downwards (i.e., with an expectation less than 1). Therefore, the remaining environment uncertainty in the brand’s belief is reduced by the belief updating in each period (as long as the service provider continues to choose non-disclosure). Since in each period the uncertainty in the brand’s belief is strictly decreasing when \( d^R_i \) approaches \( d_i \), we have \( \lim_{i \to \infty} R_{\xi^b_i} \geq 1 \xi^b_i dF_i(\xi^b_i) \to 0 \). As such, there must exist a finite threshold \( i \) beyond which \( R_{\xi^b_i} \geq 1 \xi^b_i dF_i(\xi^b_i) \leq 1/\alpha \) for any \( i \geq i \), assuming that the service provider continues to choose non-disclosure. □

**Proof of Proposition 7.**

**Part (a).** The statement follows from the assumption that if the provider reports the true environment, the report is truthful. It follows that the brand knows the true environment whenever the provider reports, and thus we must have \( y^*_i = 0 \). If on the other hand the provider does not report, then the brand can infer that \( d^b_i > d_i \), where monitoring creates potential value by calibrating its belief with the true service environment.

**Part (b).** We prove the proposition statement through backward induction. First consider the last period \( T \). If the brand does not exert monitoring effort and the provider does not report the true environment, then its belief is updated downwards from \( d^b_T \) to \( d^*_T = d^b_T - c^R/(\alpha C \beta k) \). Without monitoring, the brand’s payment to the provider is given by \( c^S(k, d^*_T) \). In contrast, with monitoring, the brand’s payment depends on actual service environment \( d_T \) and it incurs a lower payment whenever \( d_T \leq d^*_T \) (and incurs the same payment \( d^*_T \) when \( d^*_T < d_T \leq d^R_T \) since the provider absorbs the reporting cost \( c^R \)). Therefore, the value difference between monitoring and not monitoring can be derived using (3), which can be simplified as

\[
MV_T(y_T = 1) = c^S(\bar{k}, d^*_T) - \int_{d_T \leq d^*_T} c^S(\bar{k}, d_T) \hat{F}_T(d_T) - M
= c^S(\bar{k}, d^*_T) - \int_{\xi^b_T \leq 1} c^S(\bar{k}, d^*_T \xi^b_T) dF_T(\xi^b_T) - M,
\]

where we use \( \hat{F}_T(d_T) \) to denote the distribution of \( d_T \), \( F_T(d_T) \) to denote the distribution of \( \xi^b_T \), where \( \xi^b_T \) to denote the cumulative environment uncertainty not learned by the brand (after adjusting its belief downward due to the service provider’s non-disclosure). Recall that \( c^S(k, d) = \alpha C k (1 + \beta d) \).

Substituting into the above expression, we have

\[
MV_T(y_T = 1) = \alpha C \bar{k} \beta d^*_T \left( 1 - \int_{\xi^b_T \leq 1} \xi^b_T dF_T(\xi^b_T) \right) - M.
\]
Let \( s_T \) denote the variance parameter of \( \xi_T' \) and set \( H(s_T) = 1 - \int_{\xi_T' \leq 1} \xi_T' dF_T(\xi_T') \). It follows that \( y_T = 1 \) if and only if
\[
MV_T(y_T = 1) \geq 0 \iff d_T^* \geq \frac{M}{\alpha C k \beta H(s_T)} : = \tilde{d}_T^b(s_T)
\]
In addition, under the lognormal matching, \( H(s) \) is a monotone increasing function of \( s \) because \( H(s) \) can be simplified to \( H(s) = \frac{1}{2} \left(1 + \text{erf}\left( \frac{s}{\sqrt{2}} \right) \right) \), where \( \text{erf}(\cdot) \) is the standard error function and we have taken the fact that \( \mathbb{E}[\xi_T'] = 1 \) (i.e., the brand’s belief remains unbiased after updating). It follows immediately that \( H'(s) \geq 0 \) since \( \text{erf}(\cdot) \) is a monotone increasing function. Thus, we have \( \partial d_T^b(s_T)/\partial s_T \leq 0 \). This completes the proof for the last period \( T \).

Suppose the threshold policy holds for period \( i + 1 \). Consider the monitoring decision in period \( i \). The value of monitoring is given by
\[
MV_i(y_i = 1) = c^S(\tilde{t}, d_i^b) - \int_{\xi_i' \leq 1} c^S(\tilde{t}, d_i^b \xi_i') dF_i(\xi_i') - M + \gamma \left( \mathbb{E} V_{i+1}(d_i^b | d_i) - \mathbb{E} V_{i+1}(d_i^* | d_i) \right).
\]
(A-6)

Observe that the first three terms in the above expression have a similar structure to that for the last period, and thus \( c^S(\tilde{t}, d_i^b) - \int_{\xi_i' \leq 1} c^S(\tilde{t}, d_i^b \xi_i') dF_i(\xi_i') - M \) must be monotonically increasing in \( d_i^b \). It then remains to be shown that the last term is also monotonically increasing in \( d_i^b \). Note that for any random realizations of \( \xi_{i+1}^b \), we have \( V_{i+1}(d_i^b | d_i) \leq V_{i+1}(d_i^* | d_i) \) for any \( d_i^* \geq \tilde{d}_i^b \) because \( c^S(\tilde{t}, d_i^*) \geq c^S(\tilde{t}, \tilde{d}_i^b) \) (as the expected payment to the provider is higher under belief \( d_i^* \xi_i' \xi_{i+1}^b \) than that under \( \tilde{d}_i^b \xi_i' \xi_{i+1}^b \)). Because in period \( i + 1 \) a threshold policy is optimal, let \( \tilde{d}_{i+1}^b \) denote the optimal threshold. There are three cases to consider.

Case 1. \( d_i^b \xi_i' \xi_{i+1}^b > d_i^b \xi_i^b > \tilde{d}_{i+1}^b \). In this case, the belief is above the threshold in both cases and hence it is optimal to monitor. As such, we have \( E (V_{i+1}(d_i^b | d_i) - V_{i+1}(d_i^* | d_i)) = 0 \); that is, the marginal benefit of monitoring in period \( i \) does not carry over to the next period. In this case, the threshold policy must be optimal since the monitoring decision in period \( i \) is decoupled from future periods.

Case 2. \( d_i^b \xi_i' \xi_{i+1}^b > \tilde{d}_{i+1}^b > d_i^b \xi_i^b \). It is then optimal to monitor in period \( i + 1 \) if monitoring was not conducted in period \( i \) and vice versa. The carry-over marginal benefit of monitoring in period \( i \) is thus the cost savings of not having to monitor in period \( i + 1 \). Because it is optimal to monitor only if the cost of monitoring is less than the benefit obtained, it follows that \( E (V_{i+1}(d_i^b | d_i) - V_{i+1}(d_i^* | d_i)) \leq 0 \). Further, such carry-over benefit is increasing in \( d_i^b \). To see this, note that the second term does not depend on \( d_i^b \) (because \( y_{i+1}^* = 1 \)) while monitoring benefit is directly associated with \( d_i^* \xi_i^b \xi_{i+1}^b \), which is increasing in \( d_i^b \) (i.e., all else being equal, monitoring reduces the expected payment to a larger extent as \( d_i^b \) increases). It is worth pointing out that \( V_{i+1}(d_i^b | d_i) = V_{i+1}(d_i^b \xi_i' \xi_{i+1}^b) \) decreases in \( d_i^b \), but the benefit of monitoring is increasing.
Case 3. $\bar{d}_{i+1} > d_{i+1}^{bs} c_i | \xi_{i+1} > d_{i+1}^{bs}$. In this case, it is not optimal to monitor in period $i + 1$ in both cases. It then follows that $\mathbb{E} V_{i+1}(d_{i+1}^{bs} c_i | \xi_{i+1}) = \mathbb{E} V_{i+1}(d_{i+1}^{bs} | \xi_{i+1})$ monotonically increases in $d_{i+1}^{bs}$ because both $\mathbb{E} V_{i+1}(d_{i+1}^{bs} c_i | \xi_{i+1})$ and $\mathbb{E} V_{i+1}(d_{i+1}^{bs} | \xi_{i+1})$ represent the sum of discounted value from period $i + 1$ onwards, and their difference is captured by the sum of discounted cost differences that take the same structure as that in the current period $\mathbb{E}[c^S(\bar{k}, d_{i+1}^{bs} c_i) - c^S(\bar{k}, d_i)]$, which is monotonically increasing in $d_{i+1}^{bs}$.

Thus, in all three cases, the marginal value of monitoring in the current period is monotonically increasing in $d_{i+1}^{bs}$, implying that a threshold policy is also optimal for period $i$. This completes the proposition statement. □

**Proof of Corollary 2.** By the proof of Proposition 7, the value of monitoring is captured by current cost savings and the marginal benefit of reduced cost for future periods. The threshold for the last period $T$ is given by $d_T^{bs} = M/\alpha C \bar{k} H(s_T)$. The corollary statements then follow by taking derivatives of $d_T^{bs}$ with respect to $\alpha$, $\bar{k}$, and $M$, respectively. Suppose these directional results hold for period $i + 1$, and consider period $i$. By Proposition 7, the optimal monitoring policy is again a threshold type that satisfies (A-6). Define

$$
\mathcal{L}(d_i^{bs}) = c^S(\bar{k}, d_i^{bs}) - \int_{c_i^{bs}} c^S(\bar{k}, d_i^{bs} | \xi_{i+1}) dF_i(\xi_{i+1}) - M + \gamma \left( \mathbb{E} V_{i+1}(d_{i+1}^{bs} | d_i) - \mathbb{E} V_{i+1}(d_{i+1}^{bs} | d_i) \right).
$$

The corollary statements then follow by applying the implicit function theorem on $\mathcal{L}(d_i^{bs})$ and recognizing that $\partial \mathcal{L}(d_i^{bs}) / \partial d_i^{bs} \geq 0$ (from the proof of Proposition 7). Since these directional results also hold for any future periods by the supposition, the corollary statements therefore must hold for any arbitrary period. □

**Proof of Corollary 3.** The corollary statement follows from Proposition 7 by recognizing that the brand’s belief following any monitoring effort must satisfy $d_i \leq d_i^{bs}$, where the inequality follows from the fact that monitoring is optimal only if the provider does not report true environment condition (implying $d_i^{bs} \geq d_i$). All else being equal, $P(d_{i+1}^{bs} | d_i > x) \geq P(d_{i+1}^{bs} | d_i > x)$ for any given $x$ because $d_{i+1}^{bs} = d_i c_{i+1}^{bs}$ and $d_{i+1}^{bs} = d_i c_{i+1}^{bs} \xi_{i+1} b_{i+1}$. It follows then $P(d_{i+1}^{bs} | d_i > \bar{d}_{i+1}) \geq P(d_{i+1}^{bs} | d_i > \bar{d}_{i+1})$. Thus for any realized belief path, the probability that the true environment condition is more challenging than the brand’s belief is higher after the brand’s monitoring effort, implying that the provider is more likely to report the true environment. Conversely, whenever the provider reports true environment, we have $d_i \geq d_i^{bs}$. Therefore, we have $P(d_{i+1}^{bs} | d_i > \bar{d}_{i+1}) \geq P(d_{i+1}^{bs} | d_i > \bar{d}_{i+1})$, implying that the brand is more likely to conduct monitoring efforts following provider’s reporting effort. □

**Proof of Proposition 8.** The key impact of partial observability is when the brand overpays the service provider based on its own belief of the service environment, $d_i^{bs}$, as opposed to the true
environment condition \(d_i\). Under the passive monitoring strategy, the service provider updates the brand if and only if the true service environment exceeds the threshold \(d_i^R\) (i.e., only when \(d_i \geq d_i^R\)). Therefore, the brand suffers only when \(d_i < d_i^R\). Consider the cost implications under passive monitoring when \(d_i < d_i^R\). The brand will update its belief to \(d_i^{\text{new}} = d_i^R - c_{\alpha C/\beta} / (\alpha C/\beta)\) (per Proposition 5). For any given target resource level \(\bar{k}\), the cost incurred for the brand is given by \(c^S(\bar{k}, d_i^{\text{new}}) = \alpha c(\bar{k}, d_i^{\text{new}})\). If the brand were to provide the service directly, on the other hand, the cost incurred is \(c(\bar{k}, d_i)\). The expected cost over-run with outsourcing is therefore given by

\[
\Delta_c = c^S(\bar{k}, d_i^{\text{new}}) - E[c(\bar{k}, d_i)] = \alpha c(\bar{k}, d_i^{\text{new}}) - c(\bar{k}, d_i^{\text{new}}) E[\xi_i^{\text{new}} | \xi_i^{\text{new}} \leq 1] P(\xi_i^{\text{new}} \leq 1) \\
= \alpha c(\bar{k}, d_i^{\text{new}}) - c(\bar{k}, d_i^{\text{new}}) \Phi \left( -\frac{1}{2} s_i^2 \right) ,
\]

where the second equality follows from the matching approximation where \(\xi_i^{\text{new}}\) is lognormally distributed with \(E[\xi_i^{\text{new}}] = 1\), \(\Phi\) is the standard normal cumulative distribution, and \(s_i^2\) captures the cumulative variance parameter of the environment uncertainty that has not been learned by the brand. Notice that in the worst-case scenario we have \(\lim_{s_i^2 \to \infty} \Phi \left( -\frac{1}{2} s_i^2 \right) \to 0\), we have \(\Delta_c \leq \alpha c(\bar{k}, d_i^{\text{new}}) - c(\bar{k}, d_i^{\text{new}}) \cdot 0 = \alpha c(\bar{k}, d_i^{\text{new}}) - c(\bar{k}, 0)\). Since \(\alpha c(\bar{k}, 0) - c(\bar{k}, 0) < 0\) and \(c(\bar{k}, d_i^{\text{new}})\) is monotonically increasing in \(d_i^{\text{new}}\), there must exist a unique \(\overline{d}_i^R\) such that \(\alpha c(\bar{k}, \overline{d}_i^R) - c(\bar{k}, 0) = 0\). It follows that for all \(d_i^{\text{new}} \leq \overline{d}_i^R\), we have \(\alpha c(\bar{k}, d_i^{\text{new}}) - c(\bar{k}, 0) \leq 0\), implying that the brand is better off with outsourcing than direct service provision. Given the fact that \(d_i^{\text{new}} = d_i^R - c_{\alpha C/\beta} \), there is a unique \(\overline{d}_i^R\) such that the brand is better off whenever \(d_i^R \leq \overline{d}_i^R\).

Furthermore, by Proposition 5, there is a finite number of periods where \(d_i \leq \overline{d}_i^R\). Let \(N\) denote this finite number of periods. Define \(d_i^R = \max_{i \in N} \{\overline{d}_i^R\} = \overline{d}_i^R\), where the last equality follows from the fact that \(d_i^R\) is non-increasing in \(i\) (the number of periods the service provider has not reported). Since for all \(i \in N\), we have \(\overline{d}_i^R \leq d_i^R\), we must have \(\alpha c(\bar{k}, \overline{d}_i^R) - c(\bar{k}, 0) \leq 0\) for all \(i \in N\). Since any sequence of the provider’s non-disclosure periods can be viewed as a renewal process (as the service environment evolution is martingale), the above argument holds for any sequence of non-disclosure periods. For the sensitivity statement, it follows from the fact that \(\alpha c(\bar{k}, d_i^{\text{new}})\) is decreasing in \(\alpha\) and therefore a decrease in \(\alpha\) (improved efficiency) implies a larger \(\overline{d}_i^R\) and hence \(d_i^R\). This completes the proof. □

**Proof of Proposition 9.** For any given myopic policy, the optimal monitoring policy can be derived from the optimal policy characterized in Proposition 7. In particular, it is optimal to set \(y^* = 0\) if the provider reports true environment condition in period \(i\). In contrast, if the provider does not report, then the marginal benefit of monitoring for the current period is given by (see proof of Proposition 7)

\[
E[c^S(\bar{k}, d_i^{\text{new}}) - c^S(\bar{k}, d_i^{\text{new}} \xi_i^{\text{new}} | \xi_i^{\text{new}} \leq 1)] - M = \alpha C/\beta d_i^{\text{new}} E[(1 - \xi_i^{\text{new}}) | \xi_i^{\text{new}} \leq 1] - M
\]
\[ = 2\alpha Ck\beta b_i \left( \frac{\text{erf} \left( \frac{s_i}{\sqrt{2}} \right)}{1 + \text{erf} \left( \frac{s_i}{\sqrt{2}} \right)} \right) - M, \]

where the last equality follows from the matching lognormal approximation. Observe that the benefit of monitoring is increasing in \( s_i \). Let \( s_i \to \infty \), we have \( \mathbb{E}[c^S(k, d_i^b) - c^S(k, d_i^b \xi_i^b)] - M \to \alpha Ck\beta d_i^b - M \). Setting \( \alpha Ck\beta d_i^b - M \geq 0 \) yields the threshold \( d_i^b = M(\alpha Ck\beta)^{-1} \). Next, for any myopic policy that considers only the current period, the benefit of monitoring cannot exceed \( 2\alpha Ck\beta d_i^b H(s_i) - M \), where \( H(s_i) \) is defined in the proof of Proposition 7. Note that \( 2\alpha Ck\beta d_i^b H(s_i) \leq 2\alpha Ck\beta d_i^b H(1) = \alpha Ck\beta d_i^b \). When future marginal benefits are considered, then the global optimal threshold \( d_i^b \) must satisfy \( d_i^b \leq M(2\alpha Ck\beta d_i^b H(s_i))^{-1} \leq d_i^b \). Finally, observe that the marginal benefit of monitoring in the current period on future periods cannot exceed the monitoring cost \( M \) discounted one period back. To see this, note that any marginal benefit carried over to the next period is embedded in the potential lower \( d_{i+1}^b \) for future periods. However, one can always recover the lowered \( d_{i+1}^b \) in the next period by monitoring in period \( i + 1 \) with a monitoring cost \( M \). As such, the maximum marginal benefit for monitoring in period \( i \) cannot exceed \( 2\alpha Ck\beta d_i^b H(s_i) + M(1 + r)^{-1} \), and hence the global optimal threshold cannot be lower than \( 2\alpha Ck\beta d_i^b H(s_i) + M(1 + r)^{-1} - M \geq 0 \to d_i^b \geq r/(1 + r)M(2\alpha Ck\beta H(s_i))^{-1} \). The proposition statement then follows by setting \( s_i \to \infty \). □