Retailer Inventory Data Sharing in a Fresh Product Supply Chain

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This research explores the value of data sharing in a two-echelon, fresh product supply chain consisting of a retailer and manufacturer. Demand is stochastic and price sensitive. Decision-making is decentralized and the supply chain operates with a simple linear wholesale price contract. We employ a game theoretic model in which the manufacturer is a Stackelberg leader. Although fresh product supply chains are ubiquitous in the marketplace, academic guidance is limited, particularly in the context of data sharing and when price is determined endogenously.

Our analysis shows that although data sharing helps to improve product freshness and reduce waste, sufficiently improved product freshness is the key for both suppliers and retailers to benefit financially. Otherwise, either party or the entire supply chain can be worse off. We demonstrate that data sharing is most valuable when product is somewhat perishable and demand is volatile. For highly perishable or non-perishable products, the manufacturer benefits but the retailer is harmed. Our results suggest that the drivers that motivate retailers to share inventory data in a fresh product supply chain are distinct from extant knowledge. The results provide insight into strategies that not only improve both parties’ performance, but also reduce food waste at the same time.

Key words: perishable, data sharing, pricing, inventory

1. Introduction
This research examines the role of data sharing among echelons in improving the performance of fresh product supply chains, which has important implications on food price as well as food waste. Data sharing is a simple but potentially effective tool to improve fresh product supply chain performance. For example, Hingley et al. (2005) note that “Good data exchange is vital in helping us manage the day-to-day volumes successfully, which can incidentally be very volatile” [p. 69]. Recently, Vessella (2018), finding that 71% of suppliers want to receive data from retailers to help improve their planning, concluded that “Both retailers and suppliers know the vital role that data plays in retail execution. What they don’t always know is how sharing data points and key insights with each other can help correct problems on the shelf.”

A substantial body of literature evaluates how demand or sales information can be used to improve non-perishable supply chain performance and the conditions under which information is most valuable. An important insight from this literature is that, under linear wholesale price and linear production cost, a manufacturer has an incentive to exploit shared information by increasing wholesale price, which hurts the retailer and reduces the supply chain profit overall. In
such a setting, information sharing worsens the double marginalization effect, hence, the retailer has no incentive to share information (Li and Zhang 2002). The literature further suggests that diseconomies in production as well as competition can mitigate the double marginalization effect of information sharing such that the supply chain can be better off with information sharing (see, for example, Li and Zhang (2008), Ha et al. (2011), Shang et al. (2016)).

This study complements the above literature by examining the fresh product supply chain with short shelf life (e.g., prepared salads, meals, baked goods, etc.). Short product shelf life has several implications for the value of information sharing. First, data sharing influences the amount of outdating, which is in general not an issue with stable shelf-life products. Second, sharing demand or sales data is sufficient for supply chain partners to know inventory levels for stable shelf-life products, but is not so for fresh products due to outdating. Third, sharing inventory data has implications for freshness of production and inventory, which has more nuanced implications for the wholesale price. Indeed, we are not aware of any literature on a fresh product supply chain that addresses the impact of data sharing on pricing decisions, both wholesale and retail.

The above distinctions help to capture key concerns in fresh product supply chains that are often absent in stable shelf-life product supply chains. In particular, our study shows that retailer behavior, being affected by freshness upon delivery as well as price changes, can reverberate back to the supplier. The entanglement of supplier and retailer behavior makes it unclear which members of a supply chain benefits more from data sharing. Because data sharing helps suppliers run more efficient inventory systems, would suppliers charge a higher price to improve profit margins (Kulp et al. 2004), or, because data sharing reduces the supplier’s inventory cost, would it be in their best interest to lower the price to stimulate sales? Further, data sharing improves product freshness, creating another incentive for retailers to share inventory data. All things considered, would retailers be better off sharing inventory data? These are the key research questions that we address.

Additionally, there is a waste and sustainability aspect to our research. Because fresh products represent more than 50% of supermarket sales (Progressive Grocer 2017), a better understanding of fresh product supply chains is critical to help reduce food waste, estimated at 1.3 billion metric tons lost annually worldwide. The loss in the United States alone accounts for 10% of the total food supply (Gustavsson et al. 2011, Gunders 2012). An understanding of data-sharing implications for food loss would be incomplete and even misleading, however, were one to simply infer from insights derived from non-perishable product supply chains.

We study a two-echelon serial supply chain consisting of a manufacturer and a retailer, through which a perishable product is sold to consumers. Demand is stochastic and price sensitive. Decision-making is decentralized and the supply chain operates with a simple linear wholesale price contract. We employ a game theoretic model in which the manufacturer is a Stackelberg leader. The product
has a finite shelf life of two periods, with constant utility throughout. Any remaining inventory after the lifetime expires is salvaged. Both price and ordering policies are endogenous decisions at each echelon. In this setting, we explore the value of inventory data sharing by the retailer.

The following are among our key findings. (1) Inventory data sharing can be valuable for retailers, but only when the product is somewhat perishable. Retailers cannot benefit from data sharing when products are highly perishable or non-perishable. Improved product freshness is key to retailers benefiting from inventory data sharing, yet even when the product is somewhat perishable retailer benefit is not guaranteed. (2) Retailers typically benefit in settings in which demand uncertainty and consumer price sensitivity are high. The reason is that such settings are associated with higher supplier inventory and, if retailers do not share inventory data with suppliers, reduced product freshness upon delivery. In such settings, inventory data sharing enables suppliers to better match production with retailer demand and thereby increase product freshness. (3) Data sharing enables suppliers to increase product freshness upon delivery, which translates to increased product freshness at point of sale. As a result, data sharing can significantly reduce the cost of outdated and food waste. Such improvements, however, may not benefit retailers financially because of data sharing’s influence on wholesale price.

The rest of the paper is organized as follows. In §2 we position our research with respect to the literature, and in §3 we define a general model that reflects real-world dynamics of manufacturer-retailer perishable-product supply chain. While realistic, the complexity of the general model resists analytical characterization. In order to gain insight into system behavior, we define an approximate model and characterize conditions under which data sharing is beneficial. Then, in §4, we test the robustness of our conclusions from the approximate model by implementing the general model in a numerical study. We conclude the study and discuss future research directions in §5. Computational algorithms and proofs are placed within lettered appendices located in the online supplement. In the rest of the paper, in alignment with the extant academic literature we largely use the term “manufacturer” in lieu of “supplier”.

2. Literature Review
We position our research at the intersection of the literature on perishable inventory management and on the value of information. Below we summarize each stream using representative examples from the literature and position our research with respect to them.

A major distinction in the literature on perishable inventory systems is whether the product has a fixed or random lifetime. Much of the early work focuses on fixed lifetime problems under periodic review with seminal work by Nahmias and Pierskalla (1973) that considered the problem of determining optimal ordering policies under a periodic review system for a product with a
two-period lifetime. This early work was then extended by Nahmias (1975) and Fries (1975) who, separately, but contemporaneously, derive and evaluate optimal policies for the general $n$-period lifetime model. The often-referenced literature review by Nahmias (1982) provides an excellent overview of this early work. Other than to find approximations to the optimal replenishment policies in the fixed lifetime case, much of the work that follows has shifted to the analysis of random lifetime models. Raafat (1991) provides a review of early work in this area and it was later followed up by Dave (1991), Goyal and Giri (2001), and Li et al. (2010). Karaesmen et al. (2011) and Bakker et al. (2012) provide more recent updates on both the fixed and random lifetime literature.

Ostensibly, our research falls squarely in the fixed lifetime literature since in our model the supplier produces all units with a deterministic lifetime of two periods. Nevertheless, the retailer faces a random lifetime problem, since the age class of units available at the supplier varies over time. Even so, because we assume a periodic, discrete time model, the stochastic dynamic programming formulations of the fixed lifetime research are much more closely related than the formulations in the random lifetime literature. Indeed, Nahmias (1977) analyzes the problem of a random lifetime product with stationary stochastic demand, no fixed order cost, and backlogged demand. Ferguson and Ketzenberg (2006) extends the analysis to the lost sales case. In our model, the assumptions governing the retailer’s behavior are similar: periodic review and lost sales, but we extend our analysis to include price as a decision variable. With regard to pricing, there are a few contributions that make advances in this direction to which we proceed.

The importance of pricing and the opportunity it presents for research on perishables reaches back to Nahmias (1982). However, it has only been recently that contributions have appeared that address coordinated pricing and replenishment decisions. Ferguson and Koenigsberg (2007) address the joint decisions of price and quantity. The authors consider a two-period problem where the focal point of interest is the second period in which leftovers from the first period compete with new inventory. Li et al. (2010) consider a dynamic joint pricing and inventory control problem for a perishable product that has a lifetime $n \geq 2$ periods over an infinite planning horizon. Chen and Sapra (2013) consider a finite-horizon periodic review model for a perishable product with a two-period lifetime with backlogging. They analytically investigate properties of an optimal pricing and ordering policy for a firm. Collectively, these contributions to the literature analytically investigate properties of the optimal pricing and ordering policy for a single firm. In contrast, we study the interaction of optimal decisions and consequent profits in a two-echelon supply chain. Our focus is on understanding how retailer information sharing affects supply chain outcomes.

Now, research interest on information sharing has grown in tandem with the rise of e-commerce and information technologies that promise better supply chain coordination. An alternative way of coordinating a decentralized supply chain is through contracts. In this literature, outcomes
from centralized decision-making serve as a benchmark and the objective is to identify contractual mechanisms like revenue sharing and price guarantees that will achieve the same system-wide performance. See Cachon (2003) for a general review of this literature, while Wang and Webster (2009) and Chung and Erhun (2013) serve as illustrative examples that explore contracts in the context of perishable inventory. As for information sharing, which is the approach taken in our study, excellent reviews can be found in Sahin and Robinson (2002), Huang et al. (2003), Chen (2003), Ketzenberg et al. (2007), and Viet et al. (2018), among others.

A preponderance of existing research that addresses information sharing is focused on non-perishable products (see, for example, Bourland et al. (1996), Gavirneni et al. (1999), Lee et al. (2000), Gilioni et al. (2014), and Cui et al. (2015)). Retailers benefit from data sharing for products with stable shelf life and fixed wholesale price, as it improves the reliability of supply or service levels with no downside implications (Gavirneni et al. 1999, Cachon and Fisher 2000, Lee et al. 2000). Data sharing benefits could thus, in theory, be negotiated between the two parties to induce the retailer to share data. When prices are endogenous, existing research finds that data sharing generally hurts both retailer and supply chain performance by creating an incentive for manufacturers to exploit shared data to charge a higher wholesale price, thereby exacerbating the double marginalization effect (Li and Zhang 2002). This effect can be mitigated given diseconomies in production or intense competition (Li and Zhang 2008, Ha et al. 2011, Shang et al. 2016).

Recently, Roy et al. (2019) examined whether manufacturers can benefit from retailer inventory data when retailers hold inventory strategically to mitigate double marginalization. Interestingly, they found manufacturers, under certain conditions, to prefer not to observe retailer inventory, and the retailer to be willing to share inventory data only when inventory holding cost is sufficiently high. Because inventory data sharing also influences the freshness of delivered product, the value of retailer inventory data sharing is more nuanced for perishable than for non-perishable product.

As for the value of information for the management of perishable products, the existing stream of literature is focused primarily on comparing the performance of different inventory issuing policies facilitated by knowing a product’s time and temperature history as it flows through the supply chain. With this information, inventory can be issued based on a first-to-expire-first-out as opposed to first-in-first-out basis (see, for example, Koutsoumanis et al. (2005), Ketzenberg et al. (2015), Chen et al. (2021)). Although all of these contributions concern single facility, single location models such that the interaction of supply chain partners’ decisions are not relevant. A partial consideration of manufacturer-retailer interaction can be found in Ferguson and Ketzenberg (2006) and Ketzenberg and Ferguson (2008) that explore information sharing in both directions (up- and down-stream). These studies, however, assume exogenous pricing, thus the impact of information sharing on profit margins cannot be assessed.
Indeed, in contrast to the above literature, we believe ours to be the first study on information sharing to explore the joint pricing and replenishment decision for both manufacturer and retailer in a two-echelon fresh product supply chain that serves stochastic, price sensitive demand. This approach enables us to more completely assess the impact of inventory data sharing in the important sector of perishable products, in which waste is a significant concern.

3. Model
Consider a supply chain consisting of one manufacturer and one retailer selling fresh product that has a fixed, two-period shelf life, after which the product is outdated (e.g., packaged salads, sandwiches and grab-and-go meals). Product utility is constant throughout its lifetime. The two-period shelf life assumption, the simplest representation of products that are somewhat perishable, captures the essential tradeoffs in a fresh product supply chain and yet is easy to understand and compute numerically. A key challenge in such a supply chain setting is to improve operational efficiency between manufacturer and retailer in order to reduce product outdating. A natural and simple mechanism that could help to improve supply chain efficiency is data sharing. We focus on a retailer sharing inventory data with a manufacturer, which is similar to the inventory data sharing studied in Roy et al. (2019), although the settings and foci are different. Note that because we allow the retailer to endogenously set retail price, inventory information is more relevant than sales information (since price can be adjusted to affect sales) for the manufacturer to plan production. Additionally, although a reverse data sharing mechanism could be explored in which the manufacturer shares inventory data with the retailer, such a mechanism leaves the manufacturer’s production planning uncertain because the manufacturer continues to see random orders from the retailer. With retailer inventory data sharing, the manufacturer can sharply reduce demand uncertainty by deducing more accurate retailer orders that in turn help to reduce product waste.

We now describe our model setup.

3.1. Problem description and model formulation
In this section, we describe a general, infinite horizon model that, although analytically intractable, helps to illustrate the problem context and motivate the subsequent myopic approximate model. The approximate model enables us to develop crisp analytical results. We implement the general model and numerically test the robustness of our analytical results in §4.

For ease of reference, we first summarize the notations used in various parts of the proofs below. Note that for the retailer, instead of \( p \) and \( q \), it is sometimes more convenient to treat the retailer’s decisions as choosing mean demand \( d \) and stocking factor \( \theta : q = d(p, \theta) \).

- \( s \) = manufacturer produce-up-to level in a period
• \( q \) = retailer order quantity in a period
• \( d(p, \tilde{\epsilon}) \) = random retail demand at retailer price \( p \) in a period
• \( d = Ed(p, \tilde{\epsilon}) \) = mean demand in a period. \( d \) is bounded and \( d \in D \)
• \( \tilde{\epsilon} \) = random demand error with pdf \( f \), cdf \( F \), support \([\epsilon_L, \epsilon_H]\), and independent over time (e.g., random demand = \( d + \tilde{\epsilon} \) and \( E[\tilde{\epsilon}] = 0 \))
• \( \theta = q - d \) = stocking factor (e.g., surplus or shortage relative to expected demand)
• \( p(d) = a - bd \) = linear price-demand function, \( a, b > 0 \)
• \( w = \) retailer’s unit ordering cost (i.e., manufacturer’s wholesale price)
• \( h = \) inventory holding cost per unit-period for party \( k \), \( k \in \{m, r\} \) (i.e., \( m = \) manufacturer and \( r = \) retailer)
• \( \alpha_k \) = one-period discount factor for party \( k \)
• \( \zeta = \) fraction of fresh product delivered by the manufacturer (an approximate value)
• \( i_k = \) inventory at the beginning of the period for party \( k \) (i.e., \( i_k \) units are one period old at start of period for party \( k \))
• \( y = i_r + q = \) retailer’s stocking level
• \( \pi_k(\cdot|i_k) = \) expected profit in the current period for party \( k \)
• \( v_k(i_k) = \) optimal expected profit to go for party \( k \)
• \( G_k(\cdot|i_k) = \) expected profit to go for party \( k \) given current period decisions
• \( \phi_k = \) probability distribution of starting inventory \( i_k \) estimated by \( 3 - k \) (i.e., \( \phi_m \) is the distribution of \( i_m \) estimated by the retailer while \( \phi_r \) is the distribution of \( i_r \) estimated by the manufacturer)

In each period, the manufacturer decides its produce-up-to level \( s \) at a unit production cost \( c \), and sells at a steady-state unit wholesale price \( w \). This reflects scenarios in which the retailer signs long term contracts with the manufacturer to keep wholesale prices stable (Short 2020). It is worth pointing out that an alternative, a state-dependent wholesale pricing policy where \( w \) depends on the manufacturer’s and, if available, retailer’s inventory, introduces additional indirect effects such as the retailer accounting for the impact of current period order quantity on next period wholesale price. To isolate the direct (as opposed to indirect) effect of data sharing, we focus on the stable wholesale price scenario and leave the state-dependent wholesale price scenario to future studies. Nonetheless, we note that a preliminary analysis of the more complex state-dependent wholesale pricing policy yields results consistent with our analysis here. The order quantity from the retailer, denote as \( q_i \), is uncertain. If there are shortages, the manufacturer expedites delivery of \((q - s)^+\) units at unit cost \( e_m > c \). If there are leftovers, a unit holding cost of \( h_m \) is assessed. Leftover units that are two periods old at the end of the period are disposed of at unit salvage value \( \gamma_m < c \).
The retailer decides retail price \( p \) and order quantity \( q \). The retailer faces uncertain, price-dependent demand specified through the function \( d(p, \epsilon) = d(p) + \epsilon \), where \( d'(p) < 0 \) and \( \epsilon \) is a realization of stochastic noise \( \hat{\epsilon} \), which has a mean zero and support over \([\epsilon_L, \epsilon_U]\). The retailer’s unit shortage cost is \( c_r \), unit holding cost is \( h_r \), and unit salvage value is \( \gamma_r < c \). Under the manufacturer’s steady state wholesale price \( w \), the timing of events in each period is as follows.

1. Manufacturer produces \((s - i_m)\) units at unit cost \( c \), where \( i_m \) is the starting inventory level.
2. Retailer chooses retail price \( p \) and orders \( q \) units from the manufacturer. If \( q > s \), the manufacturer expedites production of \((q - s)\) units at unit cost \( e_m \).
3. Retailer takes delivery where \( \min\{q, i_m\} \) units are one-period old at time of receipt. The retailer’s on hand inventory is \( i_r + q \), where \( i_r \) is the retailer’s starting inventory level.
4. Retailer observes demand and satisfies demand up to available inventory; unsatisfied demand is lost at a unit penalty cost of \( c_r \) and leftover inventories incurs a unit holding cost \( h_r \). Units that are two periods old at the end of the period are disposed of at unit salvage value \( \gamma_r < w \).
5. Manufacturer pays a unit holding cost \( h_m \) on leftover inventories. Units that are two periods old at the end of the period are disposed of at unit salvage value \( \gamma_m < c \).

We now formulate the manufacturer’s and the retailer’s decision problem. To emphasize that the manufacturer sees an uncertain order from the retailer, we use \( \tilde{q}(w) \) to denote the random retailer order, which depends on the manufacturer’s wholesale price decision \( w \). The manufacturer’s expected profit in a period is

\[
\pi_m(s,w,i_m) = w E[\tilde{q}(w)] - c(s-i_m) - e_m E[(\tilde{q}(w) - s)^+] - h_m E[(s - \tilde{q}(w))^+] + \gamma_m E[(i_m - \tilde{q}(w))^+].
\]

In the above expression, the first term \( w E[\tilde{q}(w)] \) captures expected revenue, the second term \( c(s-i_m) \) production cost, the third term \( e_m E[(\tilde{q}(w) - s)^+] \) emergency shipping cost, the fourth term \( h_m E[(s - \tilde{q}(w))^+] \) holding cost, and the last term \( \gamma_m E[(i_m - \tilde{q}(w))^+] \) disposal cost. Random inventory at the start of the next period is \( \tilde{i}_{m'} = (s - \max\{i_m, \tilde{q}(w)\})^+ \). Let \( \alpha_m \in (0,1] \) denote the discount factor. The manufacturer’s problem in a period is

\[
v_m(i_m|w) = \max_{s \geq i_m} \left\{ \pi_m(s,w,i_m) + \alpha_m E\left[v_m\left((s - \max\{i_m, \tilde{q}(w)\})^+|w\right)\right] \right\}, \tag{1}
\]

and at time zero the manufacturer solves \( \max_{w} E\left[v_m(\tilde{i}_m(w)|w)\right] \), where the expectation is over the (steady state) random starting inventory level \( \tilde{i}_m(w) \) which itself is a function of the unit wholesale price \( w \). Analogously, the retailer’s expected profit in a period is

\[
\pi_r(p,q|i_r) = p E[\min\{i_r + q, d(p, \epsilon)\}] - wq - h_r E[(i_r + q - d(p, \epsilon))^+] - e_r E\left[(d(p, \epsilon) - q - i_r)^+\right] + \gamma_r E\left[(i_r + \min\{q, \tilde{i}_m\} - d(p, \epsilon))^+\right].
\]
Interpretations of each term in the above expression are similar to those for the manufacturer. Random inventory at the start of the next period is $i' = (i_r + q - (i_r + \min\{q, \tilde{i}_m\} - d(p, \tilde{\epsilon}))^+ - d(p, \tilde{\epsilon}))^+$. Let $\alpha_r \in (0, 1]$ denote the discount factor. The retailer’s problem in a period is

$$v_r(i_r) = \max_{p,q} \left\{ \pi_r(p, q|i_r) + \alpha_r E\left[v_r\left((i_r + q - (i_r + \min\{q, \tilde{i}_m\} - d(p, \tilde{\epsilon}))^+ - d(p, \tilde{\epsilon}))^+\right)\right] \right\}.$$  \hspace{1cm} (2)

Observe that (1) and (2) are inter-dependent through the retailer’s order quantity $\tilde{q}(w)$ and manufacturer’s inventory level $\tilde{i}_m$. The manufacturer can only anticipate $\tilde{q}(w)$, whereas the retailer anticipates $\tilde{i}_m$, and both depend on the manufacturer’s wholesale price $w$ and retailer’s retail price $p$. Thus, the manufacturer and retailer problems are tightly coupled and neither admits a closed form solution. We can nevertheless obtain certain analytical insights.

### 3.2. Benchmark properties: Non-perishable product

It is useful to consider a benchmark case in which product is non-perishable. The following lemma establishes a useful property that simplifies the analysis for non-perishable product.

**Lemma 1** \(a\) If product is non-perishable, then the manufacturer’s and the retailer’s infinite-horizon problems can be reduced to a sequence of single-period problems. \(b\) A stationary (but endogenous) wholesale price policy is optimal.

Note that for highly perishable product (one-period shelf life), the above lemma trivially holds. Thus, when Lemma 1 applies, analysis for the non-perishable directly applies as well for the highly perishable product. With Lemma 1, the manufacturer’s and retailer’s profit functions simplify to price-setting newsvendor models. From the manufacturer’s perspective, the retailer order quantity can be viewed as random price-dependent demand. If the retailer shares its inventory data, the manufacturer knows what the retailer will order for any given price (because product is non-perishable and demand noise $\tilde{\epsilon}$ is common knowledge). The manufacturer’s pricing problem is therefore structurally similar to the retailer’s price setting problem (with known $\epsilon$). We analyze such a retailer model below, recognizing that it can be adapted to the manufacturer problem under retailer data sharing.

Unlike the order of events adopted in Section 3.1, now consider the case that the retailer sets order quantity after observing demand. Supposing price to remain stationary over time and defining $E[d(p, \tilde{\epsilon})] = d(p)$, the optimal stationary price satisfies

$$p^o = \arg \max_p (p - w) d(p).$$  \hspace{1cm} (3)

**Lemma 2** Under a stationary price policy, \(a\) if price is set after $\tilde{\epsilon}$ is realized, the optimal price satisfies $(p^o - w)/p^o = \left(\frac{-p^o d(p^o)}{d(p^o)}\right)^{-1}$, \(b\) if price is set before $\tilde{\epsilon}$ is realized and $d(p) = a - bp$, the optimal price is $p^*(\theta) = p^o - E\left[(\tilde{\epsilon} - \theta)^+\right]/(2b)$ for any given stocking factor $\theta = q - d(p)$, \(c\) $p^*(\theta) < p^o$. 

The above result shows the optimal price under stochastic demand to be less than that under deterministic demand. If we interpret the above result from the manufacturer’s perspective, data sharing helps to reduce demand (retailer order) uncertainty and thereby create an incentive to increase wholesale price. Leveraging Lemma 2, we can characterize the effect of retailer inventory data sharing (i.e., retailer sharing $i_r$) on the manufacturer’s optimal wholesale price.

**Proposition 1** If the product is non-perishable or highly perishable (one-period shelf-life), the optimal wholesale price $w$ decreases in demand uncertainty. That is, all else being equal, inventory data sharing hurts the retailer.

This result suggests that the retailer cannot benefit from data sharing for non-perishable or highly perishable products. For a partially perishable product, the retailer’s optimal price and order policy reduces the demand uncertainty seen by the manufacturer (see Proposition A1 in Appendix C). This means that the manufacturer can better match production with demand and the retailer, in turn, will receive fresher deliveries from the manufacturer. Because the retailer benefits from fresher delivery (see Proposition A2 in Appendix C), sharing inventory data potentially benefits the retailer. Nevertheless, data sharing may also influence the wholesale price. Thus, whether the retailer benefits from data sharing depends on whether savings from fresher product can offset a potentially increased wholesale price. If demand is sensitive to price, the retailer can use minor price adjustments to maintain steady sales so that the manufacturer sees reduced demand uncertainty regardless of whether data is shared or not. In such a case the retailer is more likely to benefit from data sharing. If, however, demand is insensitive to price, data sharing is likely to hurt the retailer.

The above analysis illustrates the effect of price sensitivities on manufacturer demand uncertainty, but does not characterize equilibrium behaviors for somewhat perishable products. The equilibrium analysis with the general model is intractable because the retailer receives product of mixed age, and the manufacturer and retailer decisions are tightly coupled with no closed form solution. Thus, to derive further insights on equilibrium behavior we explore a simplified myopic model. The goal is to identify and analyze a model that retains the key tradeoffs in the general model, yet is amenable to analytical treatment. We then conduct a comprehensive numerical study using the general model to test the robustness of, and expand upon, findings from the myopic model.

### 3.3. A simplified, myopic model

In this section, we make simplifications and apply approximations to the general model so as to obtain sharper analytical results. We first simplify the retailer’s demand function. We assume that the retailer faces a linear demand function $d(p, \tilde{\epsilon}) = a - p + \tilde{\epsilon}$, where $\tilde{\epsilon}$ follows a Bernoulli distribution.
with $\bar{\epsilon} = E[\bar{\epsilon}]$, $P(\bar{\epsilon} = \epsilon_L) = \rho_L$, $P(\bar{\epsilon} = \epsilon_H) = \rho_H = 1 - \rho_L$, and $a + \epsilon_L \geq 0$. (We chose Bernoulli as an upper bound test in that is has the highest variation among discrete distributions. Later, in Section 4, we consider a negative binomial distribution.) Both retailer and manufacturer know the distribution of $\bar{\epsilon}$.

Next, instead of analyzing the detailed dynamics of the manufacturer’s production and inventory processes, we approximate the key consequences of the manufacturer’s system dynamics. Note that in our general model the manufacturer fills the retailer’s order from a mix of existing inventory and fresh production, and delivered product is fresher if a higher proportion is from fresh production. Let $\zeta$ approximate the fraction of the manufacturer’s delivery that can be carried to the next period (which will be that amount delivered that was freshly produced by the manufacturer in that period). Consequently, $(1 - \zeta)$ can be interpreted as the fraction to be salvaged if they remain unsold at the end of the current period. When the retailer shares inventory data, the manufacturer sees reduced demand uncertainties (see Proposition A1 in Appendix C) and hence is able to avoid inventory-related holding costs, including spoilage. As a consequence, the manufacturer will satisfy a higher proportion of retailer orders from fresh production. That is, $\zeta$ is higher when the retailer shares its inventory data and lower otherwise. Given that data sharing helps the manufacturer better match production with retailer demand, the broad effect of inventory data sharing is that the manufacturer delivers a higher proportion of fresh production and hence a higher fraction of leftovers can be carried over to the next period.

We simplify the retailer’s decision problem using a myopic approximation. A key complication with the retailer’s decision problem is the complex computation of the starting inventory for the next period (see equation (2)). Note that the key effect of starting inventory is that some of the leftover inventories have residual value that benefits the retailer in the next period. We approximate and account for this residual value in the current period, thus decoupling the retailer’s decision from future periods. Specifically, let $A(p, q|i_r)$ approximate the residual value of leftover inventories:

$$A(p, q|i_r) = \zeta w E[(i_r + q - d(p, \bar{\epsilon})]^+.$$ 

Given FIFO (first-in, first-out) inventory issuing, the fraction of the manufacturer’s delivery that can be carried to the next period, approximated by $\zeta$, also approximates the fraction of leftover inventory that can be sold in the future before it spoils. From this perspective, $\zeta w$ approximates the marginal value of units carried over to the next period. Such an approximation is myopic and sub-optimal, but does capture the key impact of residual inventories on retailer profit. We drop the disposal cost $\gamma_r$ with the understanding that any such cost is reflected in $\zeta w$. With the above approximations, the retailer’s one period profit function can be simplified to

$$\pi_r(p, q|i_r) = pE[\min\{i_r + q, d(p, \bar{\epsilon})\}] - wq - h_r E[(i_r + q - d(p, \bar{\epsilon})]^+] - e_r E[(d(p, \bar{\epsilon}) - q - i_r)^+] + A(p, q|i_r).$$
where in each period the retailer solves \( v_r(i_r) = \max_{p,q} \pi_r(p,q|i_r) \).

We analyze below manufacturer and retailer optimal profits under the above simplified model for two scenarios: the retailer shares, and does not share, inventory level \( i_r \) with the manufacturer. In our approximate model, we treat \( i_r \) as the manufacturer’s point estimate of retailer inventory.

### 3.4. Basic properties

We begin the analysis backwards by first analyzing the retailer’s pricing decision given the manufacturer’s wholesale price \( w \) and stocking level \( i_r + q \). With the optimal pricing decision fully characterized, we examine the retailer’s optimal ordering decision \( q \) for any given \( w \). Treating \( q \) as a function of the manufacturer’s wholesale price \( w \), we characterize the manufacturer’s optimal wholesale pricing decision. For notational ease, we define \( y = i_r + q \) as the retailer’s stocking level and let \( G(p,y) = pE[min\{y,d(p,\tilde{\epsilon})\}] - h_rE[(y-d(p,\tilde{\epsilon}))^+] - e_rE[(d(p,\tilde{\epsilon})-y)^+] \). Thus, we can rewrite the retailer’s objective function as \( \pi_r(y,p|i_r) = -w(y-i_r) + G(p,y) + A(p,y) \), where \( A(p,y) = \zeta w E(y-d(p,\tilde{\epsilon}))^+ \).

#### 3.4.1. Retailer pricing decision

The retailer’s pricing decision depends only on \( G(p,y) + A(p,y) \). Assuming Bernoulli distribution on demand noise and a linear demand function, we have

\[
G(p,y) + A(p,y) = \sum_{i \in \{H,L\}} \rho_i (p \min(a+\epsilon_i - p,y) - (h_r - \zeta w)(y - (a+\epsilon_i - p))^+ - e_r((a+\epsilon_i - p) - y)^+) .
\]

The retailer’s optimal price must fall into one of three regions: \( \circ \) \( p \leq a + \epsilon_L - y \), \( \mathbb{B} \) \( a + \epsilon_L - y < p \leq a + \epsilon_H - y \), and \( \mathbb{C} \) \( p > a + \epsilon_H - y \). The following proposition characterizes the retailer’s optimal pricing decision.

**Proposition 2** The retailer’s optimal pricing decision satisfies the following conditions. (i) \( p^* \) cannot lie in region \( \mathbb{B} \), i.e., \( p^* \geq a + \epsilon_L - y \). (ii) An interior \( p^* \) falls into one of the following two regions

\[
p^* = \begin{cases} \frac{1}{2} \left( a + \epsilon_L - (h_r - \zeta w) + \frac{\rho_H}{\rho_L} (y + e_r) \right), & \text{region } \mathbb{B}; \\ \frac{1}{2} (a + \tilde{\epsilon} - (h_r - \zeta w)), & \text{region } \mathbb{C}. \end{cases}
\]

The optimal price cannot simultaneously satisfy regions \( \mathbb{B} \) and \( \mathbb{C} \) because, by definition, the optimal price in these two regions must be increasing, that is, the price in region \( \mathbb{B} \) must be lower than that in region \( \mathbb{C} \). The optimal interior prices in these two regions, however, are non-increasing, which suggests that at least one of the two candidate solutions must lie on the border of their respective regions. A formal proof can be found in Corollary A1 in Appendix A. Having characterized the retailer’s optimal pricing decision for different values of \( y \), we are now in a position to analyze the retailer’s optimal stocking decision \( y^* \).
### 3.4.2. Retailer stocking decision

The below proposition proves there are two possible optimal stocking levels, both lying at corner points, which we denote as \( L \) and \( H \).

**Proposition 3**

(i) The retailer’s optimal stocking decision \( y^* \) can only lie on two corner points, which we label as strategy points \( L \) and \( H \). (ii) Furthermore, if the optimal stocking decision lies in \( L \), then the stocking and pricing decisions as well as the resulting expected profit are given by the following.

\[
y^L = \frac{a + \epsilon_L - w}{2}, \quad p^L = \frac{a + \epsilon_L + w}{2}, \quad \pi_r^L(y^*, p^*|i_r, w) = wx + \left(\frac{a + \epsilon_L - w}{2}\right)^2 - e_r (\epsilon - \epsilon_L).
\]

If the optimal stocking decision lies in \( H \), then the stocking and pricing decisions as well as the resulting expected profit are given by the following.

\[
y^H = \frac{1}{2} \left( a + \epsilon_H + \rho_L (\epsilon_H - \epsilon_L) - w \right), \quad p^H = \frac{1}{2} \left( a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) + w \right), \quad \pi_r^H(y^*, p^*|i_r, w) = wx + \left(\frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) - w}{2}\right)^2 - (h_r + (1 - \zeta)w)\rho_L (\epsilon_H - \epsilon_L).
\]

The retailer’s optimal stocking strategy can therefore be characterized as two opposing strategies: stocking low to cover low market demand only (strategy point \( L \)), or stocking high to cover all market demand (strategy point \( H \)). The former strategy minimizes waste (there being no leftovers), but incurs a high shortage penalty cost if market demand turns out to be high. The latter strategy maximizes market coverage, but generates waste and incurs holding cost if market demand turns out to be low.

From the retailer’s perspective, the relative merits of these two strategies are critically influenced by the wholesale price \( w \) charged by the manufacturer. The following proposition shows that there exists a critical threshold value of \( w \) such that one strategy is preferred to the other.

**Proposition 4**

The retailer’s optimal stocking decision lies in strategy point \( H \) if \( w \leq \bar{w} \) and in strategy point \( L \) otherwise, where

\[
\bar{w}(\zeta) = \frac{\rho_H (a + \frac{1}{2} (\epsilon + \epsilon_L)) + 2(\rho_H e_r - \rho_L h_r)}{\rho_H + 2\rho_L (1 - \zeta)}.
\]

The above proposition indicates that the retailer prefers to maximize market coverage (strategy \( H \)) when the wholesale price is below a threshold level, and to minimize waste (strategy \( L \)) otherwise. This has interesting implications from the manufacturer’s perspective. Charging a higher wholesale price improves the manufacturer’s unit margin, but the retailer ordering less to minimize waste hurts the manufacturer due to a smaller order quantity. Charging a lower wholesale price reduces the manufacturer’s unit margin, but encourages the retailer to stock more and hence place a larger
order. For any given \( w \), the manufacturer, if it could dictate which strategy the retailer chooses, would always prefer that the retailer choose strategy \( \mathcal{H} \). The manufacturer may have a strong incentive to set a wholesale price \( w \) lower than the threshold level as there will be a precipitous drop in the retailer’s order quantity once the wholesale price exceeds this threshold. Having characterized the retailer’s optimal behavior, we now examine the manufacturer’s pricing decision.

### 3.4.3. Manufacturer pricing decision

Leveraging Proposition 4, we know that the manufacturer can induce the retailer to choose strategy \( \mathcal{H} \) by setting wholesale price \( w \leq \bar{w}(\zeta) \), and lead the retailer to choose strategy \( \mathcal{L} \) by setting \( w > \bar{w}(\zeta) \). We first consider the case when \( w \leq \bar{w}(\zeta) \), and then the case when \( w > \bar{w}(\zeta) \).

When \( w \leq \bar{w}(\zeta) \), the retailer’s stocking decision falls into strategy choice \( \mathcal{H} \). Under data sharing, the manufacturer knows the retailer’s starting inventory. Otherwise, the manufacturer views retailer inventory as uncertain and makes decisions based on its expected value. We use \( i_r \) for actual inventory in the case of data sharing and expected inventory in the case of no data sharing. The manufacturer’s expected profit is

\[
\pi_m^H(w|i_r) = (w - c) \left( y^{H*}(w) - i_r \right).
\]

The profit function being linear in inventory, we replace the random variable by its expectation. The manufacturer’s profit function \( \pi_m(w|i_r) \) is concave in \( w \) and the optimal interior solutions are given by

\[
w^{H*} = \frac{a + \epsilon_H + \rho_L (\epsilon_H - \epsilon_L) + c}{2} - i_r,
\]

\[
\pi_m^H(w^*|i_r) = \frac{1}{2} \left( \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) - c}{2} - i_r \right)^2.
\]

It is worth pointing out that the above solutions are valid if \( w^{H*} \) does not exceed the threshold value \( \bar{w}(\zeta) \). Otherwise, the optimal \( w^* \) in this region is bounded from above by \( \bar{w} \). For interior solutions, note that the optimal wholesale price decreases in retailer starting inventory level \( i_r \). Later we will show that the freshness of replenishment will influence the amount of starting inventory the retailer carries from the prior period.

Now consider the case in which \( w > \bar{w}(\zeta) \) such that the retailer’s optimal stocking strategy falls into strategy \( \mathcal{L} \). By Proposition 3, the manufacturer sees an order quantity of \( y^{L*}(w) - i_r \), hence, its objective function is \( \pi_m^L(w|i_r) = (w - c) \left( y^{L*}(w) - i_r \right) \). The manufacturer’s profit function \( \pi_m(w|i_r) \) is concave in \( w \) and the optimal interior solutions are given by

\[
w^{L*} = \frac{a + \epsilon_L + c}{2} - i_r,
\]

\[
\pi_m^L(w^*|i_r) = \frac{1}{2} \left( \frac{a + \epsilon_L - c}{2} - i_r \right)^2.
\]

Only one of the above candidate solutions to the manufacturer’s wholesale pricing decision can be optimal because, by definition, the wholesale prices are valid only if \( w^H \leq \bar{w}(\zeta) \leq w^L \), but the optimal interior solutions are such that \( w^{H*} > w^{L*} \).
**Proposition 5**

(i) If \( w^{L*} \leq \bar{w}(\zeta) \), then the manufacturer’s optimal wholesale price \( w^* = \min\{w^{H*}, \bar{w}(\zeta)\} \).  
(ii) If \( w^{L*} > \bar{w}(\zeta) \), then the manufacturer’s optimal wholesale price \( w^* = \arg \max_{w \in \{w^{L*}, \bar{w}(\zeta)\}} \pi_m(w|\bar{i}) \).

The manufacturer’s optimal wholesale price is influenced directly by the retailer’s starting inventory level \( i_r \) (because both \( w^{L*} \) and \( w^{H*} \) linearly depend on \( i_r \)) and the freshness of the product (because \( \bar{w}(\zeta) \) depends on the freshness parameter \( \zeta \)). In particular, both \( w^{L*} \) and \( w^{H*} \) decrease in retailer starting inventory level \( i_r \) whereas \( \bar{w}(\zeta) \) increases in \( \zeta \). Hence, all else being equal, the manufacturer sees a larger order quantity from the retailer as freshness increases. Fresher product, however, enables the retailer to carry over more inventory, hence the starting inventory level \( i_r \) is also likely to be higher, exerting downward pressure on the manufacturer’s optimal wholesale price. The net effect is therefore ambiguous. We explore whether the retailer and manufacturer are better off with fresher product when the retailer shares inventory data.

### 3.5. Impact of sharing inventory data

For perishable products, improving freshness is a key reason for the retailer to share inventory data with the manufacturer. With a better understanding of the retailer’s inventory, the manufacturer can operate its production system more efficiently and therefore deliver fresher product. The manufacturer may, however, adjust the wholesale price to exploit the shared data. Although fresher delivery benefits the retailer, it is unclear whether this benefit will outweigh the manufacturer’s tendency to exploit the shared data. In what follows, we leverage the myopic model described in §3.3 to clarify the effect of retailer inventory data sharing on the manufacturer and retailer profits.

Note that sharing inventory data is not relevant when the retailer adopts strategy \( L \) because, in this case, the retailer’s stocking quantity only covers low market demand realizations and incurs a shortage cost for high market demand realizations. The retailer consequently does not incur leftovers and its starting inventory is always zero. If the optimal wholesale price induces the retailer to operate with strategy \( L \), the manufacturer knows the retailer’s starting inventory is always zero, so data sharing has no effect. The following proposition summarizes the above observation.

**Proposition 6** Let \( \zeta_s \) denote product freshness with inventory data sharing. There exists a threshold value of \( \bar{\zeta}_s \) such that for any \( \zeta_s \leq \bar{\zeta}_s \), the retailer never carries inventory and hence data sharing is not relevant. In such settings, the manufacturer sets a fixed wholesale price and product freshness has no impact on the manufacturer’s or retailer’s expected profit.

Certain categories of fresh products are so highly perishable that the best possible freshness that can be achieved by the manufacturer is less than \( \bar{\zeta}_s \). For products in such categories, it is natural for the retailer to minimize stocking in each order cycle and hence have little inventory at the
beginning of each period. The manufacturer, knowing the retailer’s order quantity (which is equal to \( y_L^\ast(w) \) since \( i_r = 0 \)), charges a fixed wholesale price to maximize its profit. Product freshness thus has no impact on manufacturer or retailer profit.

Sharing inventory data is consequently of interest only if the retailer adopts strategy \( \mathcal{H} \). We focus below on the case in which \( \zeta_s > \bar{\zeta}_s \) such that the retailer adopts strategy \( \mathcal{H} \). To analyze the impact of data sharing on manufacturer and retailer performance, we consider three possible scenarios with regard to the manufacturer’s optimal wholesale price decisions. Let subscripts \( n \) and \( s \) denote no inventory sharing and inventory sharing, respectively. There are three possibilities: (a) \( w_n^\ast = \bar{w}(\zeta_n) \) and \( w_s^\ast = \bar{w}(\zeta_s) \), (b) \( w_n^\ast = \bar{w}(\zeta_n) \) and \( w_s^\ast = w^{\mathcal{H}^\ast} \), and (c) \( w_n^\ast = w_s^\ast = w^{\mathcal{H}^\ast} \).

We consider first the case in which the manufacturer’s optimal wholesale price is constrained by the retailer’s strategy choice, that is, \( w^\ast = \bar{w}(\zeta) \) such that the retailer chooses strategy \( \mathcal{H} \).

**Proposition 7** If \( \zeta_s > \zeta_n \) and \( w_s^\ast = \bar{w}(\zeta_s) \), then \( w_n^\ast = \bar{w}(\zeta_n) \). There exists a market volatility bound \( \Delta \) such that both manufacturer and retailer are strictly better off with inventory sharing if market volatility exceeds \( \Delta \), i.e., if \( \epsilon_H - \epsilon_L \geq \Delta \).

The above proposition shows that when sharing inventory data enables the manufacturer to improve product freshness (\( \zeta_s > \zeta_n \)), and the manufacturer’s wholesale price is bounded by the retailer’s strategy choice \( \mathcal{H} (w_s^\ast = \bar{w}(\zeta_s)) \), sharing inventory data benefits both manufacturer and retailer when market volatility is high. The retailer is more likely to benefit when the market is volatile and product freshness can be improved by sharing inventory data. It is worth noting with regard to the retailer benefiting from data sharing that improved product freshness, which is not considered by the extant literature, is critical. The following corollary shows that if product freshness cannot be improved (i.e., \( \zeta_s = \zeta_n \)) data sharing cannot benefit the retailer.

**Corollary 1** For highly perishable or non-perishable products, the retailer cannot benefit from inventory data sharing.

The retailer can benefit from data sharing only when product is somewhat perishable. Improved product freshness is thus key for the retailer to share inventory data.

We next consider the case in which the manufacturer’s optimal wholesale price is constrained when inventory data is not, but interior when inventory data is, shared.

**Proposition 8** If \( \zeta_s > \zeta_n \) and \( w_n^\ast = \bar{w}(\zeta_n) \), but \( w_s^\ast = w^{\mathcal{H}^\ast} \), then the manufacturer is strictly better off. The retailer is better off only if product freshness is sufficiently improved (\( \zeta_s - \zeta_n \)) by data sharing.
The above proposition reinforces the insight that improved product freshness is key to the retailer benefiting from sharing inventory data. Because, by definition, $\zeta_s \leq 1$, a sufficiently large $\zeta_s$ may not exist for the retailer to be better off in this case. Echoing our earlier observations around Proposition 6, if product shelf life is inherently limited, the retailer is less likely to benefit from sharing inventory data.

We consider last the case in which the manufacturer’s optimal wholesale price is not constrained regardless of whether the retailer shares inventory data.

**Proposition 9** If $w^*_n = w^*_s = w^{H^*}$, then the manufacturer benefits from retailer inventory data. The retailer may benefit only if product freshness is sufficiently improved after sharing inventory data.

We now summarize the above three propositions to highlight our key findings. The retailer can only benefit from sharing inventory data if three conditions hold jointly:

Condition (a): the product is *somewhat perishable*,

Condition (b): product freshness can be sufficiently improved by the manufacturer, and

Condition (c): market demand is sufficiently volatile.

Condition (a) ensures that there is opportunity for the manufacturer to improve product freshness with retailer inventory data. Condition (b) ensures that improved product freshness must be sufficient to offset the manufacturer’s inventory-dependent wholesale price due to data sharing. Condition (c) ensures that the retailer is able to carry over a sufficient amount of inventory, which serves as an important restraint on the manufacturer’s tendency to raise wholesale price. The following theorem encapsulates these insights.

**Theorem 1** All else being equal, the retailer has no incentive to share inventory data when product is highly perishable or non-perishable. The retailer has an incentive to share inventory data if (a) the product is somewhat perishable, (b) product freshness can be sufficiently improved by the manufacturer, and (c) market demand is sufficiently volatile.

It is useful to contrast the above results with the extant literature on the value of data sharing under endogenous pricing. It is well recognized that data sharing hurts the retailer and can leave the supply chain as a whole worse off (see, for example, Li and Zhang 2002, 2008, Ha et al. 2011, Shang et al. 2016). These studies assume that the retailer can set price and order quantity after observing realized demand and investigate whether it is beneficial to share realized demand data. An important implication of the above assumption is that the retailer never carries any inventory, therefore the effect of retailer inventory on the manufacturer’s pricing decision is not relevant. Casual observation suggests that most retailers do carry inventory, as pricing and stocking
decisions typically cannot be made after demand is realized. This study complements the extant literature by considering the more realistic setting in which a retailer’s inventory data becomes relevant in influencing the manufacturer’s pricing decision. More important, the foregoing studies do not consider the impact of data sharing on product freshness, hence the drivers that can benefit the retailer are also significantly different (e.g., the existing literature examines drivers like order volatility or economies of scale in contrast to the present research’s focus on product freshness).

Whether improving product freshness through the sharing of inventory data reduces the amount of outdated product at the retailer level is also of interest. Intuition suggests that fresher product should enable the retailer to outdate fewer leftovers, thereby reducing food waste in the supply chain. The following corollary confirms this intuition.

**Corollary 2** *If the retailer always adopts strategy $H$, sharing inventory data will help the retailer to reduce the amount of outdated leftovers.*

In general, sharing inventory data affects the manufacturer in two ways. It enables the manufacturer to improve the efficiency of its production process and hence deliver fresher product to the retailer. It also enables the manufacturer to extract more revenue from the retailer by charging a higher wholesale price. The combined effect on the retailer’s profit is ambiguous, as demonstrated by propositions 7 through 9. Because the amount of food waste is reduced whether the retailer benefits financially or not, from a sustainability perspective, sharing inventory data is always beneficial in terms of reducing food waste.

To sum up, we show that product freshness can provide an incentive for the retailer to voluntarily share inventory data with the manufacturer, and that both parties can benefit when product freshness can be sufficiently improved. For highly perishable or non-perishable product however, the retailer cannot benefit from data sharing. The above results are established using a simplified, approximate model that captures the essential tradeoffs in a fresh product supply chain. But because the model abstracts away the detailed operations of the manufacturer, it is necessary to test the robustness of the above findings to the general model introduced at the beginning of this section and build upon the insights generated. Toward this end, we perform a comprehensive numerical analysis of the general model described in §3.1.

4. **Numerical Analysis**

The numerical analysis examines the impact of endogenous pricing and product perishability on the value of information, measured as the change in profit that accrues to data sharing relative to the case without data sharing. In §4.1 we describe our experimental design. We present the results for the non-perishable case in §4.2, and for the perishable case in §4.3. We present the non-perishable and perishable cases separately to help disambiguate the differing effects of product perishability and endogenous pricing on the value of information sharing.
4.1. Study design

We set the price-dependent, random demand $D(p, \hat{\epsilon})$ as a negative binomial random variable $NB(r, \rho)$. The parameters $r$ and $\rho$ are set such that mean demand is $d(p) = (a - bp)$. Letting $\tau = 1/(1 - \rho)$ denote the variance-to-mean ratio, for any given $\tau$ the parameters $r$ and $\rho$ can be determined by recognizing that $pr/(1 - \rho) = a - bp$. The negative binomial distribution possesses several appealing properties including non-negative, discrete values, and enables specific control of demand variability that can automatically scale for any pricing decision. That is, by holding $\tau$ constant for a given scenario demand variability will scale up or down proportionally based on mean demand, which is determined by price. Across all experiments, market size, $a$, is held constant at 40. The unit holding cost rates, $h_r$ and $h_m$, are set such that $h_r = hw$ and $h_m = hc$. The salvage value for the manufacturer and retailer is set to zero, that is, $\gamma_m = \gamma_r = 0$, as the effect of positive or negative values will only enhance or diminish the effect of perishability. To simplify the study, we assume that $e_r = 0$, as the margin from lost sales already reflects a penalty cost.

We focus on five key experimental parameters: price sensitivity $b$, variance-to-mean ratio $\tau$, unit production cost $c$, holding cost rate $h$, and expediting cost rate $e \equiv e_m$. The study consists of a randomized set of experiments for each parameter of which the value of a parameter is a realization of a uniform random variable. Specifically, we choose $b \in [2.0, 4.0]$, $\tau \in [2.0, 7.0]$, $c \in [2.0, 4.0]$, $h \in [0.01, 0.04]$, and $e \in [c, 2c]$. In general, the values for each parameter reflect a range broad enough to readily assess model sensitivity. The apparent low values for the holding rate correspond to our assumption of short time-periods, on the order of days, weeks, or at most months, which are typical for most perishables. Given the short time periods, we set a discount factor of $\alpha = 1.0$ across our numerical experiments and solve each using an average cost criterion through value iteration.

We solve a total of 1,000 random scenarios for four model permutations: non-perishable and perishable products, with and without data sharing. A complete grid search over the feasible decision space for order quantity and price at each echelon is conducted to find the optimal decisions. The order quantity decision is restricted to integer values and price to values in $\$0.10$ increments. For each scenario, it is necessary to iterate successive solutions for manufacturer and retailer until the expected ending inventories for both echelons, from one iteration to the next, converge to the same values. Details of the computational algorithm can be found in Appendix B.

4.2. Analysis of the non-perishable case

As indicated by Corollary 1 and Proposition 1, data sharing hurts the retailer in the context of decentralized decision-making and a simple linear wholesale price contract wherein the manufacturer is a Stackelberg leader. This analytical result does not speak to the magnitude of retailer loss, however. Further, although the analytical results hint that the manufacturer is better off, as it
charges a higher price and can better match production with retailer orders, it is unclear whether the manufacturer’s gain can offset the retailer’s loss such that supply chain performance is better off. Although the existing literature suggests that, with exogenous pricing, supply chain performance is always better off with data sharing, our analysis is positioned to extend understanding to the case of endogenous pricing. In what follows, we define *value of information* as the difference in profit from sharing inventory data minus the non-sharing cases.

Overall, our results indicate that with data sharing, a) the retailer is never better off, b) the manufacturer is always better off, and c) the gains by the manufacturer rarely offset the loss for the retailer such that the supply chain is worse off most of the time (98.5%). On average, retailer profit declines by 49.2% and manufacturer profit increases by 19.6%. These aggregate results are reflected in Figure 1, using box-whisker plots, and showing the averages and ranges for the value of information at each echelon and for the supply chain as a whole. The upper and lower bounds of each box denote the upper and lower quantiles for the change in profit. Within each box, the dashed lines denote mean values and the solid lines median values. The whiskers extend each quantile by an additional 150% or the extreme value, whichever is closer to the median. Outliers are omitted from the plot.

![Box-whiskers plots for the value of information in the non-perishable case.](image)

Figure 1 shows that incorporating endogenous pricing reverses the commonly held belief that data sharing is always beneficial, at least at a system level. True, the manufacturer is always better off, but the retailer and supply chain generally suffer as a consequence. A key driver behind Proposition 1, and the reason why the retailer is worse off, is that data sharing enables the manufacturer to increase the wholesale price. Overall, both manufacturer and retailer prices are higher in 99% of the scenarios with data sharing compared to the scenarios without data sharing. The remaining 1% of scenarios reflect no change in the underlying prices. Therefore, we do not observe any instances in
which prices decrease. On average, the manufacturer increases price by 32% and the retailer by only 10%. Due to price sensitive demand, the retailer is unable to fully pass along the manufacturer’s price increase to customers. This phenomenon also lends indirect support to the USDA economic research services’ finding that retailers absorb much of the wholesale price volatility (Short 2020), with the retailer’s price increase being much more measured than that of the manufacturer.

The relative difference in price increase between retailer and manufacturer exerts a major effect on the distribution of profits in the supply chain and signifies a reversal of fortune for the retailer. Without data sharing, retailer profit exceeds that of the manufacturer in 68.5% of the scenarios. There is not a single such case with data sharing. The split in profit without data sharing, 52% for the retailer and 48% for the manufacturer, shifts to 32% and 68%, respectively, with data sharing. Essentially, data sharing enables the manufacturer to extract from the retailer higher rents that the retailer is unable to fully pass on to customers. Higher prices reduce demand (from an average of 10.9 units down to 7.9 units, per period), leaving the retailer worse off.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Factor effects on the value of information for non-perishables.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Δ Retailer Profit</td>
</tr>
<tr>
<td>Intercept</td>
<td>-19.117</td>
</tr>
<tr>
<td>Variance-to-Mean Ratio $\tau$</td>
<td>-5.076</td>
</tr>
<tr>
<td>Production Cost $c$</td>
<td>4.689</td>
</tr>
<tr>
<td>Holding Cost $h$</td>
<td>1.752</td>
</tr>
<tr>
<td>Expediting Cost $e$</td>
<td>0.580</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Scaled estimates (centered by mean and scaled by range/2). All estimates are significant ($p$-value ≤ 0.01) except for shaded values.

As one would expect, the value of information for each echelon and for the supply chain as a whole is sensitive to model parameters. To assess model sensitivity of the parameters and the relative magnitude of their influence on the value of information, we follow Wagner (1995) in conducting a global sensitivity analysis. Global sensitivity analysis involves regressing model parameters (i.e., independent variables) on an outcome measure of interest (i.e., dependent variable), which in our case is the value of information. This approach is used by Souza et al. (2004), Ferguson et al. (2009), and Subramanian et al. (2013), among others. Table 1 reports the results of our regression models. To facilitate comparison and interpretation of effects, we report scaled coefficients and their corresponding $p$-values, where each regressor has been scaled to have a range from -1 to 1. Given our scaled coefficients, the values of the intercept terms correspond exactly to the average change in profit across all 1,000 numerical scenarios.
The values of the coefficients reported in Table 1 generally reinforce our understanding that with data sharing the manufacturer gains at the expense of the retailer. Note that for the retailer, the value of information increases with respect to price sensitivity $b$ and production cost $c$ and decreases with respect to variance-to-mean ratio $\tau$, the converse being true for the manufacturer. The value of information is most sensitive to the demand parameters $b$ and $\tau$, as indicated by the magnitudes of their respective (scaled) coefficients relative to the others. We explore the impact of these two parameters further, in Figure 2, by plotting their corresponding regression lines. Note that the slopes of the lines in Figure 2 (and later in Figures 5 and 6) reflect the actual, not scaled, parameter values.

![Figure 2](image_url)

**Figure 2** Value of information with respect to variance-to-mean ratio $\tau$ (left) and price sensitivity $b$ (right).

The left panel of Figure 2 shows that the manufacturer indeed benefits more as $\tau$, our measure of demand uncertainty, increases. It is intuitive that information, as it explains increasingly higher levels of demand uncertainty, becomes more valuable to the manufacturer. The manufacturer benefits through the elimination of holding and expediting costs; information enables the manufacturer to satisfy retail orders exactly without underage or overage. Additionally, as indicated by Theorem 1, information puts upward pressure on wholesale prices that harms the retailer. This pressure is heightened at high levels of demand uncertainty, wholesale prices being increasingly lower without data sharing. As expected, assessing the impact of the model parameters on wholesale price reveals a positive and statistically significant relationship between $w$ and $\tau$ with data sharing (details not reported). Hence, as demand uncertainty increases, the retailer becomes progressively worse off, and, as a result, supply chain performance declines. This last result runs entirely counter to the premise that sharing data reduces uncertainty and, hence, that supply chain profits should increase through enhanced ability to match supply with demand.
An important managerial implication is that it cannot be taken for granted that if data sharing is beneficial to the manufacturer the system is better off. The above observation shows that if one examines performance from a supply chain level, the reverse can be true; it is better off not to share data. This insight, however, does not mean that managers should shun data sharing in a volatile market. Instead, supply chain parties may adopt alternative contract forms, such as revenue sharing or price guarantees, to ensure that data sharing benefits not only the manufacturer, but also the retailer and the supply chain.

The right panel of Figure 2 shows that higher price sensitivity mitigates the loss suffered by the retailer and the supply chain. Part of the intuition is that higher price sensitivity limits the manufacturer’s ability to raise prices (which would result in a decline in retail sales). The manufacturer is still better off because it can better match production with demand. The retailer and supply chain loss are mitigated, and the manufacturer’s benefit remains mostly intact. Consequently, mechanisms that limit the manufacturer’s ability to raise prices are key levers for ensuring that data sharing benefits the retailer and supply chain. Product perishability is one such mechanism, which we explore in the next section.

4.3. Analysis of the perishable case

We now present the results for the perishable product case, using the insights from our analysis of the non-perishable case, to separately address the impact of endogenous pricing and product perishability on the value of information. We first examine how perishability shifts the benefits or losses incurred by each party due to data sharing. Figure 3 presents, for the case of perishables, the change in expected profit for each echelon and for the supply chain. The results presented in Figure 3 are noticeably different from those for non-perishables (Figure 1). Although overall, and in a preponderance of the cases, data sharing harms the retailer and benefits the manufacturer, either entity may benefit or be harmed, and the same holds for the supply chain as a whole.

The key difference between a perishable and non-perishable product is the influence of product freshness on supply chain performance. In the context of non-perishables, product freshness is irrelevant. In the case of perishables, however, delivering fresher product can reduce spoilage. Measured as the percentage of units produced and delivered in the same period, product freshness increases by an average of 32.6%, and spoilage is in turn reduced by 66.6%. Data sharing is thus highly effective at reducing product waste. Supply chain managers should treat data sharing as a tool for reducing product waste and, potentially, a managerial lever for implementing alternative contract forms to ensure that retailers are not worse off with data sharing.

We find that improvement in product freshness, in addition to reducing spoilage, enables the retailer to increase inventory (20.0%) and thereby improve service levels (1.1%). Even with these
improvements, we find that average profit decreases for both retailer (17.7%) and for the supply chain as a whole (2.9%). For the manufacturer, average profit increases (6.2%), but far less than the average increase (19.6%) observed for the case of non-perishables. It turns out that even with higher levels of retailer inventory and improved service levels, the decrease in spoilage results in a net reduction (17.7%) in retailer orders. This, paradoxically, represents a lost revenue stream as spoiled units at the retailer translate into unit sales for the manufacturer. This result is consistent with findings in Ketzenberg and Ferguson (2008) and Beullens and Ghiami (2022). Note that, here, a reduction in retailer ordering puts downward pressure on wholesale prices, lest manufacturer sales decline further. With data sharing, the average increase in wholesale price is 14.5% in the case of perishables and 18.3% in the case of non-perishables.

Less spoilage due to product freshness, together with a smaller increase in wholesale price, means that the retailer may benefit from data sharing, whereas smaller retail orders combined with a diminished ability to raise prices means that the manufacturer may be harmed by data sharing. That either echelon may be harmed by data sharing indicates that there are conditions under which the retailer will not be willing to share inventory data, or, even if it is willing to do so, the manufacturer may be unwilling to use the data in its decision-making. Moreover, sharing and use of data may not be Pareto improving for both echelons. We explore in Figure 4 the breakdown across scenarios in which the retailer, manufacturer, and supply chain are better or worse off with data sharing. The figure distinguishes cases in which the retailer is better off (top row, +) and worse off (bottom row, -) and the manufacturer is better off (left column, +) and worse off (right column, -). Within this two-by-two table, each cell reports the percentages of the 1,000 scenarios in which the supply chain is better off (left bar, +) and worse off (right bar, -).

A number of interesting findings are observed in Figure 4. First, in only 13.3% of the cases profits improve for both echelons and the supply chain as a whole. Second, the supply chain is always
better off when the retailer is better off or when the manufacturer is worse off. Lastly, in 54% of the cases the manufacturer increases its profit at the expense of the retailer’s profit and that of the supply chain as a whole. Overall, these results indicate that in the vast majority of scenarios data sharing benefits one echelon at the expense of the other. We elucidate the mechanisms that explain these observations by regressing, as in the case for non-perishables, the five model parameters on the changes in retailer, manufacturer, and overall supply chain profit that accrue to data sharing. We report the results of these regressions in Table 2.

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Δ Retailer Profit</th>
<th>Δ Manufacturer Profit</th>
<th>Δ Supply Chain Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-4.054</td>
<td>2.327</td>
<td>-1.727</td>
</tr>
<tr>
<td>Price Sensitivity b</td>
<td>4.229</td>
<td>-1.204</td>
<td>3.025</td>
</tr>
<tr>
<td>Variance-to-Mean Ratio τ</td>
<td>7.499</td>
<td>-2.938</td>
<td>4.561</td>
</tr>
<tr>
<td>Production Cost c</td>
<td>2.168</td>
<td>-0.287</td>
<td>1.881</td>
</tr>
<tr>
<td>Holding Cost h</td>
<td>-0.985</td>
<td>0.412</td>
<td>-0.573</td>
</tr>
<tr>
<td>Expediting Cost e</td>
<td>2.058</td>
<td>-0.881</td>
<td>1.177</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.62</td>
<td>0.52</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Scaled estimates (centered by mean and scaled by range/2). All estimates are significant (p-value ≤ 0.01) except for shaded values.
In Table 2, as expected from the observations gleaned from Figure 4, the signs of the coefficients for retailer profit are opposite those for manufacturer profit and the same as those for supply chain profit. Hence, operating conditions under which information benefits the retailer also benefit the supply chain. The reversal in signs of the coefficients between retailer and manufacturer indicate that, to a large extent, data sharing will benefit one echelon at the expense of the other. These insights are largely the same as those observed for the case of non-perishables, but the magnitudes of the effects differ. Moreover, unlike the case for non-perishables, in many scenarios both echelons increase profitability through data sharing. We do not observe any case for non-perishables in which data sharing is Pareto improving.

For the retailer, the value of information is greatest under high demand uncertainty as indicated by the coefficient for the variance-to-mean ratio $\tau$ in Table 2. This result is opposite that for non-perishables. To highlight this difference, we reproduce Figure 2 for the case of perishables. Although the right panel of Figure 5 is similar to that of Figure 2, the left panels are completely opposite one another. The finding that increased demand uncertainty in the presence of data sharing can in fact hurt the manufacturer is in stark contrast to the literature, which holds that data sharing is more beneficial to manufacturers under high demand uncertainty (Lee et al. 2000). The retailer and the supply chain, on the other hand, benefit from data sharing under high demand uncertainty. Note that this finding is consistent with Condition (c) of §3.5 and supports the expected impact of demand volatility in our myopic model through Proposition 7. The perplexing result that the manufacturer can be harmed under high demand uncertainty can be explained, in part, as follows. High demand uncertainty typically promotes the use of safety stock by the retailer as a hedge,
but higher safety stock leads to more outdating which is also part of manufacturer sales to the retailer. With information sharing, fresher product will have a more significant reduction in waste, than when demand uncertainty is low, and this may result in a net decrease in retail orders to the manufacturer.

The importance of product freshness and its role in determining the value of information for a perishable product can also be seen in the behavior of the manufacturer in response to changes in supply and demand mismatch costs. In absence of data sharing, decreasing holding cost (increasing expediting cost) makes satisfying retailer replenishment orders with inventory as opposed to expediting more (less) profitable. The manufacturer’s use of inventory has a direct effect on the freshness of retailer replenishment and the subsequent level and cost of retailer outdating. Because increasing unit production cost $c$ amplifies the mismatch costs and cost of product outdating, just as with demand uncertainty, we observe a reversal between echelons as the supply and demand mismatch costs change. Changes that increase the freshness of replenishment benefit the retailer to the detriment of the manufacturer, whereas a decrease in freshness benefits the manufacturer at the expense of the retailer. Hence, the value of information to the retailer (manufacturer) is greatest (least) in operating environments with a low level of product freshness. The converse is true as well.

![Figure 6](image.png)

**Figure 6** Value of Information Conditional on Retail Product Freshness without Information Sharing.

The observation that product freshness is a critical determinant of the value of information (as alluded to in Theorem 1 and Lemma A2) and to whom it is valuable is further demonstrated through a set of simple regressions. We regress retail product freshness in absence of data sharing on the change in retailer, manufacturer profit, and overall supply chain profit attributable to data sharing. The three models are statistically significant, explaining approximately 50% of the variance.
in each case. The corresponding regression lines plotted in Figure 6 make clear that as retail product freshness increases (meaning the opportunity to further improve product freshness through data sharing decreases) the value of information declines for the retailer and supply chain and increases for the manufacturer. Conversely, information sharing is only valuable to the retailer when product freshness can be sufficiently improved, consistent with Condition (b) in §3.5, and hence supports the expected impact of product freshness from our myopic model through Propositions 8 and 9.

5. Conclusion
This research explores the impact of data sharing on fresh product supply chain performance. To do so, we study a two-echelon serial supply chain consisting of a manufacturer and a retailer. Key modeling assumptions include stochastic, price sensitive demand, and lost sales. Decision-making is decentralized and the supply chain coordinates through a simple linear wholesale price contract. Both price and ordering policies are endogenous decisions made at each echelon and we employ a game theoretic model in which the manufacturer is a Stackelberg leader.

The research is motivated by a concern with product perishability in retail business and the potential data sharing to improve freshness and reduce waste. A more refined understanding of how data sharing influences the manufacturer and retailer performance is a first step in crafting a mutually beneficial strategy that not only improves both parties’ performance, but also reduces food waste.

Contrary to the existing literature on data sharing under endogenous pricing, we find that when product freshness is considered data sharing does not necessarily always hurt the retailer. In fact, data sharing can be mutually beneficial if product freshness can be significantly improved and market volatility is high. This differs from the case for highly perishable or non-perishable product supply chains, in which data sharing always benefits (hurts) the manufacturer (retailer) because data sharing worsens the double marginalization effect when the manufacturer increases wholesale price as a result of data sharing. An important managerial implication is that inventory data sharing agreements must address a manufacturer’s ability to sufficiently improve product freshness.

When product is perishable, data sharing enables the retailer, through the manufacturer’s ability to deliver fresher product, to significantly reduce product waste, maintain a higher level of inventory, and satisfy more customer demand. Sharing inventory data generally harms the retailer when the manufacturer extracts higher rents in the form of higher wholesale prices, although price increases are considerably lower relative to the case for non-perishable products. Interestingly, reduced spoilage signifies a gross misalignment of incentives for the manufacturer, as units outdated by the retailer constitute part of the manufacturer’s revenue stream.

In addition to yielding financial benefits, data sharing also significantly reduces product spoilage and waste. Although it is difficult to measure precise levels of externality with reduced product
waste, the mere fact that waste currently amounts to more than one billion tons annually should serve as sufficient motivation for managers to devise strategic contracts that ensure a mutually beneficial data sharing agreement. The important lesson here is that data sharing cannot be regarded to be beneficial by default. It is critical that product freshness be sufficiently improved.

A number of interesting research questions invite future study. An important extension is to consider cases in which a manufacturer serves multiple competing retailers. It would be interesting to investigate, for example, whether a subset of competing retailers could be better off sharing inventory information, and whether retailers that did not do so were worse or better off. Another interesting avenue to explore is the opposite setting in which a powerful retailer is the Stackelberg leader. Other forms of information sharing, for example, the manufacturer sharing its inventory information with the retailer, are of interest. Moreover, we only explore the wholesale price contract as a coordinating mechanism. While this is the most prevalent arrangement in industry, other coordinating mechanisms may better and more meaningfully capture the benefits of information sharing and would be an intriguing line of research. A broader, holistic understanding of these different settings would foster better management of the fresh product supply chain, both financially and with respect to food waste reduction. Finally, we note that our models assume that demand is not a function of freshness at the retailer. To a large extent, retailers can control the influence of product freshness on demand (e.g., issuing policies, product positioning, shelving, and gravity wells). Nevertheless, there is evidence that consumer behavior is influenced by product freshness and as such it would be interesting to see how such behavior augments our results.

References


Online Supplements for “Retailer Inventory Data Sharing in a Fresh Product Supply Chain”

This document contains list of proofs, algorithms, as well as some additional analyses and results related to the more general model setup.

Appendix A: Proofs

Proof of Lemma 1. First consider the manufacturer’s problem. Because product is non-perishable, no inventory is disposed at the end of any period. For now, assume that $s \geq i_m$. We will show that this assumption is not needed in the end. Due to this assumption, we have

$$v_m(i_m|w) = \max_{s \geq i_m} \left\{ \pi_m(s|w, i_m) + \alpha_m E \left[ v_m \left( (s - \tilde{q}(w))^+ | w \right) \right] \right\} = \pi_m(s^*|w, i_m) + \alpha_m E \left[ v_m \left( (s^* - \tilde{q}(w))^+ | w \right) \right].$$

Note that if $s^* - \tilde{q}(w^*) > 0$, then it follows from the above assumption that, compared to $(s^* - \tilde{q}(w^*))^+ = 0$, the production cost in period $t+1$ is reduced by amount $c(s^* - \tilde{q}(w^*))$; there is no other change in the profit-to-go, i.e., no units are disposed in any period due to non-perishability and each period interior solution $s^*$ satisfies $s^* \geq i_s = s^* - \tilde{q}(w^*)$. Therefore

$$v_m(i_m|w) = \pi_m(s^*|w, i_m) + \alpha_m c E(s^* - \tilde{q}(w^*))^+ + \alpha_m v_m(0|w)$$

$$= \max_{s \geq i_m} \left\{ \pi_m(s|w, i_m) + \alpha_m c E(s - \tilde{q}(w))^+ \right\} + \alpha_m v_m(0|w)$$

$$= \max_{s \geq i_m} \left\{ \tilde{\pi}_m(s|w, i_m) \right\} + \alpha_m v_m(0|w),$$

where $\tilde{\pi}_m(s|w, i_m) = wE\tilde{q}(w) - c(s - i_m) - \alpha_m E(\tilde{q}(w) - s)^+ - (h_m - \alpha_m c) E(s - \tilde{q}(w))^+$. The manufacturer’s infinite-horizon optimization problem therefore reduces to a sequence of single-period optimization problems. Since demand is stationary, it follows that the manufacturer’s optimal policy is independent of time and inventory, i.e.,

$$(w^*, s^*) = \arg \max_{w, s \geq i_m} \left\{ wE\tilde{q}(w) - c(s - i_m) - \epsilon E(\tilde{q}(w) - s)^+ - (h_m - \alpha_m c) E(s - \tilde{q}(w))^+ \right\}$$

$$= \arg \max_{w, s} \left\{ wE\tilde{q}(w) - cs - \epsilon_m E(\tilde{q}(w) - s)^+ - (h_m - \alpha_m c) E(s - \tilde{q}(w))^+ \right\}.$$

Hence, the assumption that $s \geq i_m$ holds naturally.

Next consider the retailer. Instead of $p$ and $q$, here it is more convenient to treat the retailer’s decisions as choosing mean demand $d$ and stocking factor $\theta : q = d(p, \theta)$. Following similar logic as in the manufacturer’s case, assume for now that $d + \theta - i_r \geq 0$ (i.e., $q \geq 0$). Due to this assumption,

$$v_r(i_r) = \max_{d, \theta \geq i_r - d} \left\{ \pi_r(d, \theta|i_r) + \alpha_r E \left[ v_r \left( (\theta - \tilde{\epsilon})^+ \right) \right] \right\} = \pi_r(d^*, \theta^*|i_r) + \alpha_r E \left[ v_r \left( (\theta^* - \tilde{\epsilon})^+ \right) \right].$$

Note that if $\theta^* - \tilde{\epsilon} > 0$, then it follows from the above assumption that, compared to $(\theta^* - \tilde{\epsilon})^+ = 0$, the purchase cost in any period is reduced by amount $w(\theta^* - \tilde{\epsilon})$; there is no other change in the profit-to-go (i.e., no units are disposed in period $t$ due to non-perishability and the interior solution $d^* + \theta^*$ satisfies $d^* + \theta^* \geq i_r = \theta^* - \tilde{\epsilon}$). Therefore

$$v_r(i_r) = \pi_r(d^*, \theta^*|i_r) + \alpha_r wE(\theta^* - \tilde{\epsilon}) + \alpha_r v_r(0)$$

$$= \max_{d, \theta \geq i_r - d} \left\{ \pi_r(d, \theta|i_r) + \alpha_r wE(\theta - \tilde{\epsilon}) \right\} + \alpha_r v_r(0) = \max_{d, \theta \geq i_r - d} \left\{ \tilde{\pi}_r(d, \theta|i_r) \right\} + \alpha_r v_r(0)$$
where \( \hat{\pi}_r (d, \theta | i_r) = p_d (d + E \min \{ \hat{\varepsilon}, \theta \}) - w (d + \theta - i_r) - (h_r - \alpha_r w) E(\theta - \hat{\varepsilon}) + e_r E (d (p, \hat{\varepsilon}) - q)^+ \). The retailer’s infinite-horizon optimization problem therefore reduces to a sequence of single-period optimization problems. Suppose now that the manufacturer uses a stationary wholesale price, then it follows that the retailer’s optimal policy is independent of time and inventory, i.e.,

\[
(d^*, \theta^*) = \arg \max_{d, \theta \geq i_r - d} \left\{ p_d (d + E \min \{ \hat{\varepsilon}, \theta \}) - w (d + \theta - i_r) - (h_r - \alpha_r w) E(\theta - \hat{\varepsilon})^+ \right\}.
\]

Thus, the manufacturer observes a stationary retailer demand process for which a policy with stationary wholesale price is optimal. This means that the stationary fixed wholesale price policy is mutually reinforcing in this two-level supply chain. Furthermore, the above assumption of \( d + \theta - i_r \geq 0 \) holds naturally. □

**Proof of Lemma 2.** (a) The lemma statement follows from applying the first order condition to (3). (b) The lemma statement follows from the fact that the retailer’s profit function is given by \( (p - w)E[\min(q, d(p) + \epsilon)] \), which can be simplified to

\[
(p - w)E[\min(q, d(p) + \epsilon)] = (p - w)d(p) + (p - w) \int_{\epsilon < q - d(p)} (q - d(p) - \epsilon) dF(\epsilon).
\]

By the definition of stocking factor \( \theta \), the above expression can be further simplified as

\[
(p - w)E[\min(q, d(p) + \epsilon)] = (p - w)d(p) + (p - w) \int_{\epsilon > q - d(p)} (\theta - \epsilon) dF(\epsilon) = (p - w)d(p) - (p - w)E[(\epsilon - \theta)^+].
\]

Observe that \( p^* \) solves the first part of the revenue function \( (p - w)d(p) \), that is, \( p^* = \arg \max_{p} (p - w)d(p) \). Applying \( d(p) = a - bp \) obtains the lemma statement that \( p^*(\theta) = p^* - E [(\epsilon - \theta)^+] \). (c) The inequality follows from \( b > 0 \) and \( E(\epsilon - \theta)^+ \geq 0 \). □

**Proof of Proposition 1.** Let \( p^*(w) \) denote the retailer’s optimal price and \( \theta^*(w) \) denote the optimal stocking factor. The retailer’s optimal base-stock level is \( d (p^*(w), \theta^*(w)) \). The random retailer order quantity from the perspective of the manufacturer is \( \tilde{q}(w) = d (p^*(w), \theta^*(w)) - \tilde{i}_r \). The retailer’s random inventory at optimal price \( p^*(w) \) is \( \tilde{i}_r = (d(p^*(w), \theta^*(w))) - d(p^*(w), \hat{\varepsilon})^+ \). Therefore,

\[
\tilde{q}(w) = q(w, \hat{\varepsilon}) := \min \{ d(p^*(w), \hat{\varepsilon}), d(p^*(w), \theta^*(w)) \} = d(p^*(w)) + \min \{ \hat{\varepsilon}, \theta^*(w) \}.
\]

The manufacturer’s production quantity is \( (s - i_m) = q(w, \beta) \) such that its profit function can be simplified to \( \pi_m(w, \beta) = (w - c) \tilde{q}(w) - L(w, \beta) \), where

\[
L(w, \beta) = (w + e_m - c) E (q(w, \hat{\varepsilon}) - q(w, \beta))^+ + (c + h_m) E(q(w, \beta) - q(w, \hat{\varepsilon}))^+.
\]

To evaluate the impact of data sharing on optimal wholesale price, we examine the cases of with and without data sharing. If the retailer shares data, the manufacturer’s demand function is \( q(w, \epsilon) = \min \{ d(p^*(w), \epsilon), d(p^*(w), \theta^*(w)) \} \), which is also the manufacturer’s optimal production quantity. Thus, the manufacturer’s profit function is \( \pi_m(w) = (w - c) q(w, \epsilon) \). The optimal stationary price, denoted as \( w^o \), is given by \( w^o = \arg \max_{w} E [(w - c) q(w, \hat{\varepsilon})] \). Letting \( \tilde{q}(w) \) denote the expected retailer order given price \( w \), we have

\[
\tilde{q}(w) = E[q(w, \hat{\varepsilon})] = E \min \{ d(p^*(w), \hat{\varepsilon}), d(p^*(w), \theta^*(w)) \} = d(p^*(w)) + E \min \{ \hat{\varepsilon}, \theta^*(w) \}
\]
From Lemma 2, it follows that the manufacturer’s optimal wholesale price satisfies
\[ \frac{w^o - c}{w^o} = \left( -\frac{w^o \bar{q}^o}{\bar{q}(w^o)} \right)^{-1}. \]

If the retailer does not share data, the manufacturer sees random demand, where the random term min \{\tilde{\epsilon}, \theta^*(w)\} stochastically decreases in \( w \) (Raz and Proteus 2006). Leveraging Lemma 2, we can characterize the effect of data sharing on the manufacturer’s optimal wholesale price.

First consider the case where \( \beta < \theta^*(w) \). Adopting similar approach as that in Raz and Proteus (2006), their result for pure additive demand noise can be extended to our mixed demand noise case. Let the marginal expected cost of a unit sold be denoted as
\[ C(w, \beta) = \frac{\partial q(w, \beta)}{\partial w} \frac{\partial w}{\partial \beta}, \]
where \( \bar{q}(w, \beta) = E_q q(w, \beta) \). Observe that
\[ \frac{\partial q(w, \beta)}{\partial w} = d'(p^*(w)) \frac{dp^*(w)}{dw} + \frac{d\beta^*(w)}{dw} P(\beta > \beta^*(w)) = d'(p^*(w)) \frac{dp^*(w)}{dw}, \]
where the last equality follows from the fact that \( \beta \leq \beta^*(w) \). Similarly,
\[ \frac{\partial q(w, \beta)}{\partial \beta} = d'(p^*(w)) \frac{dp^*(w)}{dw} = \frac{\partial q(w, \beta)}{\partial w}. \]
This implies that \( C(w, \beta^*(w)) \) is not affected by a mean-preserving spread in \( \tilde{\epsilon} \). In contrast, let the elasticity of the average quantity ordered by the retailer with respect to \( w \) be denoted as
\[ \eta = \frac{\partial \bar{q}(w, \beta)}{\partial w} \frac{w}{\bar{q}(w, \beta)}. \]
We have
\[ \eta = d'(p^*(w)) \frac{dp^*(w)}{dw} \frac{w}{E_q q(w, \beta)}. \]
Observe that a mean-preserving spread of \( \tilde{\epsilon} \) influences \( \eta \) by causing \( E_q q(w, \beta) \) to decrease (as more demand is cut off by \( \beta \)). The magnitude of \( \eta \) therefore increases, leading to a reduction in \( w \). Note that \( \eta \leq 0 \) because \( dp^*(w)/dw \geq 0 \) by Lemma 2. The proposition statement then follows by applying Equation (6) in Salinger and Ampudia (2011), i.e., \( w(1 + 1/\eta) = C(w, \beta) \).

Next consider the case where \( \beta = \theta^*(w) \). Observe that
\[ \frac{\partial q(w, \beta)}{\partial w} = \frac{\partial q(w, \theta^*(w))}{\partial w} = d'(p^*(w)) \frac{dp^*(w)}{dw} + \frac{d\theta^*(w)}{dw}, \]
which is independent of \( \tilde{\epsilon} \), and
\[ \frac{\partial q(w, \beta)}{\partial \beta} = \frac{\partial q(w, \theta^*(w))}{\partial \beta} = d'(p^*(w)) \frac{dp^*(w)}{dw} + \frac{d\theta^*(w)}{dw} P(\tilde{\epsilon} > \theta^*(w)) \]
\[ - \theta^*(w) f(\theta^*(w)) \frac{d\theta^*(w)}{dw} + \theta^*(w) f(\theta^*(w)) \frac{d\theta^*(w)}{dw} \]
\[ = d'(p^*(w)) \frac{dp^*(w)}{dw} + \frac{d\theta^*(w)}{dw} P(\tilde{\epsilon} > \theta^*(w)) \]
which decreases in a mean-preserving spread of \( \tilde{\epsilon} \). This implies that \( C(w, \beta) = C(w, \theta^*(w)) \) decreases in a mean-preserving spread of \( \tilde{\epsilon} \). In contrast,
\[ \eta = \left( d'(p^*(w)) \frac{dp^*(w)}{dw} + \frac{d\theta^*(w)}{dw} P(\tilde{\epsilon} > \theta^*(w)) \right) \frac{w}{E_q q(w, \theta^*(w))}. \]
Observe that a mean-preserving spread of $\epsilon$ causes $E_q(w, \theta^*(w))$ to decrease (as more demand is cut off by $\theta^*(w)$) and $P(\epsilon > \theta^*(w))$ to increase for the same reason. Note that the generalized Lerner relationship (equation (6) in Salinger and Ampudia (2011)) can be re-written as

$$p \left( \frac{\partial \bar{q}(w, \theta^*(w))}{\partial w} + \frac{\partial \bar{q}(w, \theta^*(w))}{\partial w} \right) = c \frac{\partial \bar{q}(w, \theta^*(w))}{\partial w}. \quad (A-1)$$

By the above observations, we know that the LHS of (A-1) decreases in a mean-preserving spread of $\epsilon$ but the RHS is unaffected. It follows that $w$ decreases in a mean-preserving spread of $\epsilon$. Note that $dp^*(w)/dw \geq 0$ and $d\theta^*(w)/dw \leq 0$ by Lemma 2. □

**Proof of Proposition 2.** We prove the proposition statements by analyzing the retailer’s pricing decision in each region. To simplify notation, let $G(p, y) = G(p, y) + A(p, y)$.

**Region (3).** We have $p \leq a + \epsilon_L - y \Rightarrow a + \epsilon_L - p \geq y$. Hence,

$$G(p, y) = p(p_L (a + \epsilon_L - p) + \rho_H) - (h_r - \zeta w)p_L(y - (a + \epsilon_L - p)) - e_r \rho_H((a + \epsilon_L - p) - y).$$

It follows that $G'(p, y) = y + e_r > 0$. Therefore the retailer’s objective is always increasing in $p$ when $p \leq a + \epsilon_L - y$, suggesting that the optimal price cannot lie in region (3).

**Region (5).** We have $a + \epsilon_L - y < p \leq a + \epsilon_L - y \Rightarrow a + \epsilon_L - p < y \leq a + \epsilon_H - p$. Hence,

$$G(p, y) = p(p_L(a + \epsilon_L - p) + \rho_H) - (h_r - \zeta w)p_L(y - (a + \epsilon_L - p)) - e_r \rho_H((a + \epsilon_L - p) - y).$$

It follows that $G'(p, y) = p_L(a + \epsilon_L - p) + \rho_H - p_L(h_r - \zeta w) + \rho_H e_r$, and $G''(p, y) = -2p_L < 0$. Hence the retailer’s objective is concave in $p$ when $p$ is in region (5), and the optimal interior price $p^*$ satisfies

$$G'(p^*, y) = 0 \Rightarrow p^* = \frac{1}{2} \left( a + \epsilon_L - (h_r - \zeta w) + \frac{\rho_H}{\rho_L}(y + e_r) \right).$$

Note that if $\rho_L = 0$ then the optimal $p^*$ cannot be in region (5) since $G(p, y)$ strictly increases in $p$. We therefore assume that $\rho_L > 0$ to avoid the degenerate case. In addition, $p^*$ is interior only if $a + \epsilon_L - y < p^* \leq a + \epsilon_H - y$. It follows that $p^*$ is interior if

$$p^* \geq a + \epsilon_L - y \Rightarrow y \geq \frac{a + \epsilon_L + (h_r - \zeta w) - \frac{a + \epsilon_H - e}{\rho_L}}{2 + \frac{\rho_H}{\rho_L}}; \quad (A-2)$$

$$p^* \leq a + \epsilon_H - y \Rightarrow y \leq \frac{a + 2\epsilon_H - \epsilon_L + (h_r - \zeta w) - \frac{a + \epsilon_H - e}{\rho_L}}{2 + \frac{\rho_H}{\rho_L}}. \quad (A-3)$$

**Region (6).** We have $p > a + \epsilon_L - y \Rightarrow y > a + \epsilon_H - p$. Hence,

$$G(p, y) = p(p_L(a + \epsilon_L - p) + \rho_H(a + \epsilon_H - p)) - (h_r - \zeta w)\rho_L(y -(a + \epsilon_L - p)) + \rho_H(y - (a + \epsilon_H - p)))$$

$$= p(a + \epsilon - p) - (h_r - \zeta w)(y - (a + \epsilon - p)).$$

It follows that $G'(p, y) = a + \epsilon - 2p - h$ and $G''(p, y) = -2 < 0$. Hence the retailer’s objective is concave in $p$ when $p$ is in region (6), and the optimal interior price $p^*$ satisfies

$$G'(p^*, y) = 0 \Rightarrow p^* = \frac{1}{2} \left(a + \epsilon - (h_r - \zeta w)\right).$$

Note that $p^*$ is interior only if $p^* \geq a + \epsilon_H - y$. It follows that $p^*$ is interior if

$$p^* \geq a + \epsilon_H - y \Rightarrow y \geq \frac{1}{2} \left(a + 2\epsilon_H - \epsilon + (h_r - \zeta w)\right). \quad (A-4)$$

This completes the proof of Proposition 2. □
Corollary A1 The optimal retail price can be uniquely found by partitioning the inventory position $y$ into four regions:

1. $y \leq \frac{a + \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L} \epsilon_r}{2 + \frac{\rho H}{\rho L}}$,
2. $\frac{a + \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L} \epsilon_r}{2 + \frac{\rho H}{\rho L}} < y \leq \frac{a + 2 \epsilon_H - \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L} \epsilon_r}{2 + \frac{\rho H}{\rho L}}$,
3. $\frac{a + 2 \epsilon_H - \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L} \epsilon_r}{2 + \frac{\rho H}{\rho L}} < y \leq \frac{1}{2} \left( a + 2 \epsilon_H - \bar{\epsilon} + (h_r - \zeta w) \right)$, and
4. $y > \frac{1}{2} \left( a + \bar{\epsilon} - (h_r - \zeta w) \right)$.

The corresponding optimal prices in these regions are

1. $p^* = a + \epsilon_L - y$,
2. $p^* = \frac{1}{2} \left( a + \epsilon_L - (h_r - \zeta w) + \frac{\rho H}{\rho L}(y + \epsilon_r) \right)$,
3. $p^* = a + \epsilon_H - y$, and
4. $p^* = \frac{1}{2} \left( a + \bar{\epsilon} - (h_r - \zeta w) \right)$.

Proof of Corollary A1. The corollary statements follow from conditions (A-2), (A-3), and (A-4) in the proof of Proposition 2. These conditions partition the inventory position $y$ into four regions as in the corollary statement. In addition, the prices can be identified by these three conditions as well since they are either bounded by the regions or interior optimal solutions identified in Proposition 2.

Proof of Proposition 3. We analyze the retailer’s optimal stocking decision $y^*$ by the four different regions identified in Corollary A1.

Region (1). In this region, we have $p^* = a + \epsilon_L - y$. Recall that the retailer’s objective function can be written as $\pi_r(y, p^*|i_r) = -w(y - x) + G(p^*, y) + A(p^*, y)$. Note that $a + \epsilon_L - p^* = y$ and $a + \epsilon_H - p^* = \epsilon_H - \epsilon_L + y > y$. It follows that

\[ G(p^*, y) = (a + \epsilon_L - y) y - \epsilon_r \rho_H \left( (a + \epsilon_H - p^*) - y \right) = (a + \epsilon_L - y) y - \epsilon_r \rho_H (\epsilon_H - \epsilon_L). \]

Hence,

\[ \pi_r(y, p^*|i_r) = -w(y - x) + (a + \epsilon_L - y) y - \epsilon_r \rho_H (\epsilon_H - \epsilon_L), \tag{A-5} \]

which is clearly concave in $y$ and the optimal interior $y^*$ is given by $y^* = (a + \epsilon_L - w)/2$.

Region (2). In this region, we have $p^* = \frac{1}{2} \left( a + \epsilon_L - (h_r - \zeta w) + \frac{\rho H}{\rho L}(y + \epsilon_r) \right)$. Note that $a + \epsilon_L - p^* = \frac{1}{2} \left( a + \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L}(y + \epsilon_r) \right)$ and $a + \epsilon_H - p^* = \frac{1}{2} \left( a + 2 \epsilon_H - \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L}(y + \epsilon_r) - y \right)$. It follows that

\[
G(p^*, y) = \frac{1}{2} \left( a + \epsilon_L - (h_r - \zeta w) + \frac{\rho H}{\rho L}(y + \epsilon_r) \right) \left( \rho_L \frac{1}{2} \left( a + \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L}(y + \epsilon_r) \right) + \rho_H y \right) - (h_r - \zeta w) \rho_L \left( y - \frac{1}{2} \left( a + \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L}(y + \epsilon_r) \right) \right) - e_r \rho_H \left( \frac{1}{2} \left( a + 2 \epsilon_H - \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L}(y + \epsilon_r) - y \right) - y \right).
\]

Observe that

\[
\frac{\partial \pi_r(y, p^*|i_r)}{\partial y} = -w + \frac{1}{2} \rho_H \rho_L \left( \rho_L \frac{1}{2} \left( a + \epsilon_L + (h_r - \zeta w) - \frac{\rho H}{\rho L}(y + \epsilon_r) \right) + \rho_H y \right) + \frac{1}{2} \left( a + \epsilon_L - (h_r - \zeta w) + \frac{\rho H}{\rho L}(y + \epsilon_r) \right) \frac{1}{2} \rho_H - (h_r - \zeta w) \rho_L \left( 1 + \frac{1}{2} \rho_H \right) - e_r \rho_H \left( \frac{1}{2} \left( \frac{\rho H}{\rho L} - 1 \right) - 1 \right).
\]
Observe that $G$ is clearly concave in $y$, and therefore the optimal $y^*$ cannot be in this region.

Region (3). In this region, we have $p^* = a + \epsilon_H - y$. Note that $a + \epsilon_H - p^* = y$ and $a + \epsilon_L - p^* = y - (\epsilon_H - \epsilon_L) < y$. It follows that

$$G(p^*, y) = (a + \epsilon_H - y) (\rho_L (y - (\epsilon_H - \epsilon_L)) + \rho_H y) - (h_r - \zeta w) \rho_L (\epsilon_H - \epsilon_L).$$

Observe that

$$\frac{\partial \pi_r(y, p^*|i_r)}{\partial y} = -w - (\rho_L (y - (\epsilon_H - \epsilon_L)) + \rho_H y) + (a + \epsilon_H - y) = -w - (y - \rho_L (\epsilon_H - \epsilon_L)) + (a + \epsilon_H - y).$$

The objective function is clearly concave in $y$, and the optimal $y^*$ is given by

$$y^* = \frac{1}{2} (a + \epsilon_H + \rho_L (\epsilon_H - \epsilon_L) - w).$$

Region (4). In this region, we have $p^* = \frac{1}{2} (a + \bar{e} - (h_r - \zeta w))$. Note that $a + \epsilon_H - p^* = a + \epsilon_L - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) < y$ and $a + \epsilon_H - p^* = a + \epsilon_H - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) < y$. It follows that

$$G(p^*, y) = \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \left( \rho_L \left( a + \epsilon_L - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \right) + \rho_H \left( a + \epsilon_H - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \right) \right)$$

$$- h \left( \rho_L \left( y - \left( a + \epsilon_L - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \right) \right) + \rho_H \left( y - \left( a + \epsilon_H - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \right) \right) \right)$$

$$= \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \left( a + \epsilon_L - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \right) - (h_r - \zeta w) \left( y - \left( a + \bar{e} - \frac{1}{2} (a + \bar{e} - (h_r - \zeta w)) \right) \right).$$

Observe that $\frac{\partial \pi_r(y, p^*|i_r)}{\partial y} = -w - (h_r - \zeta w) < 0$, suggesting that the optimal $y$ cannot be in region (4).

Combining the above four regions, the optimal stocking decision $y^*$ can only lie in region (1) or region (3).

To sum up, the possible two regions have the following solutions.

Region (1).

$$y^* = \frac{a + \epsilon_L - w}{2}, \quad p^* = a + \epsilon_L - y^* = \frac{a + \epsilon_L + w}{2}.$$ 

It follows that

$$\pi_r^* (i_r) = \pi_r (y^*, p^*|i_r) = -w(y^* - x) + p^* y^* - e(a + \bar{e} - p^* - y^*)$$

$$= -w \left( \frac{a + \epsilon_L - w}{2} - x \right) + \frac{a + \epsilon_L + w}{2} \frac{a + \epsilon_L - w}{2} - e_r (a + \bar{e} - (a + \epsilon_L))$$

$$= wi_r + \left( \frac{a + \epsilon_L - w}{2} \right)^2 - e_r (\bar{e} - \epsilon_L).$$

Region (3).

$$y^* = \frac{1}{2} (a + \epsilon_H + \rho_L (\epsilon_H - \epsilon_L) - w), \quad p^* = a + \epsilon_H - y^* = \frac{1}{2} (a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) + w).$$
Note that \( a + \epsilon_L - p^* = y - (\epsilon_H - \epsilon_L) < y \). It follows that
\[
\pi^*_r(i_r) = \pi_r(y^*, p^* | i_r) = -w(y^* - x) + p^*(\rho_L(a + \epsilon_L - p^*) + \rho_H y^*) - (h_r - \zeta w) \rho_L(y^* - (a + \epsilon_L - p^*)) \\
= -w \left( \frac{1}{2} (a + \epsilon_H + \rho_L(\epsilon_H - \epsilon_L) - w) - x \right) \\
+ \frac{1}{2} (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) + w) \left( \rho_L(y - (\epsilon_H - \epsilon_L)) + \rho_H \frac{1}{2} (a + \epsilon_H + \rho_L(\epsilon_H - \epsilon_L) - w) \right) - (h_r - \zeta w) \rho_L(\epsilon_H - \epsilon_L) \\
= wi_r - \frac{1}{2} w (a + \epsilon_H + \rho_L(\epsilon_H - \epsilon_L) - w) \\
+ \frac{1}{2} (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) + w) \frac{1}{2} (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w) - (h_r - \zeta w) \rho_L(\epsilon_H - \epsilon_L) \\
= wi_r + \left( \frac{a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w}{2} \right)^2 - (h_r + (1 - \zeta) w) \rho_L(\epsilon_H - \epsilon_L).
\]

This completes the proof of Proposition 3. □

**Proof of Proposition 4.** Observe that the expected profit in region (1) is higher than that in region (3) (as identified in the proof of Proposition 3) if and only if
\[
\left( \frac{a + \epsilon_L - w}{2} \right)^2 - e_r (\bar{\epsilon} - \epsilon_L) \geq \left( \frac{a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w}{2} \right)^2 - (h_r + (1 - \zeta) w) \rho_L(\epsilon_H - \epsilon_L) \\
\Rightarrow \left( \frac{a + \epsilon_L - w}{2} \right)^2 - \left( \frac{a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w}{2} \right)^2 \geq (\rho_L e_r - \rho_L (h_r + (1 - \zeta) w)) (\epsilon_H - \epsilon_L) \\
\Rightarrow - (2a - 2w + \epsilon_L + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L)) \rho_H(\epsilon_H - \epsilon_L) \geq 4 (\rho_H e_r - \rho_L (h_r + (1 - \zeta) w)) (\epsilon_H - \epsilon_L) \\
\Rightarrow - (2a - 2w + \epsilon_L + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L)) \geq 4 \left( e - \frac{\rho_L}{\rho_H} (h_r + (1 - \zeta) w) \right) \\
\Rightarrow w \geq a + \bar{\epsilon} + \epsilon_L + 2 \left( e - \frac{\rho_L}{\rho_H} (h_r + (1 - \zeta) w) \right) \\
\Rightarrow w \geq \frac{\rho_H (a + \frac{\epsilon + \epsilon_L}{2}) + 2 (\rho_H e_r - \rho_L h_r)}{\rho_H + 2 (1 - \zeta) \rho_L}.
\]
The proposition statement therefore follows directly. □

**Proof of Proposition 5.** First observe that
\[
\pi^*_m(w^* | i_r) - \pi^*_m(w_r^* | i_r) = \frac{1}{2} \left( \frac{1}{2} (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - c) - i_r \right)^2 - \frac{1}{2} \left( \frac{1}{2} (a + \epsilon_L - c) - i_r \right)^2 \\
= \frac{1}{4} \left( a + \frac{1}{2} (\epsilon_H + \epsilon_L - \rho_L(\epsilon_H - \epsilon_L)) - c - 2i_r \right) \rho_H (\epsilon_H - \epsilon_L) \\
\geq 0,
\]
where the inequality follows from the fact that \( i_r \leq \frac{1}{2} (a + \epsilon_L - c) \), because otherwise the retailer order quantity is zero under strategy point \( \mathcal{L} \). Note that if \( w_{c^*} \leq \bar{w} \) then \( w_{c^*} \) cannot be an interior solution, suggesting that \( w_{c^*} = \bar{w} \). However, for any \( w \leq \bar{w} \), the retailer’s optimal stocking decision is given by strategy point \( \mathcal{H} \) and so the manufacturer’s optimal profit is given by \( \pi^*_m(w^* | i_r) \). In addition, the optimal profit under strategy point \( \mathcal{H} \) dominates that under strategy point \( \mathcal{L} \). The manufacturer’s optimal wholesale pricing decision therefore must be the smaller of \( \bar{w} \) and \( w_{c^*} \). Now, \( w_{c^*} \leq \bar{w} \) implies that
\[
\frac{a + \epsilon_L + c}{2} - i_r \leq \frac{\rho_H (a + \frac{1}{2} (\bar{\epsilon} + \epsilon_L)) + 2 (\rho_H e_r - \rho_L h_r)}{\rho_H + 2 \rho_L (1 - \zeta)}
\]
The proposition statement then follows. □
Proof of Proposition 6. Define \( L(\zeta) = \pi_m(w^e) - \pi_m(\bar{w}(\zeta)) \). Observe that \( \bar{w}(\zeta) \) increases in \( \zeta \), and hence if \( w^e > \bar{w}(\zeta) \) at \( \tilde{\zeta}_a \), \( \zeta_a \) satisfies \( L(\tilde{\zeta}_a) = 0 \), then for any \( \zeta \leq \tilde{\zeta}_a \) we must have \( w^* = w^e \). Consequently, the retailer orders just enough to cover the demand for low market realizations and pay for penalty for high market realizations. Therefore the retailer incurs no leftovers and hence its starting inventory level is always zero. The manufacturer knows the retailer’s starting inventory level is 0 and hence can ship fresh product to the retailer. \( \Box \)

Proof of Proposition 7. First observe that \( \bar{w}(\zeta) \) increases in \( \zeta \), and because \( \zeta_a < \zeta^* \), it follows that if \( w^*_a = \bar{w}(\zeta_a) \) then we must have \( w^*_a = \bar{w}(\zeta_a) \). Further, it is straightforward to note that the manufacturer is strictly better off with inventory data sharing since the interior optimal wholesale price is greater than \( \bar{w}(\zeta) \). Therefore, it remains to examine the impact of inventory data sharing on the retailer’s expected profit. Since \( \bar{w}(\zeta) \) is a function of \( \zeta \), it is suffice to examine the impact of freshness parameter \( \zeta \) on the retailer’s performance. Observe that

\[
\frac{\partial \pi_r}{\partial \zeta} = \frac{\partial \pi_r}{\partial w} \frac{\partial w}{\partial \zeta} + \frac{\partial \pi_r}{\partial \zeta} = \left\{ i_r - \frac{a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w}{2} - (1 - \zeta)\rho_L(\epsilon_H - \epsilon_L) \right\} \frac{\partial w}{\partial \zeta} + \rho_L(\epsilon_H - \epsilon_L) \tag{A-6}
\]

Notice that

\[
\frac{\partial w}{\partial \zeta} = \frac{\rho_H \left( a + \frac{1}{2}(\epsilon_H + \epsilon_L - \rho_L(\epsilon_H - \epsilon_L)) + 2(\rho_H \epsilon_r - \rho_L h_r) \right)}{(\rho_H + 2(1 - \zeta)\rho_L)^2} \rho_L(\epsilon_H - \epsilon_L) \tag{A-7}
\]

and

\[
w\rho_L(\epsilon_H - \epsilon_L) = \frac{\rho_H \left( a + \frac{1}{2}(\epsilon_H + \epsilon_L - \rho_L(\epsilon_H - \epsilon_L)) + 2(\rho_H \epsilon_r - \rho_L h_r) \right)}{(\rho_H + 2(1 - \zeta)\rho_L)^2} \rho_L(\epsilon_H - \epsilon_L) (\rho_H + 2(1 - \zeta)\rho_L) \tag{A-8}
\]

Substituting (A-7) and (A-8) back into (A-6), and defining

\[
A = \frac{\rho_H \left( a + \frac{1}{2}(\epsilon_H + \epsilon_L - \rho_L(\epsilon_H - \epsilon_L)) + 2(\rho_H \epsilon_r - \rho_L h_r) \right)}{(\rho_H + 2(1 - \zeta)\rho_L)^2} \rho_L,
\]

we have

\[
\frac{\partial \pi_r}{\partial \zeta} = A \left\{ 2i_r - (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w) - 2(1 - \zeta)\rho_L(\epsilon_H - \epsilon_L) + (\epsilon_H - \epsilon_L)(\rho_H + 2\rho_L(1 - \zeta)) \right\}
\]

\[
= A \left\{ (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w) + 2i_r + (\epsilon_H - \epsilon_L)(1 - 2(1 - \zeta)\rho_L + \rho_H + 2\rho_L(1 - \zeta)) \right\}
\]

\[
= A \left\{ (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w) + 2i_r + (\epsilon_H - \epsilon_L)\rho_H \right\}.
\]

Observe that the above expression holds for any given \( i_r \). Now, since the retailer adopts strategy \( H \), its starting inventory level \( i_r \) in the steady state follows a Bernoulli distribution: \( x = 0 \) with probability \( \rho_H \) and \( x = \epsilon_H - \epsilon_L \) with probability \( \rho_L \). Therefore,

\[
\frac{\partial E_x \pi_r}{\partial \zeta} = A \left\{ (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w) + 2\rho_L(\epsilon_H - \epsilon_L) + (\epsilon_H - \epsilon_L)\rho_H \right\}
\]

\[
= A \left\{ (a + \epsilon_H - \rho_L(\epsilon_H - \epsilon_L) - w) + (\epsilon_H - \epsilon_L)(1 + \rho_L) \right\}.
\]

Observe that $\partial E_s \pi_r / \partial \zeta$ is increasing in $(\epsilon_H - \epsilon_L)$ and that $\partial E_s \pi_r / \partial \zeta < 0$ if $(\epsilon_H - \epsilon_L) = 0$. It therefore follows that there exists a unique $\Delta = \epsilon_H - \epsilon_L$ such that $\partial E_s \pi_r / \partial \zeta$ is positive. This concludes the proposition statement. □

Proof of Corollary 1. For highly perishable product, the result follows from Proposition 6. For non-perishable products, $\zeta$ is constant irrespective of whether the retailer share inventory data. As such, the wholesale price remains the same and hence the retailer cannot benefit from data sharing. □

Proof of Proposition 8. The manufacturer is better off can be seen by the fact that its optimal wholesale price is constrained by $\bar{w}(\zeta_n)$ when the retailer does not share its inventory data. In contrast, the optimal price is interior when the retailer shares inventory data. The manufacturer’s objective function is therefore increasing at $\bar{w}(\zeta)$. Now consider the retailer’s profit. Before sharing inventory data, the manufacturer’s expected profit is fixed at $\bar{w}$ and hence the retailer’s expected profit is given by

$$\pi_{r,n} = \bar{w} E[i_r] + \left( \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) - \bar{w}}{2} \right)^2 - (h_r + (1 - \zeta_n)\bar{w}) \rho_L (\epsilon_H - \epsilon_L).$$

After sharing inventory data, the retailer’s profit is

$$\pi_{r,s} = \mathbb{E} \left[ w^{H(i_r)} i_r + \left( \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) - w^{H(i_r)}}{2} \right)^2 - (h_r + (1 - \zeta_s)w^{H(i_r)}) \rho_L (\epsilon_H - \epsilon_L) \right].$$

Observe that $i_r$ is a Bernoulli random variable, where $x = 0$ with probability $\rho_H$ and $x = (\epsilon_H - \epsilon_L)$ with probability $\rho_L$. In addition, we have $w^{H(i_r)} \geq \bar{w}$ for any given $i_r$. It is straightforward to show that $\pi_{r,s} \leq \pi_{r,n}$ if $\zeta_s = \zeta_n$. Notice that $\partial \pi_{r,s} / \partial \zeta_s = \mathbb{E}[w^{H(i_r)}] \rho_L (\epsilon_H - \epsilon_L) > 0$, whereas $\partial \pi_{r,s} / \partial \zeta_n = 0$. Therefore, $\pi_{r,s} \geq \pi_{r,n}$ only if $\zeta_s$ is sufficiently large. It is worth pointing out that because $\zeta_s \leq 1$, such a sufficient large $\zeta_s$ may not exist. □

Proof of Proposition 9. The manufacturer’s optimal wholesale price is not affected by product freshness.

Without data sharing, the manufacturer’s optimal wholesale price is

$$w^{H*} = \frac{a + \epsilon_H + \rho_L (\epsilon_H - \epsilon_L) + c}{2} - E[i_r] = \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) + c}{2}.$$

The manufacturer’s expected profit is

$$\pi_{m,n} = \mathbb{E}[(w^{H*} - c)(y^{H*} - x)] = \mathbb{E} \left[ (w^{H*} - c) \left( \frac{1}{2} (a + \epsilon_H + \rho_L (\epsilon_H - \epsilon_L)) - w - x \right) \right].$$

In contrast, with inventory data sharing, the manufacturer’s optimal wholesale price depends on retailer’s inventory level $i_r$. That is,

$$w^{H(x=0)*} = \frac{a + \epsilon_H + \rho_L (\epsilon_H - \epsilon_L) + c}{2}, \quad w^{H(x=\epsilon_H-\epsilon_L)*} = \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) + c}{2}.$$

The manufacturer’s expected profit is

$$\pi_{m,s} = \rho_H \left( w^{H(x=0)*} - c \right) y^{H*} + \rho_L \left( w^{H(x=\epsilon_H-\epsilon_L)*} - c \right) \left( y^{H*} - (\epsilon_H - \epsilon_L) \right) > \pi_{m,n}.$$

The manufacturer is therefore strictly better off with retailer sharing inventory data. In contrast, the retailer’s expected profit without data sharing is given by

$$\pi_{r,n} = \mathbb{E} \left[ w^{H*} i_r + \left( \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) - w^{H*}}{2} \right)^2 - (h_r + (1 - \zeta_n)w^{H*}) \rho_L (\epsilon_H - \epsilon_L) \right]$$

$$= w^{H*} \rho_L (\epsilon_H - \epsilon_L) + \left( \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) - w^{H*}}{2} \right)^2 - (h_r + (1 - \zeta_n)w^{H*}) \rho_L (\epsilon_H - \epsilon_L).$$
On the other hand, with inventory data sharing, the retailer’s expected profit is

\[
\pi_{r,s} = \rho_H \left( \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L)}{2} - (h_r + (1 - \zeta_s) w^H(x=0) \rho_L (\epsilon_H - \epsilon_L)) \right) - \rho_L \left( w^H(x=\delta) (\epsilon_H - \epsilon_L) + \frac{a + \epsilon_H - \rho_L (\epsilon_H - \epsilon_L) - w^H(x=\delta) \rho_L (\epsilon_H - \epsilon_L)}{2} \right) - (h_r + (1 - \zeta_s) w^H(x=\delta) \rho_L (\epsilon_H - \epsilon_L)) \right) \\
< \pi_{r,n},
\]

where \( \delta = \epsilon_H - \epsilon_L \), and the last inequality follows from the fact that \( w^H(x=\delta) = w^N \) if \( \zeta_s = \zeta_n \). Observe that \( \partial \pi_{r,s}/\partial \zeta_s > 0 \). Therefore, \( \pi_{r,s} \geq \pi_{r,n} \) only if \( \zeta_s \) is sufficiently large. Note that such a sufficiently large \( \zeta_s \) may not exist because there is a natural limit on the maximum value of \( \zeta_s < 1 \). □

**Proof of Theorem 1.** The theorem statement follows directly from Propositions 7 to 9. □

**Proof of Corollary 2.** Observe that by Propositions 7 to 9, the optimal wholesale price is non-decreasing with inventory data sharing. By Proposition 3, \( y^H \) is decreasing in the wholesale price \( w \). Therefore, the retailer’s optimal order quantity is non-increasing after retailer inventory data sharing (since the optimal wholesale price is non-decreasing). Note that the retailer’s inventory at the end of the period under strategy \( \mathcal{H} \) can be derived via Proposition 3. Specifically, \( y^H - d(p^H, \tilde{\epsilon}) = 0 \) if \( \tilde{\epsilon} = \epsilon_H \) (with probability \( \rho_H \)) and \( y^H - d(p^H, \tilde{\epsilon}) = (\epsilon_H - \epsilon_L) \) if \( \tilde{\epsilon} = \epsilon_L \) (with probability \( \rho_L \)). Therefore, the amount of the leftovers that cannot be carried over (and hence outdated) is given by \( (1 - \zeta)(\epsilon_H - \epsilon_L) \) with probability \( \rho_L \). This amount is clearly decreasing in \( \zeta \). Given the fact that data sharing improves product freshness parameter \( \zeta \), it follows that data sharing helps to reduce the amount of leftovers that are outdated. □
Appendix B: Algorithms

B.1. Manufacturer’s problem with retail information sharing

Define $N$ as the set of possible realizations of retailer inventory.

Value iteration algorithm with convergence parameter $\delta$.

Step 1. For each $w \in W$ where $W$ is the set of all possible wholesale prices, do steps 2-5.

Step 2. Set $t = 0$ and initialize the value-to-go vectors $v_{st}(i, w)$ and $v_{rt}(i, w)$ for $i \in N$.

Step 3. For each state $i$, update the value-to-go vectors as follows:

$$
v_{rt+1}(i, w) = \max_{d \in D, d + \theta \geq 0} \left\{ \pi_r(d, \theta | i, w) + \alpha_r \mathbb{E} v_{rt}(d + \theta - (d + \tilde{\epsilon} - i)^+, w) \right\}^+,$$

$$v_{st+1}(i, w) = \pi_m(i, w) + \alpha_m \mathbb{E} v_{mt}(d^*(i, w) + \theta^*(i, w) - (d^*(i, w) + \tilde{\epsilon} - i)^+, w) +,$$

where

$$(d^*(i, w), \theta^*(i, w)) = \arg \max_{d \in D, d + \theta \geq 0} \left\{ \pi_r(d, \theta | i, w) + \alpha_r \mathbb{E} v_{rt}(d + \theta - (d + \tilde{\epsilon} - i)^+, w) \right\}.$$

Step 4. If $\|v_{st+1} - v_{mt}\| \leq \delta(1 - \alpha)/(2\alpha)$ and $\|v_{rt+1} - v_{rt}\| \leq \delta(1 - \alpha)/(2\alpha)$, then go to Step 5; otherwise set $t = t + 1$ and go to Step 3.

Step 5. Set $d^*(i, w) = d^*(i, w)$, $\theta^*(i, w) = \theta^*(i, w)$. Solve the steady-state retailer inventory distribution

$$\sum_{i \in N} \phi_r(x | i, w) \phi_r(i | w) = \phi_r(x | w) \text{ for all } x \in N$$

and set

$$v^*_m(i, w) = v_{mt+1}(i, w), \quad v^*_r(i, w) = v_{rt+1}(i, w)$$

$$\mathbb{E} v^*_m(w) = \sum_x v_{mt+1}(x, w) \phi_r(x | w), \quad \mathbb{E} v^*_r(w) = \sum_x v_{rt+1}(x, w) \phi_r(x | w).$$

Step 6. Return

$$w^* = \arg \max_w v^*_m(w)$$

$$v^*_m(i, w^*), \quad v^*_r(i, w^*)$$

$$\mathbb{E} v^*_m(w^*), \quad \mathbb{E} v^*_r(w^*)$$

$$d^*(i, w^*), \quad \theta^*(i, w^*)$$

The above algorithm is assured to return $\delta$-optimal solutions for the following reasons. Given that optimal solutions are obtained for each subproblem, a value iteration algorithm is assured to converge if the reward functions are bounded (Puterman 1994), which is clearly the case here. Grid search assures that subproblems are solved to optimality (within the specified mesh size).
B.2. Manufacturer’s problem without retail information sharing

Recall that the manufacturer’s pricing decision and the retailer’s ordering decision are intertwined. Because of this, the manufacturer’s demand function is complex which thereby inhibits conditions that assure unimodal profit. As such, an iterative approach to solve the combined problems is necessary. In what follows, we sketch the iterative algorithm that solves the combined retailer and manufacturer problem. Define \( N_r \) as the set of possible realizations of retailer inventory and \( N_m \) as the set of possible realizations of manufacturer inventory.

Value iteration algorithm with convergence parameter \( \delta \).

Step 0. Initialize manufacturer inventory distribution \( \phi_m(x|w) \) for \( x \in N_m \).

Step 1. For each \( w \in W \) where \( W \) is the set of all possible wholesale prices, do steps 2-5

Step 2. Set \( t = 0 \), initialize the value-to-go vectors \( v_{i_0}(i_r, w) \) for \( i_r \in N_r \).

Step 3. For each state \( i_r \), solve the following problem (e.g., via grid search):

\[
v_{rt+1}(i_r, w) = \max_{d \in D, d + \theta \geq 0} \left\{ \pi_r(d, \theta|i_r, w) + \alpha_r \sum_x E v_{rt} \left( (d + \theta - x - (d + \bar{c} - i_r - x)^+) + w \right) \phi_m(x|w) \right\},
\]

Step 4. If \( \|v_{rt+1} - v_{rt}\| \leq \delta(1 - \alpha)/(2\alpha) \) then go to Step 5; otherwise set \( t = t + 1 \) and go to Step 3.

Step 5. Set \( d^*(i_r, w) = d^*(i_r, w), \theta^*(i_r, w) = \theta^*(i_r, w) \), and \( v^*_r(i_r, w) = v_{rt+1}(i_r, w) \). Solve the steady-state retailer inventory distribution

\[
\sum_{i_r \in N_r} \phi_r(x|i_r, w) v_r(i_r|w) = \phi_r(x|w) \text{ for all } x \in N_r
\]

Step 6. For each \( w \in W \), do steps 7-10

Step 7. Set \( t = 0 \) and initialize the value-to-go vectors \( v_{n_0}(i_m, w) \) for \( i_m \in N_m \).

Step 8. For each state \( i_m \), solve the following problem (e.g., via grid search):

\[
v_{mt+1}(i_m, w) = \max_{q_m \geq 0} \left\{ \pi_m(q_m, i_m, w) + \alpha_m \sum_x E v_{mt} \left( (q_m - (q^*_m(\tilde{i}_m, w) - i_m)^+) + w \right) \phi_r(x|w) \right\},
\]

where \( q^*_m(\tilde{i}_m, w) = d^*(\tilde{i}_m, w) + \theta^*(\tilde{i}_m, w) \) can be obtained by Step 5.

Step 9. If \( \|v_{mt+1} - v_{mt}\| \leq \delta(1 - \alpha)/(2\alpha) \) then go to Step 10; otherwise set \( t = t + 1 \) and go to Step 8.

Step 10. Set \( v^*_m(i_m, w) = v_{mt+1}(i_m, w) \) and solve the updated steady-state manufacturer inventory distribution

\[
\sum_{i_m \in N_m} \phi_m(x|i_m, w) v^*_m(i_m|w) = \phi_m^*(x|w) \text{ for all } x \in N_m
\]

Step 11. If \( \|\phi^*_m - \phi_m\| \leq \delta(1 - \alpha)/(2\alpha) \) then go to Step 12; otherwise set \( \phi_m = \phi^*_m \) and go to step 1.

Step 12. Return

\[
w^* = \arg \max_{w \in W} \sum_{i_m \in N} v^*_m(i_m, w) \phi^*_m(i_m|w)
\]

\[
q^*_m(i_m, w^*)
\]

\[
(d^*(i_r, w^*), \theta^*(i_r, w^*))
\]

\[
v^*_m(i_m, w^*)
\]

\[
v^*_r(i_r, w^*)
\]
Given a grid search (that does not require assurance of a unimodal objective function), each value iteration loop (steps 2-5 and steps 7-10) is assured to converge to an optimal solution (Puterman 1994). The combined iterative sequences, however, may not necessarily converge to a single equilibrium.

The reason is that the above algorithm is based on sequentially solving the manufacturer’s and the retailer’s value functions, and update the inventory distribution functions $\phi_r$ and $\phi_m$ iteratively. Because each iteration for the manufacturer/retailer depends on the updated inventory distributions, there is no guarantee that the algorithm will converge, where convergence is defined as both $\phi_r$ and $\phi_m$ converge to steady distributions, and neither the manufacturer or the retailer has the incentive to deviate (by changing $w^*$ or $q^*_r$). Therefore, one cannot eliminate the possibility that the system cycles between several equilibria.

For example, given retailer’s inventory distribution $\phi_r$, the manufacturer may find it optimal to charge price $w$. Given $w$, however, the retailer’s optimal response may lead to actual inventory distribution becomes $\phi'_r$, which leads the manufacturer to charge $w'$. The system then enters into an cycling equilibria if $w'$ leads to $\phi_r$. There is no existing analytical guarantee that such iterative games have unique pure-strategy equilibrium. Nevertheless, our numerical study suggests that in most cases the system converges to a single equilibrium quickly, and for the rest of the cases that does cycle between two or more equilibria, the optimal expected costs are identical across cycling equilibria. Hence, regardless of whether a single equilibrium exists, one can still gain some limited insight into the value of information for the manufacturer, the retailer, and the supply chain.
Appendix C: Supplementary Results

Proposition A1  The retailer’s optimal price and order policy reduces the demand uncertainty seen by the manufacturer.

Proof of Proposition A1. In the following proof we suppress subscript $r$ to simplify the notation. Note that the proposition statement is true if $d'(i) \leq 1$ and $d'(i) + \theta(i) \leq 0$. To prove this result, we first characterize a single period problem, and then use induction to extend the analysis to multiple periods. Then in the limit we obtain infinite horizon characterization. The following lemma characterizes a single period problem.

Define $S(\theta) = E(i + \theta - \tilde{\epsilon}) - (i + \theta)$ as the expected lost sales in a period, e.g., expected units sold is $d - S(\theta)$. Further, define $\lambda(z) = f(z)/\tilde{F}(z)$ as the failure rate of distribution $F$. In addition, let

$$G(d, \theta) = \left\{ \pi(d, \theta|i) + \alpha \text{Ev} \left( \left( d + \theta - (i + \theta) \min \{d + \theta - i, i_m\} - d - (\tilde{\epsilon} + \epsilon) \right) \right) \right\}.$$  

Lemma A1 (Single Period) If $\lambda(z)$ is sufficiently high for any $z \in [\epsilon_L, \epsilon_H]$, then (i) $d_1, \theta_1$ is the solution to

\begin{align}
   d_1 &= \frac{a - w}{b} + S(\theta_1), \\
   F(i + \theta_1) &= \frac{a - bd_1 - w}{a - bd_1 + h}.
\end{align}

(ii) $d_1'(i) = 0$, $\theta_1'(i) = -1$. (iii) $v_1'(i) = w$, $v_1''(i) = 0$.

Proof of Lemma A1. Part (i). The first-order condition $G_{1d}(d, \theta|i) = \pi_d(d, \theta|i) = -b(d - S(\theta)) + p(d) - w = 0$ and $G_{1\theta}(d, \theta|i) = \pi_{\theta}(d, \theta|i) = p(d) - w - (p(d) + h) F(i + \theta) = 0$ yields (A-9) and (A-10). Next, we prove that $G_1(d, \theta|i)$ at any stationary point is negative semidefinite, implying that any stationary point is a global maximum. Note that $G_{1dd}(d, \theta|i) = \pi_{dd}(d, \theta|i) = -2b < 0$, $G_{1\theta\theta}(d, \theta|i) = \pi_{\theta\theta}(d, \theta|i) = -(p(d) + h) f(i + \theta) \leq 0$, $G_{1d\theta}(d, \theta|i) = \pi_{d\theta}(d, \theta|i) = -bF(i + \theta)$. The determinant of the Hessian evaluated at any stationary point $(d^*, \theta^*)$ (that satisfies (A-9) and (A-10)) is

$$\Delta = G_{1dd}(d^*, \theta^*|i) G_{1\theta\theta}(d^*, \theta^*|i) - G_{1d\theta}(d^*, \theta^*|i)^2$$

\begin{align}
   &= 2b(p(d^*) + h_r) \left[ f(i + \theta^*) - \frac{b\tilde{F}(i + \theta^*)^2}{2(p(d^*) + h_r)} \right] \\
   &\geq 2b(p(d^*) + h_r)\tilde{F}(i + \theta^*) \left[ \lambda(i + \theta^*) - \frac{b}{a + b\epsilon_L + w + 2h} \right] \geq 0
\end{align}

where the last inequality follows from the fact that $\lambda$ is sufficiently high. Thus, $G_1(d, \theta|i)$ is negative semidefinite at any stationary point.

Part (ii). Note that

$$\Delta = 2b(p(d_1) + h) f(i + \theta_1) - b^2\tilde{F}(i + \theta_1)^2 \geq 0 \quad \text{(see (A-11))}$$

$$\Delta_d = b\tilde{F}(i + \theta_1)(p(d_1) + h) f(i + \theta_1) - b\tilde{F}(i + \theta_1)(p(d_1) + h) f(i + \theta_1) = 0$$

$$\Delta_{\theta} = 2b(p(d_1) + h) f(i + \theta_1) - b^2\tilde{F}(i + \theta_1)^2.$$ 

Therefore $d_1'(i) = -\Delta_d/\Delta = 0$, $\theta_1'(i) = -\Delta_{\theta}/\Delta = -1$. 


Part (iii). Note that $G_{tv}(d_1, \theta_1|i) = 0 \Rightarrow p(d_1) - (p(d_1) + h) F(i + \theta_1) = w$, and $v_1(i) = G_1(d_1, \theta_1|i) = \pi(d_1, \theta_1|i) + \alpha v_0 = \pi(d_1, \theta_1|i)$. Hence, $v_1'(i) = G_{tv}(d_1, \theta_1|i) d_1'(i) + G_{tv}(d_1, \theta_1|i) v_1'(i) + G_{1i}(d_1, \theta_1|i) = G_{1i}(d_1, \theta_1|i) = \pi_1(d_1, \theta_1|i) = p(d_1) - (p(d_1) + h) F(i + \theta_1) = w$. □

An analogous result can be proved for the two-period case. Now consider the general $t$ period. We know that $v_t'(i) \in [w(1 - \alpha), w]$ and $v_t''(i) \leq 0$ for $t \leq 2$. Assume that this property holds for period $t - 1$, i.e.,

$$v_{t-1}'(i) \in [w(1 - \alpha), w], \quad v_{t-1}''(i) \leq 0.$$  \hfill (A-12)

We will show that (A-12) holds for period $t$. In what follows, we first prove that the optimal $(d_t, \theta_t)$ can be found efficiently. We need the following intermediary results.

**Lemma A2** Let $(d^{(k)}, \theta^{(k)})$ denote stationary points where $(d^{(k)}, \theta^{(k)})$ for $k = 0, ..., N$ is the solution to

$$d^{(k)} = \frac{1}{2} \left( \frac{a - w}{b} + S(\theta^{(k)}) \right) + \frac{\alpha \sum_{x=0}^{k} v_{t-1}'(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)}) \phi_m(x)}{2b},$$

$$a - bd^{(k)} - w + \alpha \sum_{x=0}^{k} \left( \frac{v_{t-1}'(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)})}{d^{(k)} + \theta^{(k)} - x} + \int_{i+x-d^{(k)}}^{i+x} \theta^{(k)} + i - z f(z)dz \right) \phi_m(x),$$

where $\phi_m$ is the retailer’s belief about the manufacturer’s inventory distribution. Define

$$\Psi = \{(d^{(k)}, \theta^{(k)}): k < d^{(k)} + \theta^{(k)} \leq k + 1, k = 0, ..., N - 1; d^{(k)} + \theta^{(k)} > N, k = N\}.$$  \hfill Then \( (d_t, \theta_t) = \arg \max_{(d, \theta) \in \Psi} G_t(d, \theta|i) \).

**Proof of Lemma A2.** To prove the lemma statement, we begin by defining

$$g^{(k)}(d, \theta|i) = \pi(d, \theta|i) + \alpha \sum_{x \leq k} \left[ \int_{i+x-d}^{i+x} v_{t-1}'(d + \theta - x) f(z)dz \right] \phi_m(x) + \alpha v_{t-1}(0) P[i_m > k].$$

Note that $G_t(d, \theta|i) = g^{(k)}(d, \theta|i)$ for all $(d, \theta) \in \Lambda(k) = \{(d, \theta): d + \theta \in [k, k+1], k = 0, ..., N - 1; d + \theta \geq k, k = N\}$. Solving

$$g_d^{(k)}(d, \theta|i) = -b(d - S(\theta)) + a - bd - w + \alpha w \sum_{x \leq k} v_{t-1}'(d + \theta - x) F(i + x - d) \phi_m(x) = 0,$$

$$g_\theta^{(k)}(d, \theta|i) = a - bd - w - (a - bd + h) F(i + \theta) + \alpha \sum_{x \leq k} \left( \int_{i+x-d}^{i+x} v_{t-1}'(d + \theta - x) f(z)dz \right) \phi_m(x) = 0,$$

yields solution to $(d^{(k)}, \theta^{(k)})$. The candidate optimal solutions are stationary points of $G_t$, which is the following set

$$\Psi = \{(d^{(k)}, \theta^{(k)}): k < d^{(k)} + \theta^{(k)} \leq k + 1, k = 0, ..., N - 1; d^{(k)} + \theta^{(k)} > N, k = N\}.$$
We will next show that $G_t$ is negative semidefinite at any point in $\Psi$. Thus, each point in $\Psi$ is a local maximum, and furthermore, the global maximum must be in $\Psi$. Note that

\[
S(\theta) \leq -\epsilon_L
\]

\[
\bar{v}_{t-1dd}(d, \theta|i) = \sum_{x \in \Omega(d,\theta)} (v''_{t-1}(d+\theta - x)F(i + x - d) - v'_{t-1}(d+\theta - x)f(i + x - d)) \phi_m(x) \leq 0
\]

\[
\bar{v}_{t-1d0}(d, \theta|i) = \sum_{x \in \Omega(d,\theta)} (v''_{t-1}(d+\theta - x)F(i + x - d) + \int_{i+x-d}^{i+\theta} v''_{t-1}(\theta + i - z)f(z)dz + v'_{t-1}(0)f(i + \theta) \phi_m(x)
\]

\[
\bar{v}_{t-1\delta0}(d, \theta|i) = \sum_{x \in \Omega(d,\theta)} v''_{t-1}(d+\theta - x)F(i + x - d)\phi_m(x) \leq 0
\]

\[
G_{t,dd} = \pi_{dd}(d, \theta|i) + \alpha \bar{v}_{t-1dd}(d, \theta|i)
\]

\[
= -2b + \alpha \sum_{x \in \Omega(d,\theta)} (v''_{t-1}(d+\theta - x)F(i + x - d) - v'_{t-1}(d+\theta - x)f(i + x - d)) \phi_m(x) < 0
\]

\[
G_{t,d\delta} = \pi_{d\delta}(d, \theta|i) + \alpha \bar{v}_{t-1d\delta}(d, \theta|i) = -bF(i + \theta) + \alpha \sum_{x \in \Omega(d,\theta)} v''_{t-1}(d+\theta - x)F(i + x - d)\phi_m(x)
\]

\[
G_{t,\delta\delta} = \pi_{\delta\delta}(d, \theta|i) + \alpha \bar{v}_{t-1\delta\delta}(d, \theta|i)
\]

\[
= -(p(d) + h)f(i + \theta) + \alpha \sum_{x \in \Omega(d,\theta)} (v''_{t-1}(d+\theta - x)F(i + x - d) + \int_{i+x-d}^{i+\theta} v''_{t-1}(\theta + i - z)f(z)dz + v'_{t-1}(0)f(i + \theta) \phi_m(x) \leq 0
\]

where the last inequality follows from $p(d) \geq w$ for all $d \in D$, $v'_{t-1} \leq w$, and $v''_{t-1} \leq 0$. For any $(d^{(k)}, \theta^{(k)}) \in \Psi$, note that

\[
x < d^{(k)} + \theta^{(k)} \Leftrightarrow x \leq k
\]

\[
F(i + x - d^{(k)}) \leq F(i + \theta^{(k)}) \text{ for all } x \leq k \text{ (because } k < d^{(k)} + \theta^{(k)})
\]

\[
d^{(k)} = \frac{1}{2} \left( \frac{a - w}{b} + S(\theta^{(k)}) \right) + \alpha \sum_{x \leq k} v_{t-1}'(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)}) \phi_m(x)
\]

\[
\leq \frac{1}{2} \left( \frac{a - w}{b} + S(\theta^{(k)}) \right) + \frac{\alpha wF(i + \theta^{(k)}) P[m \leq k]}{2b}
\]

\[
p(d^{(k)}) = a - bd^{(k)} \geq \frac{a + w - bS(\theta^{(k)}) - \alpha w \sum_{x \leq k} F(i + x - d^{(k)}) \phi_m(x)}{2} \geq \frac{a + w - bS(\theta^{(k)}) - \alpha wF(i + \theta^{(k)})}{2}
\]

\[
2(p(d^{(k)}) + h - \alpha w) \geq a + w - bS(\theta^{(k)}) - \alpha wF(i + \theta^{(k)}) - 2\alpha w + 2h \geq a + b\epsilon_L + w(1 - 3\alpha) + 2h.
\]

Thus, letting

\[
\tau^{(k)} = P[m \leq k]
\]

\[
k_1 = -\alpha \sum_{x \leq k} v''_{t-1}(d^{(k)} + \theta^{(k)} - x)F(i + x - d^{(k)}) \geq 0
\]

\[
k_2 = \alpha \sum_{x \leq k} v_{t-1}'(0) f(i + \theta^{(k)}) \phi_m(x)
\]

\[
= \alpha \tau^{(k)} v_{t-1}'(0) f(i + \theta^{(k)}) \in [\alpha (1 - \alpha) \tau^{(k)} w f(i + \theta^{(k)}), \alpha \tau^{(k)} w f(i + \theta^{(k)})]
\]

we have

\[
\Delta = G_{t,dd}(d^{(k)}, \theta^{(k)}|i) G_{t,\delta\delta}(d^{(k)}, \theta^{(k)}|i) - G_{t,d\delta}(d^{(k)}, \theta^{(k)}|i)^2
\]
\begin{align}
\Delta & = G_{i\bar{d}d} (d^{(k)}, \theta^{(k)} | i) G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) - G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) \\
& = \left[ 2b + \alpha \sum_{x \leq k} \left( -v_{r-1}''(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)}) + v_{r-1}'(d^{(k)} + \theta^{(k)} - x) f(i + x - d^{(k)}) \right) \phi_m(x) \right] \\
& \quad \times \left[ (p(d^{(k)}) + h) f(i + \theta^{(k)}) + \alpha \sum_{x \leq k} \left( -v_{r-1}''(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)}) - \int_{i+x-d^{(k)}}^{i+x-\theta^{(k)}} v_{r-1}'(\theta^{(k)} + i - z) f(z) dz \right) \phi_m(x) \right] \\
& \quad - \left[ b \bar{F}(i + \theta^{(k)}) - \alpha \sum_{x \leq k} v_{r-1}'(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)}) \phi_m(x) \right]^2 \tag{A-13}
\end{align}

\begin{align}
\Delta_d & = G_{i\bar{d}d} (d^{(k)}, \theta^{(k)} | i) G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) - G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) \\
& = \left[ b \bar{F}(i + \theta^{(k)}) - \alpha \sum_{x \leq k} v_{r-1}'(d^{(k)} + \theta^{(k)} - x) f(i + x - d^{(k)}) \phi_m(x) \right] \\
& \quad \times \left[ (p(d^{(k)}) + h) f(i + \theta^{(k)}) + \alpha \sum_{x \leq k} \left( -v_{r-1}''(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)}) - \int_{i+x-d^{(k)}}^{i+x-\theta^{(k)}} v_{r-1}'(\theta^{(k)} + i - z) f(z) dz \right) \phi_m(x) \right] \\
& \quad - \left[ b \bar{F}(i + \theta^{(k)}) - \alpha \sum_{x \leq k} v_{r-1}'(d^{(k)} + \theta^{(k)} - x) F(i + x - d^{(k)}) \phi_m(x) \right] \tag{A-14}
\end{align}

where $\kappa = a + b\epsilon_L + w(1-3a) + 2h$ and the last inequality follows from $\lambda$ being sufficiently high. Thus, $G_1$ is negative semidefinite at $(d^{(k)}, \theta^{(k)})$. □

We are now able to show that $d_r(i) \leq 1$ and $d_r(i) + \theta_r(i) \leq 0$. Note that

$$
\Delta_d = G_{i\bar{d}d} (d^{(k)}, \theta^{(k)} | i) G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) - G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i) G_{i\bar{d}i} (d^{(k)}, \theta^{(k)} | i)
$$
\[
\times \left[ bF(i + \theta^{(k)}) - \alpha \sum_{z \leq k} v_{t-1}'' \left( d^{(k)} + \theta^{(k)} - x \right) F \left( i + x - d^{(k)} \right) \phi_{m}(x) \right]
\]

Let
\[
k_1 = -\alpha \sum_{z \leq k} v_{t-1}'' \left( d^{(k)} + \theta^{(k)} - x \right) F \left( i + x - d^{(k)} \right) \phi_{m}(x) \geq 0
\]
\[
k_2 = \alpha \tau^{(k)} v_{t-1}'(0) f \left( i + \theta^{(k)} \right) \in \left[ \alpha (1 - \alpha) \tau^{(k)} w f \left( i + \theta^{(k)} \right), \alpha \tau^{(k)} w f \left( i + \theta^{(k)} \right) \right]
\]
\[
k_3 = \alpha \sum_{z \leq k} v_{t-1}' \left( d^{(k)} + \theta^{(k)} - x \right) f \left( i + x - d^{(k)} \right) \phi_{m}(x)
\]
\[
eq \left[ \alpha (1 - \alpha) w \sum_{z \leq k} f \left( i + x - d^{(k)} \right) \phi_{m}(x), \alpha w \sum_{z \leq k} f \left( i + x - d^{(k)} \right) \phi_{m}(x) \right] \geq 0
\]
\[
h_1 = \alpha \sum_{z \leq k} v_{t-1}' \left( d^{(k)} + \theta^{(k)} - x \right) f \left( i + x - d^{(k)} \right) \phi_{m}(x) \left| G_{\theta^{(k)}} \left( d^{(k)}, \theta^{(k)} \right) \right| \geq 0
\]
\[
h_2 = (2b + k_1) \left| G_{\theta^{(k)}} \left( d^{(k)}, \theta^{(k)} \right) \right| \geq 0
\]

Then
\[
\Delta = h_1 + h_2 - (bF \left( i + \theta^{(k)} \right) + k_1)^2
\]
\[
\Delta_d = (bF \left( i + \theta^{(k)} \right) - k_3) \left| G_{\theta^{(k)}} \left( d^{(k)}, \theta^{(k)} \right) \right| \left( \left| G_{\theta^{(k)}} \left( d^{(k)}, \theta^{(k)} \right) \right| - k_1 \right) \left( bF \left( i + \theta^{(k)} \right) + k_1 \right)
\]
\[
= -h_1 + k_1 \left( bF \left( i + \theta^{(k)} \right) + k_1 - \left| G_{\theta^{(k)}} \left( d^{(k)}, \theta^{(k)} \right) \right| \right)
\]
\[
d^{(k)}' (i) = \frac{-\Delta_d}{\Delta} = \frac{h_1 + k_1 \left( \left| G_{\theta^{(k)}} \left( d^{(k)}, \theta^{(k)} \right) \right| - \left( bF \left( i + \theta^{(k)} \right) + k_1 \right) \right)}{h_1 + h_2 - \left( bF \left( i + \theta^{(k)} \right) + k_1 \right)^2}
\]

Note that
\[
\Delta = h_1 + h_2 - (bF \left( i + \theta^{(k)} \right) + k_1)^2 = h_1 + (2b + k_1) \left| G_{\theta^{(k)}} \right| \left( bF \left( i + \theta^{(k)} \right) + k_1 \right)^2
\]
\[
= h_1 + k_1 \left( \left| G_{\theta^{(k)}} \right| - \left( bF \left( i + \theta^{(k)} \right) + k_1 \right) \right) + 2b \left| G_{\theta^{(k)}} \right| - bF \left( i + \theta^{(k)} \right) \left( bF \left( i + \theta^{(k)} \right) + k_1 \right)
\]
\[
\geq -\Delta_d + |\pi_{\theta^{(k)}}| \left( |\pi_{\theta^{(k)}}| - k_2 + k_1 \right) - |\pi_{\theta^{(k)}}| \left( |\pi_{\theta^{(k)}}| + k_1 \right)
\]
\[
= -\Delta_d + |\pi_{\theta^{(k)}}| \left( |\pi_{\theta^{(k)}}| - k_2 \right) - |\pi_{\theta^{(k)}}|^2 - k_1 \left( |\pi_{\theta^{(k)}}| - |\pi_{\theta^{(k)}}| \right)
\]
\[
\geq -\Delta_d + k_1 \left( 2b - bF \left( i + \theta^{(k)} \right) \right) \quad \text{(see (A-13))}
\]
\[
\geq -\Delta_d
\]

and thus \( d^{(k)}' (i) \leq 1 \). Note that
\[
\Delta_d = (2b + k_1 + k_3) \left( \left| G_{\theta^{(k)}} \right| - \left( bF \left( i + \theta^{(k)} \right) - k_3 \right) \left( bF \left( i + \theta^{(k)} \right) + k_1 \right) \right)
\]
\[
= k_3 \left| G_{\theta^{(k)}} \right| + k_1 \left( \left| G_{\theta^{(k)}} \right| - \left( bF \left( i + \theta^{(k)} \right) + k_1 \right) \right) + 2b \left| G_{\theta^{(k)}} \right| - bF \left( i + \theta^{(k)} \right) \left( bF \left( i + \theta^{(k)} \right) + k_1 \right)
\]
\[
= -\Delta_d + 2b \left| G_{\theta^{(k)}} \right| - k_1 \left( bF \left( i + \theta^{(k)} \right) - k_3 \right) \left( bF \left( i + \theta^{(k)} \right) \right)
\]

and thus
\[
d^{(k)}' (i) + \theta^{(k)}' (i) = \frac{-\Delta_d - \Delta_a}{\Delta} = \frac{-2b \left| G_{\theta^{(k)}} \right| - k_1 \left( bF \left( i + \theta^{(k)} \right) - k_3 \right) \left( bF \left( i + \theta^{(k)} \right) \right)}{\Delta}
\]
\[
= -\frac{\left| \pi_{\theta^{(k)}} \right| - k_2}{\Delta} \sum_{z \leq k} \left( f_{i+z-\theta^{(k)}} v_{t-1}'' \left( \theta^{(k)} + i - z \right) f(z) dz \right) \phi_{m}(x)
\]
Finally, as function \( L \) manufacturer has a fixed lifetime of a single period, and for any given retailer starting inventory and a non-perishable product. Suppose that \( \phi \) end-of-life products). A change in \( \pi \) only affects the next period optimal value-to-go function \( \bar{v}\). Let \( (d^k, \theta^k) \in \Psi \) denote the optimal solution. Note that

\[
\begin{align*}
v_t'(i) &= G_t \left( d^k, \theta^k | i \right) = \pi \left( d^k, \theta^k | i \right) + \alpha \bar{v}_{t-1} \left( d^k, \theta^k | i \right) \\
v_t''(i) &= G_{td} \left( d^k, \theta^k | i \right) d^{k'}(i) + G_{sv} \left( d^k, \theta^k | i \right) \theta^{k'}(i) + G_{ts} \left( d^k, \theta^k | i \right) = G_{t}\left( d^k, \theta^k | i \right)
\end{align*}
\]

Therefore \( d_t'(i) \leq 1 \) and \( d_t'(i) + \theta_t'(i) \leq 0 \).

It is also useful to show that \( v_t'(i) \leq 0 \) and \( v_t''(i) \leq 0 \). Recall that \( (d_t, \theta_t) \in \Psi \). Let \( (d^k, \theta^k) \in \Psi \) denote the optimal solution. Note that

\[
\begin{align*}
v_t(i) &= G_t \left( d^k, \theta^k | i \right) = \pi \left( d^k, \theta^k | i \right) + \alpha \bar{v}_{t-1} \left( d^k, \theta^k | i \right) \\
v_t'(i) &= G_{td} \left( d^k, \theta^k | i \right) d^{k'}(i) + G_{sv} \left( d^k, \theta^k | i \right) \theta^{k'}(i) + G_{ts} \left( d^k, \theta^k | i \right) = G_{t}\left( d^k, \theta^k | i \right)
\end{align*}
\]

However,

\[
\begin{align*}
G_{ts} \left( d^k, \theta^k | i \right) &= p \left( d^k \right) - \left( p \left( d^k \right) + h \right) F \left( i + \theta^k \right) \\
+ \alpha \sum_{x \leq k} \left( v_{t-1}' \left( d^k + \theta^k - x \right) F \left( i + x - d^k \right) + \int_{i+x-d^k}^{i+x} v_{t-1}' \left( \theta^k + i - z \right) f \left( z \right) dz \right) \phi_m \left( x \right) = 0
\end{align*}
\]

and thus

\[
\begin{align*}
v_t'(i) &= w - \alpha \sum_{x \leq k} v_{t-1}' \left( d^k + \theta^k - x \right) F \left( i + x - d^k \right) \phi_m \left( x \right) \in \left[ w \left( 1 - \alpha \right), w \right] \\
v_t''(i) &= -\alpha \sum_{x \leq k} \left( v_{t-1}'' \left( d^k + \theta^k - x \right) f \left( i + x - d^k \right) \left( 1 - d^{k'}(i) \right) \\
- \alpha \sum_{x \leq k} \left( v_{t-1}'' \left( d^k + \theta^k - x \right) F \left( i + x - d^k \right) \left( d^{k'}(i) + \theta^{k'}(i) \right) \right) \phi_m \left( x \right) \leq 0.
\end{align*}
\]

Finally, as function \( G_t \) is continuous and function \( \pi(d, \theta|i) \) is bounded, e.g., \( \pi(d, \theta|i) < p(\min D) \max D \). All the above results extend to the infinite horizon case. \( \square \)

**Proposition A2** The retailer’s value function is highest when the product is non-perishable and lowest when the product has only one period life. The value function is in between when product has a two period life.

**Proof of Proposition A2.** We suppress subscript \( r \) to simplify the notation. The value-to-go function (that applies to case of \( L = 2 \)) is increasing in the retailer starting inventory \( i \). Note that the current-period retail profit function, \( \pi \), is not affected by the value of \( L \) (e.g., the value of starting inventory \( i \) is after disposal of end-of-life products). A change in \( L \) only affects the next period optimal value-to-go function \( \bar{v}_{t-1} \left( d_t, \theta_t | i \right) \).

By specifying conditions on the manufacturer’s inventory distribution \( \phi_m \), we transform our model with \( L = 2 \) to models that are either equivalent or provide a suitable bound for analysis of a single-period lifetime and a non-perishable product. Suppose that \( \phi_m(N) = 1 \). Then all product shipped to the retailer from the manufacturer has a fixed lifetime of a single period, and for any given retailer starting inventory \( i \), our model
with $L = 2$ is equivalent to a model with $L = 1$ (i.e., equivalent to a model where the manufacturer only ships fresh product to the retailer that expires after one period). Alternatively, suppose $\phi_m(0) = 1$. Then all product shipped to the retailer has remaining life beyond the current period. With this background, we proceed with analysis. Recall for our model with $L = 2$ that

$$v_t(i) = \pi(d_t, \theta_t | i) + \alpha \bar{v}_{t-1}(d_t, \theta_t | i)$$

$$\bar{v}_{t-1}(d_t, \theta_t | i) = \sum_{x \in \Omega(d_t, \theta_t)} \left\{ \int_{i+1}^{i+\theta_t} v_{t-1}(d_t + \theta_t - x) f(z) dz \ight. + \int_{i+\theta_t}^{i+\theta_t} v_{t-1}(d_t + \theta_t - x) f(z) dz \right\} \phi_m(x) + v_{t-1}(0) \left( 1 - \sum_{x \in \Omega(d_t, \theta_t)} \phi_m(x) \right)$$

Setting $\phi_m(0) = 1$ while keeping $(d_t, \theta_t)$ fixed yields

$$\bar{v}_{t-1}(d_t, \theta_t | i) |_{\phi_m(0) = 1} = \int_{i+1}^{i+\theta_t} v_{t-1}(d_t + \theta_t) f(z) dz + \int_{i+\theta_t}^{i+\theta_t} v_{t-1}(d_t + \theta_t - z) f(z) dz + \int_{i+\theta_t}^{i+\theta_t} v_{t-1}(0) f(z) dz.$$

Note that for $z \in [i + x - d_t, i + \theta_t]$ and $x \geq 1$,

$$\theta_t + i - z \leq d_t + \theta_t - x$$

$$v_{t-1}(0) \leq v_{t-1}(\theta_t + i - z) \leq v_{t-1}(d_t + \theta_t - x) \leq v_{t-1}(d_t + \theta_t)$$

(due to $v_t'(i) \in [w(1 - \alpha), w]$), which implies

$$\bar{v}_{t-1}(d_t, \theta_t | i) |_{L = \infty} \geq \bar{v}_{t-1}(d_t, \theta_t | i) |_{\phi(0) = 1, L = 2} \geq \bar{v}_{t-1}(d_t, \theta_t | i) |_{L = 2}$$

(the first inequality follows from the observation that the problem with $L = \infty$ is a relaxation in the problem where all product must be disposed after two periods). Furthermore, if $\phi_m(N) = 1$, then $\sum_{x \in \Omega(d_t, \theta_t)} \phi_m(x) = 0$ and

$$\bar{v}_{t-1}(d_t, \theta_t | i) |_{L = 1} = \bar{v}_{t-1}(d_t, \theta_t | i) |_{\phi(N) = 1} \leq \bar{v}_{t-1}(d_t, \theta_t | i).$$

Therefore, $v_t(i) |_{L = 1} \leq v_t(i) |_{L = 2} \leq v_t(i) |_{L = \infty}$. □