Tracking Faults and Network Model Changes Using Phasor Measurements

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Outline

Phasor Measurements

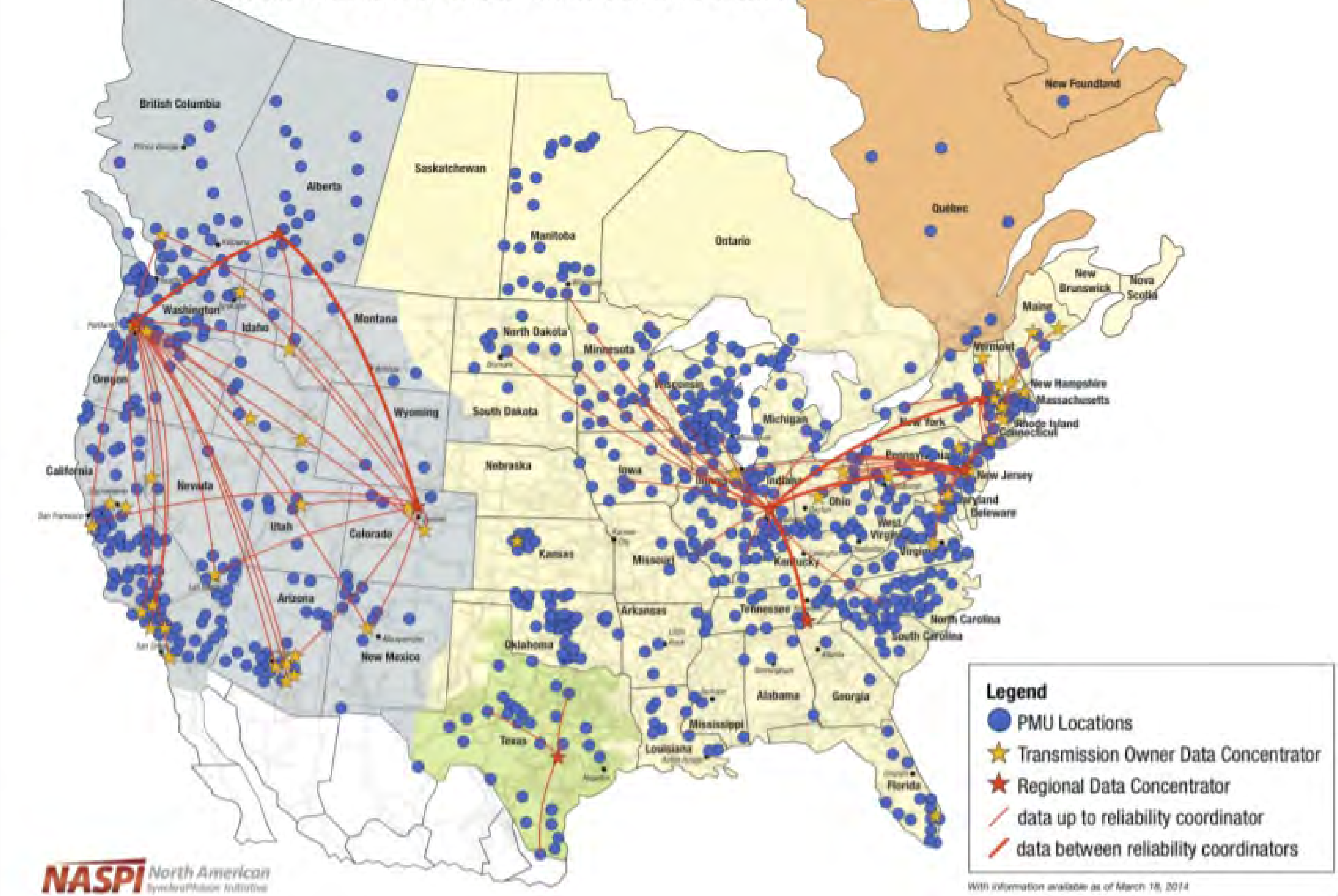
Sparse Estimation Problems

- Fault Location
- Line Outage Identification

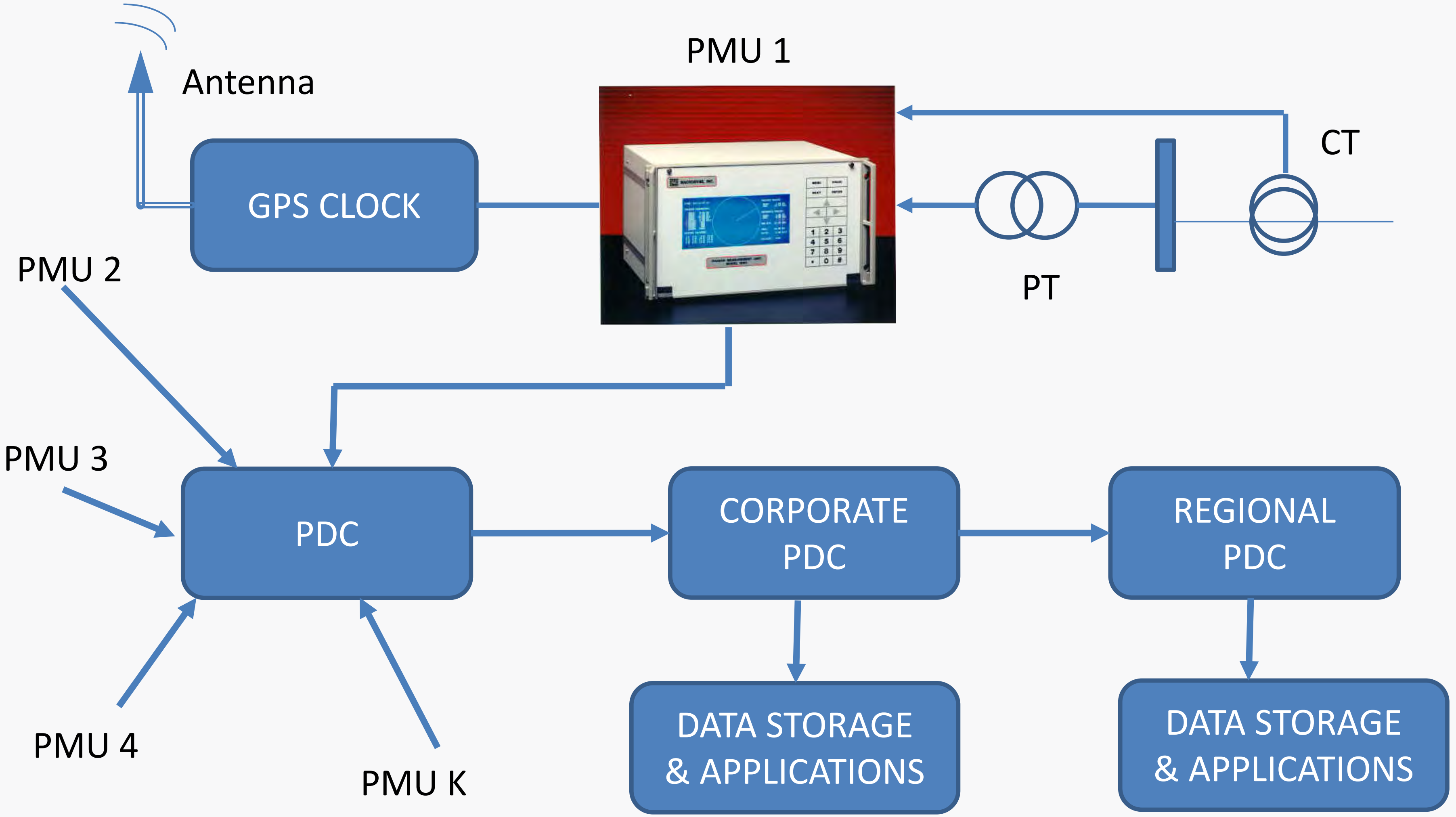
Transmission Line Parameter Tracking

Closing remarks

Phasor Measurement Units and Synchrophasor Data Flows in the North American Power Grid

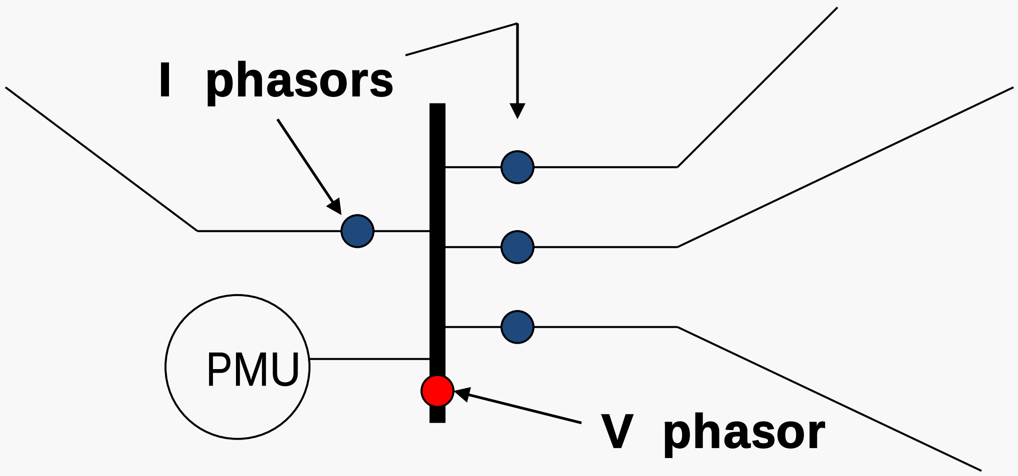


Phasor Measurement Units (PMU) Phasor Data Concentrators (PDC)[*]



[*] IEEE PSRC Working Group C37 Report

Measurements provided by PMUs



ALL 3-PHASES ARE TYPICALLY MEASURED, BUT ONLY POSITIVE SEQUENCE COMPONENTS ARE REPORTED

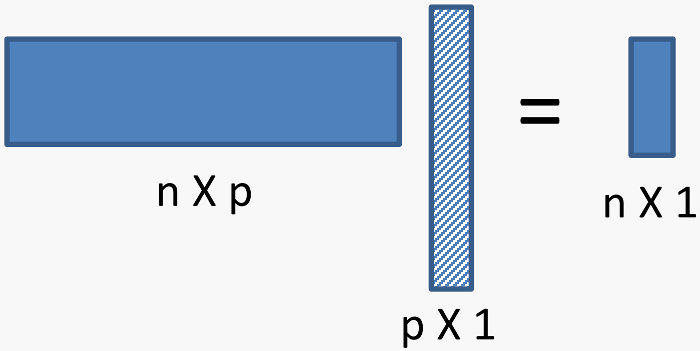
$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = [T] \begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix} \Rightarrow V_+ = \tfrac{1}{3}[V_A + aV_B + a^2\, V_C]$$

$$a = e^{j\frac{2\pi}{3}}$$

Sparse Estimation Problems

Underdetermined set of equations:

$$[\boldsymbol{X}][\boldsymbol{\beta}] = [\boldsymbol{y}]$$



n < p

It is known that β is "*k*" sparse, that is:

"*k*" out of "*p*" entries are known to be significantly larger than the remaining (*p*-*k*) entries,

However, it is NOT known which ones they are.

Fault

Line

Least Angle Regression and Shrinkage (LARS)[*]

$$\hat{\beta} \in \underset{\beta \in \mathbb{R}^p}{\operatorname{argmin}} \frac{1}{2} \| \mathbf{y} - \mathbf{X}\beta \|_2^2 + \lambda \| \beta \|_1 \qquad \lambda \geq 0$$

$$\mathbf{y} \in \mathbb{R}^n \qquad \mathbf{X} \in \mathbb{R}^{n \times p} \ (p > n)$$

[*] R. Tibshirani, "Regression shrinkage and selection via the lasso," Journal of the Royal Statistical Society. Series B (Methodological), Vol.58,No.1, pp. 267–288, 1996.
B. Efron, T. Hastie, I. Johnstone, R. Tibshirani et al., "Least angle regression," The Annals of statistics, vol. 32, no. 2, pp. 407–499, 2004.

LASSO: Least Absolute Shrinkage and Selection Operator

Return

Dantzig Selector

George Dantzig
1914-2005

THE DANTZIG SELECTOR: STATISTICAL ESTIMATION WHEN *p* IS MUCH LARGER THAN *n*

BY EMMANUEL CANDES AND TERENCE TAO
Cal Tech UCLA

The Annals of Statistics, 2007, Vol. 35, No. 6, 2313–2351

$$\min_{\tilde{\beta}\in\mathbf{R}^p} \|\tilde{\beta}\|_{\ell_1} \quad \text{subject to} \quad X\tilde{\beta} = y, \quad X \in \mathbf{R}^{n\times p}$$

LP Formulation

$$\min \sum_i u_i \quad \text{subject to} \quad -u \le \tilde{\beta} \le u \quad \text{and}$$

$$-\lambda_p \sigma \mathbf{1} \le X^*(y - X\tilde{\beta}) \le \lambda_p \sigma \mathbf{1},$$

Two Applications in Power Systems

Fault Location

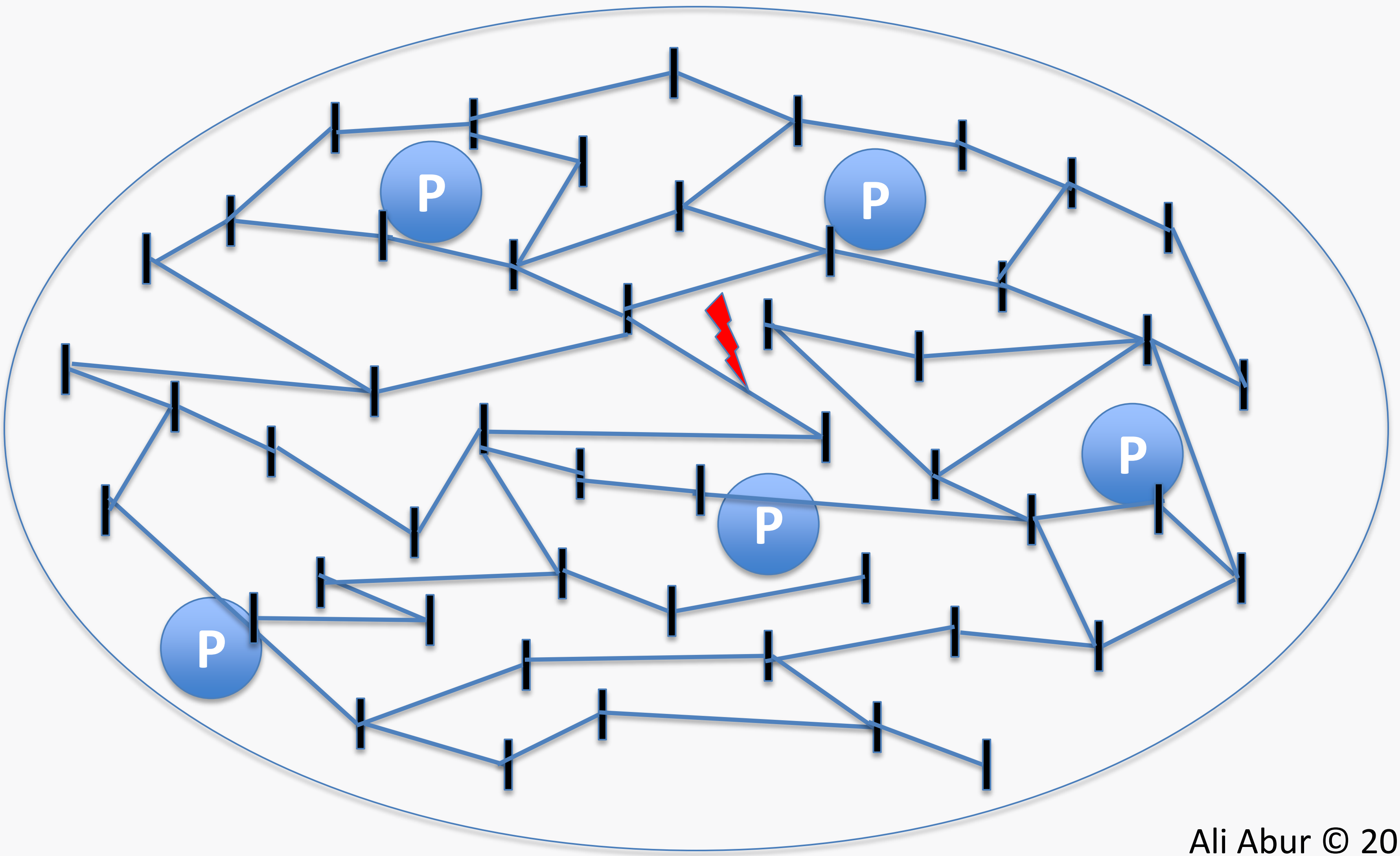
Large number of candidate branches, one of which is faulted, which one?

Further complication: unknown fault location along the suspect branch.

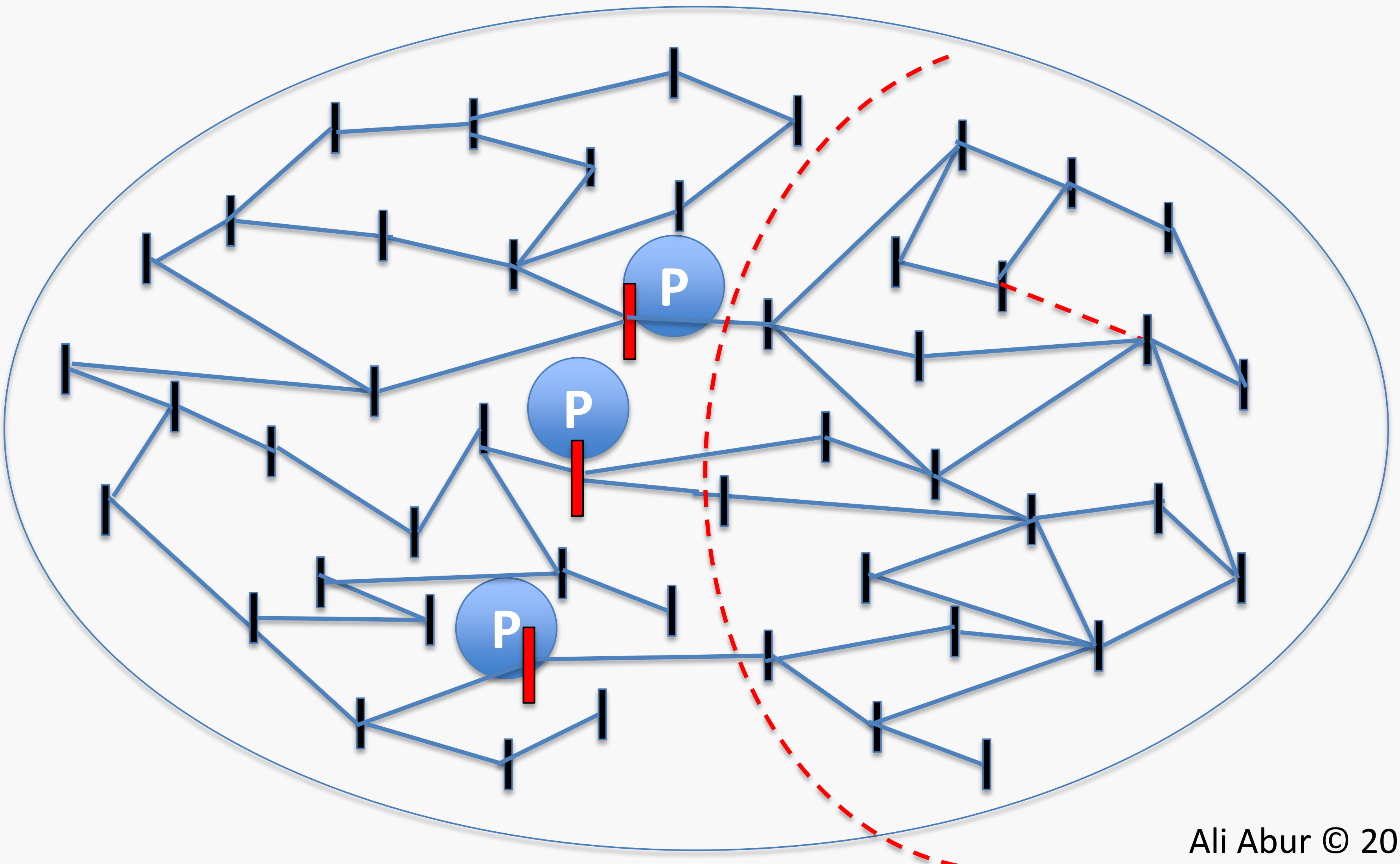
External Network Line Outage Detection

Large number of candidate lines, one of which is disconnected, which one?

Fault Location Using Few PMUs



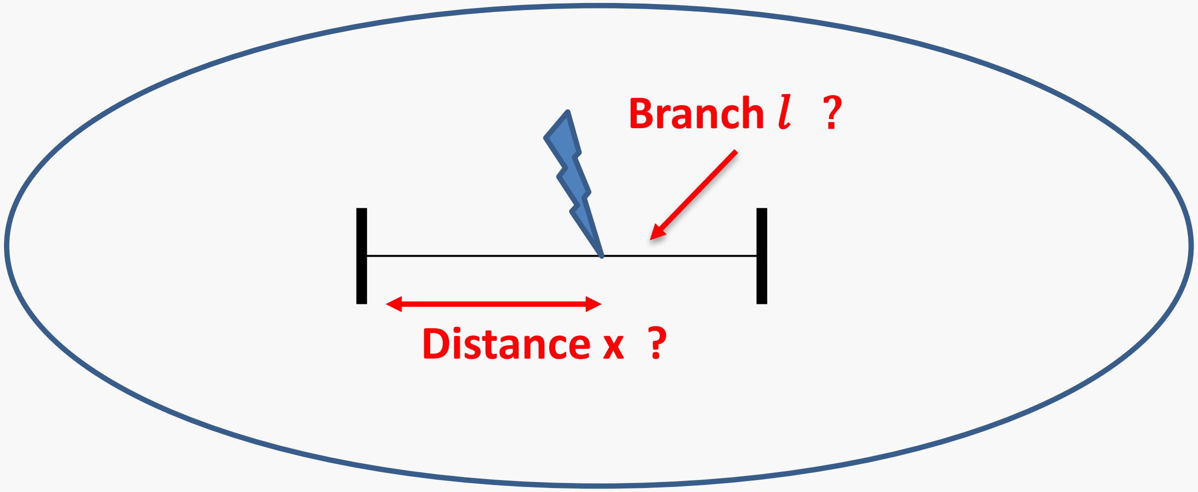
External Network Line Outage



Fault location problem

Fault occurs, typically along a transmission line, less frequently right at a bus / substation. Need to find:

- the faulted line or line segment,
- the distance between one of the line terminals and the point of fault.

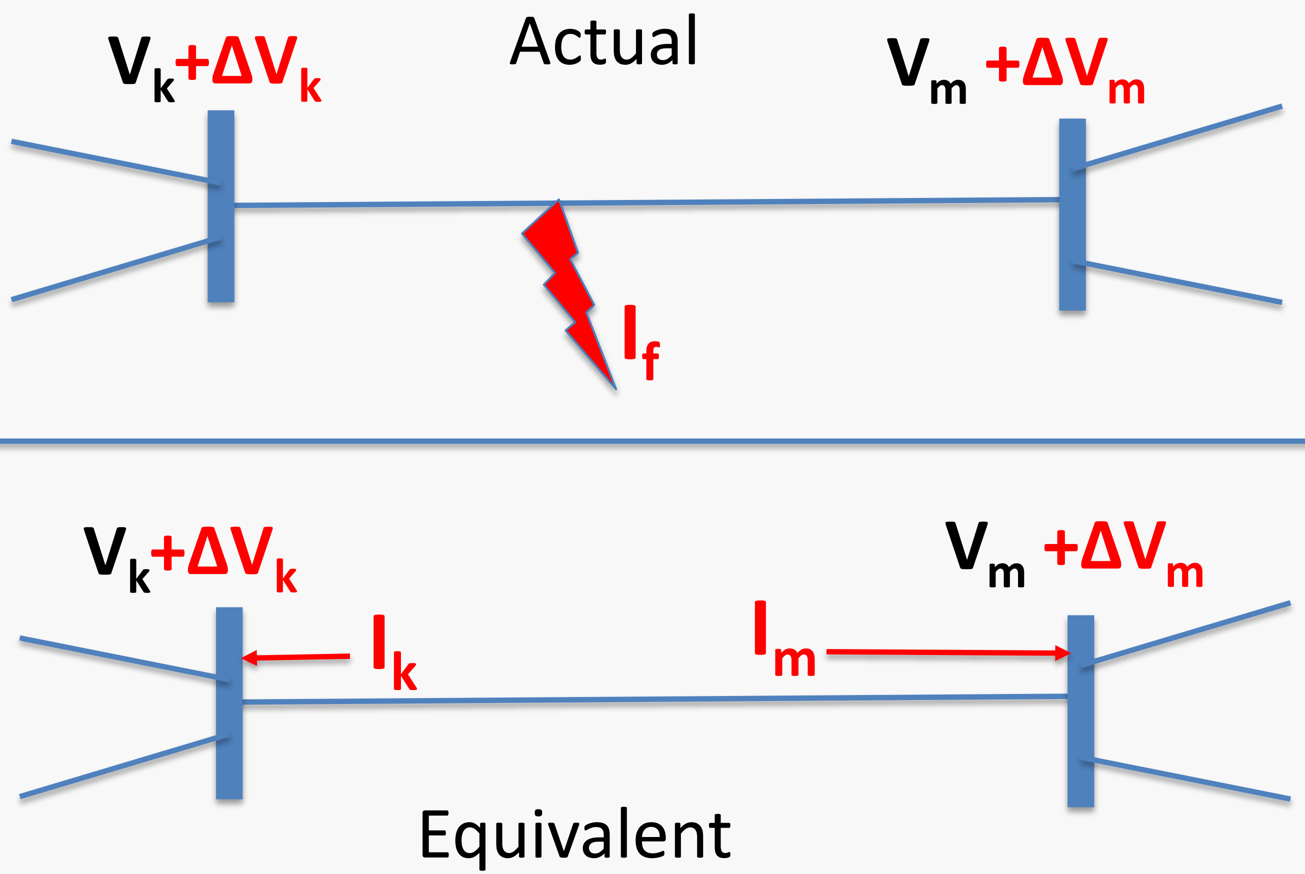


Fault location methods

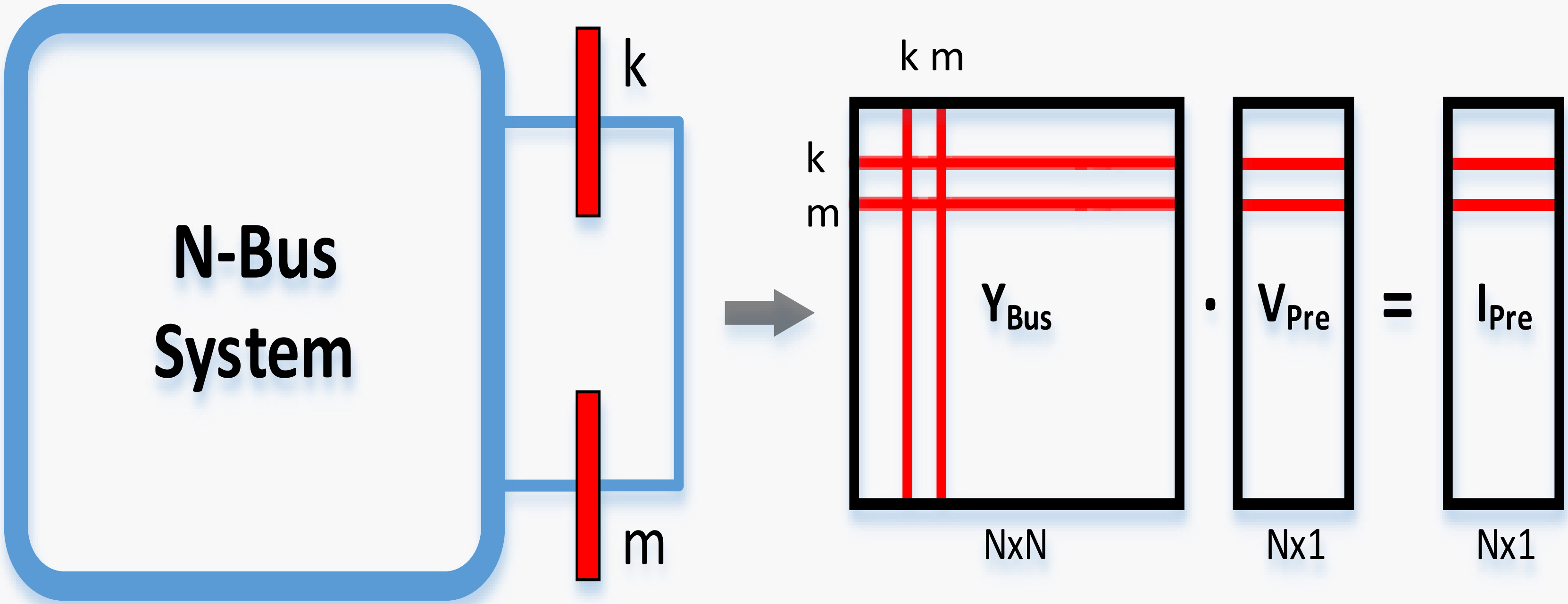
- Power frequency / impedance-based methods
  - Record fault transients at line terminals
  - Filter HF signals, estimate impedance to fault
- Traveling wave-based methods
  - High frequency sampling (> 20KHz)
  - Capture wave front arrival instants using synchronized sensors
- AI / pattern recognition / machine learning based methods

Feng, G. and Abur, A., "Fault Location Using Wide-Area Measurements and Sparse Estimation," IEEE Transactions on Power Systems, Vol. 31, No: 4, pp.2938-2945, (2016).

Equivalent Current Injections

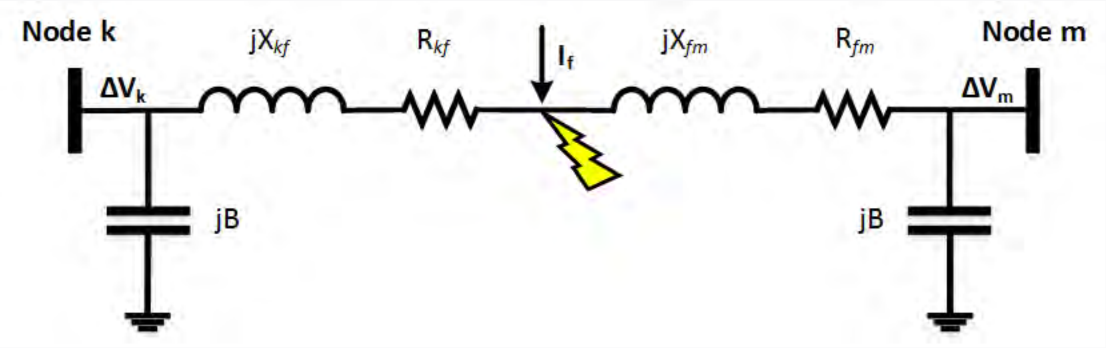


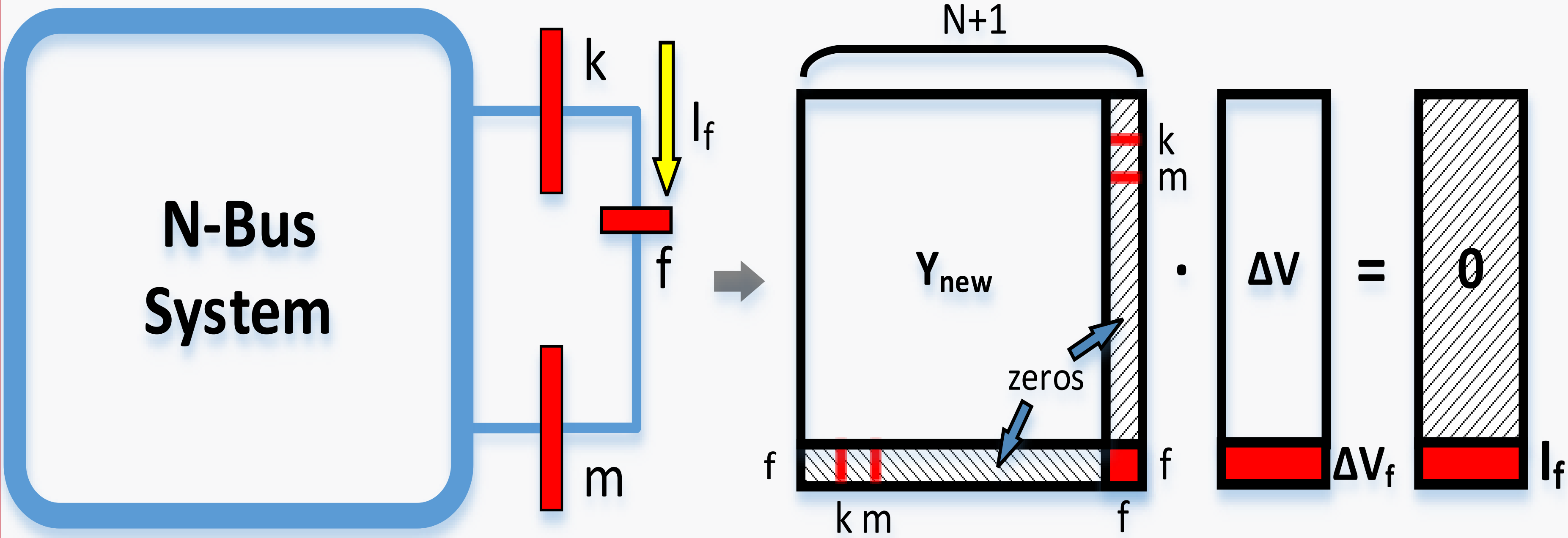
Fault Location Using PMUs



Pre-fault network equations

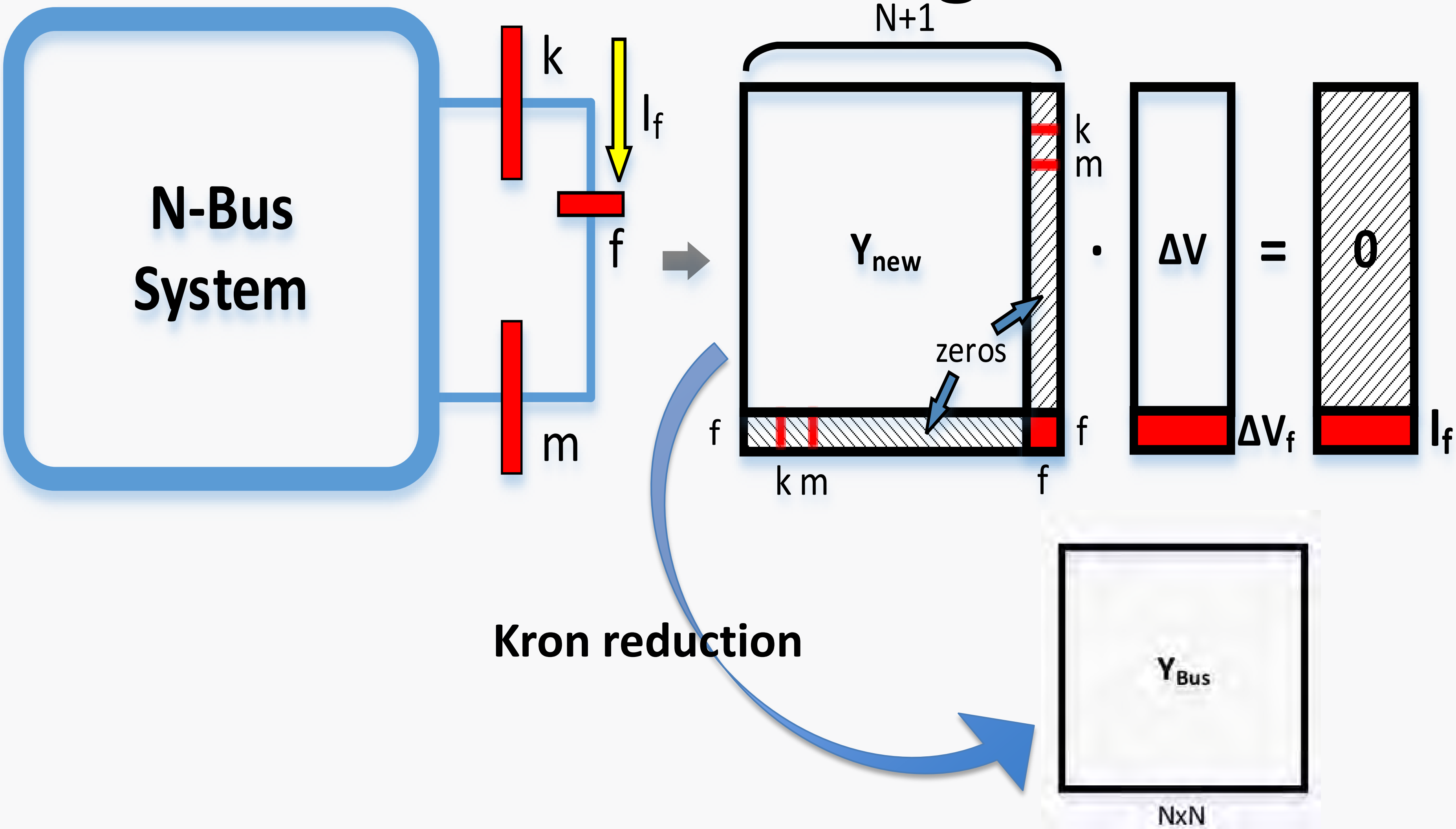
Fault Location Using PMUs



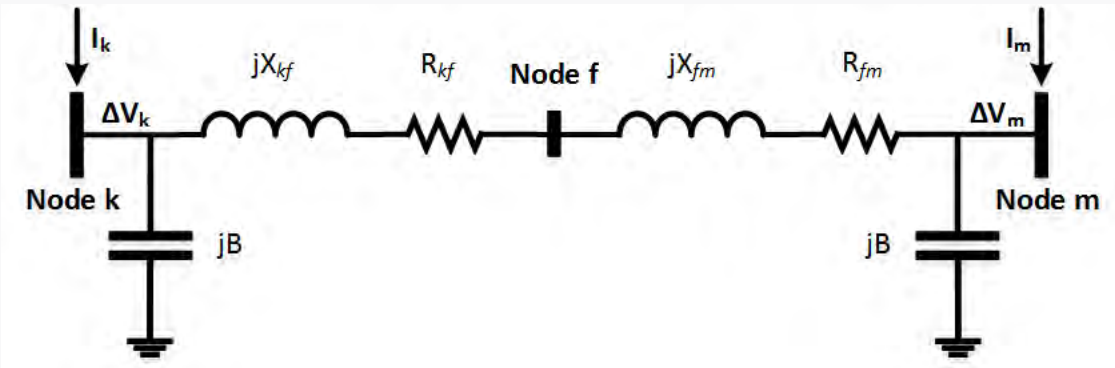


Ali Abur © 2021

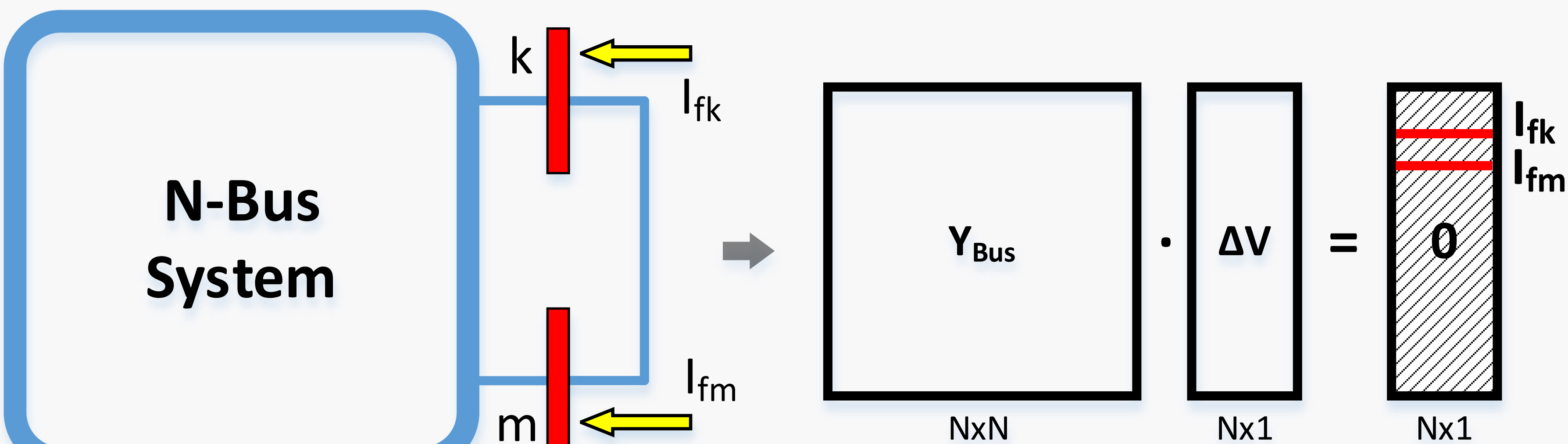
Fault Location Using PMUs



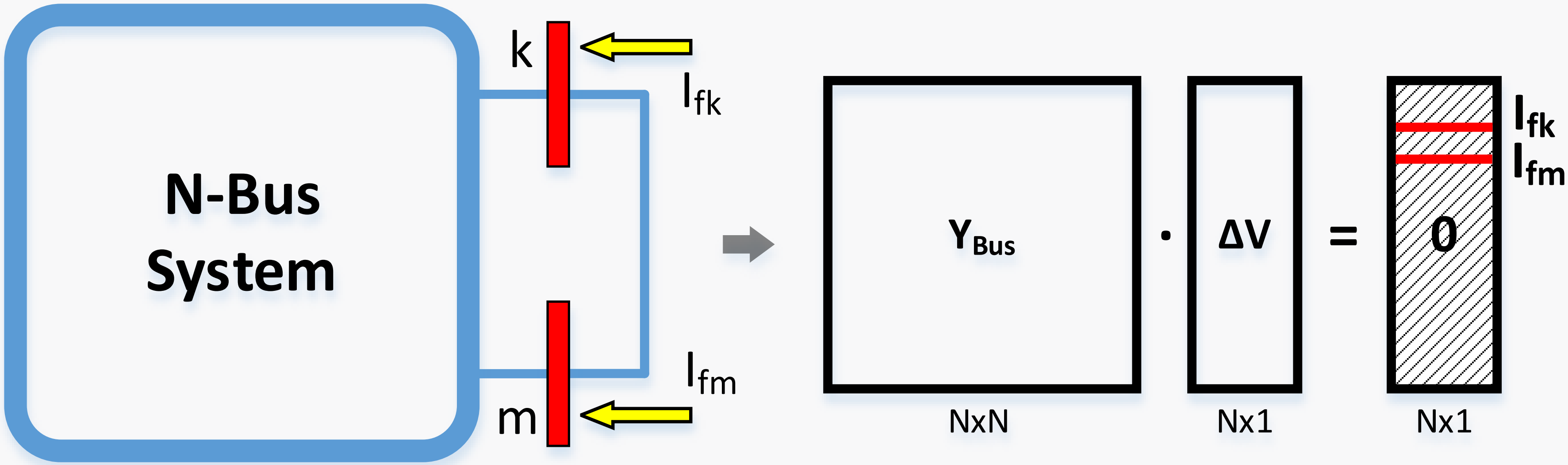
Fault Location Using PMUs



After Kron Reduction:



Fault Location Using PMUs



$$Fault\ Location(m) = \frac{I_{fk}}{I_{fk} + I_{fm}} * 100\%$$

$$Fault\ Location(k) = \frac{I_{fm}}{I_{fk} + I_{fm}} * 100\%$$

Locating Faults Using PMU Measurements

$$[\boldsymbol{Y}][\Delta \boldsymbol{V}] = [\Delta \boldsymbol{I}]$$

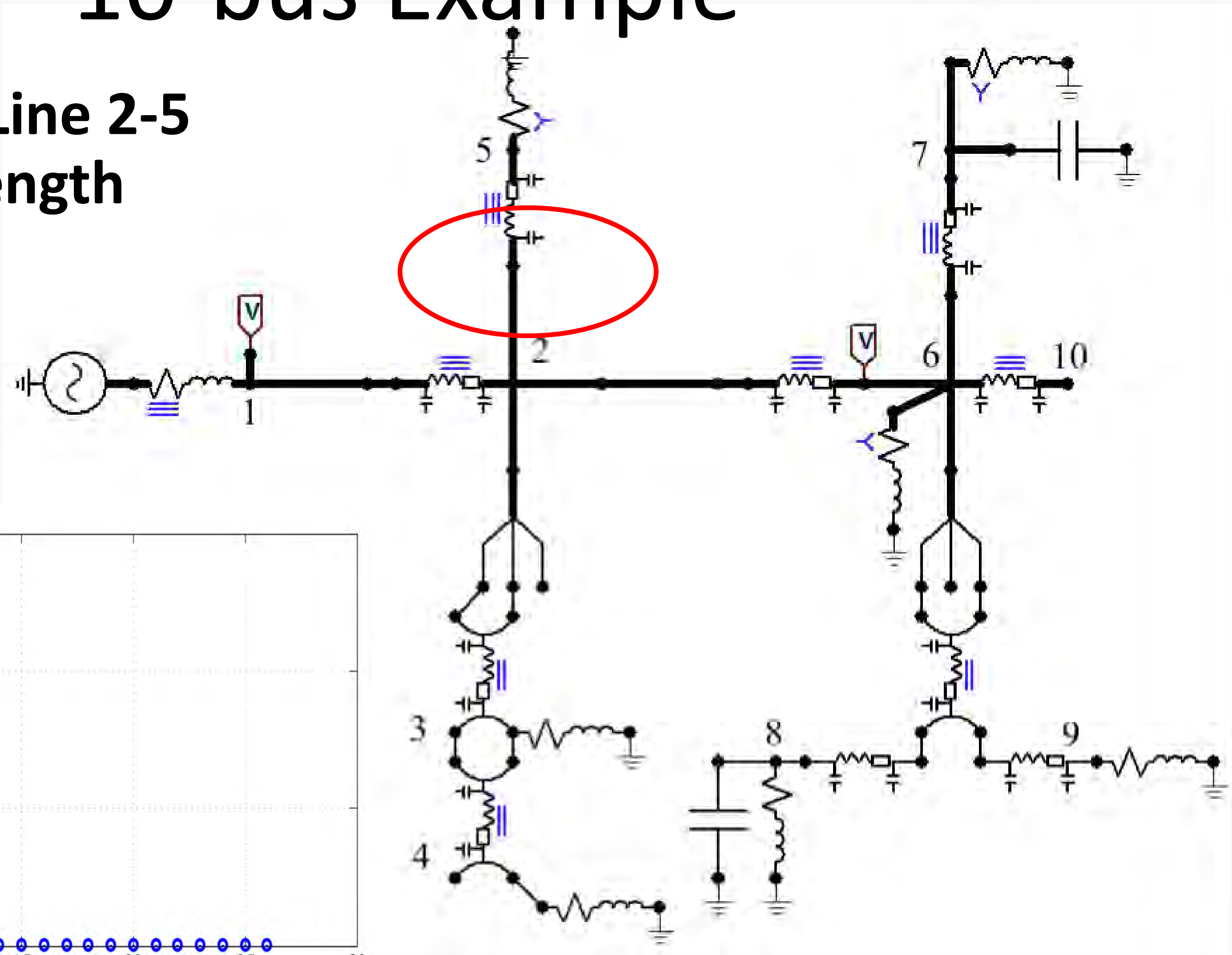
Measure ΔV = (Post fault – Pre fault) Voltage

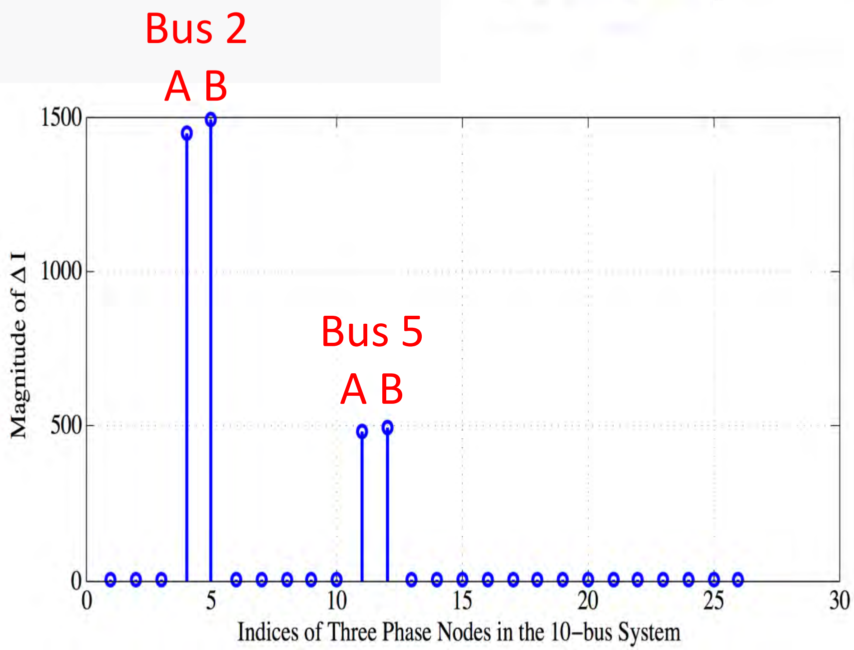
Compute ΔI

Determine distance to fault

10-bus Example

AB-G Fault on Line 2-5 at 1/4 line length





Implementation Challenges

- $[\Delta V] = V^{post} - V^{pre}$
  Post-Fault voltage V^{post} may not be accurately measured by PMUs due to the fast clearing of the fault by protective relays

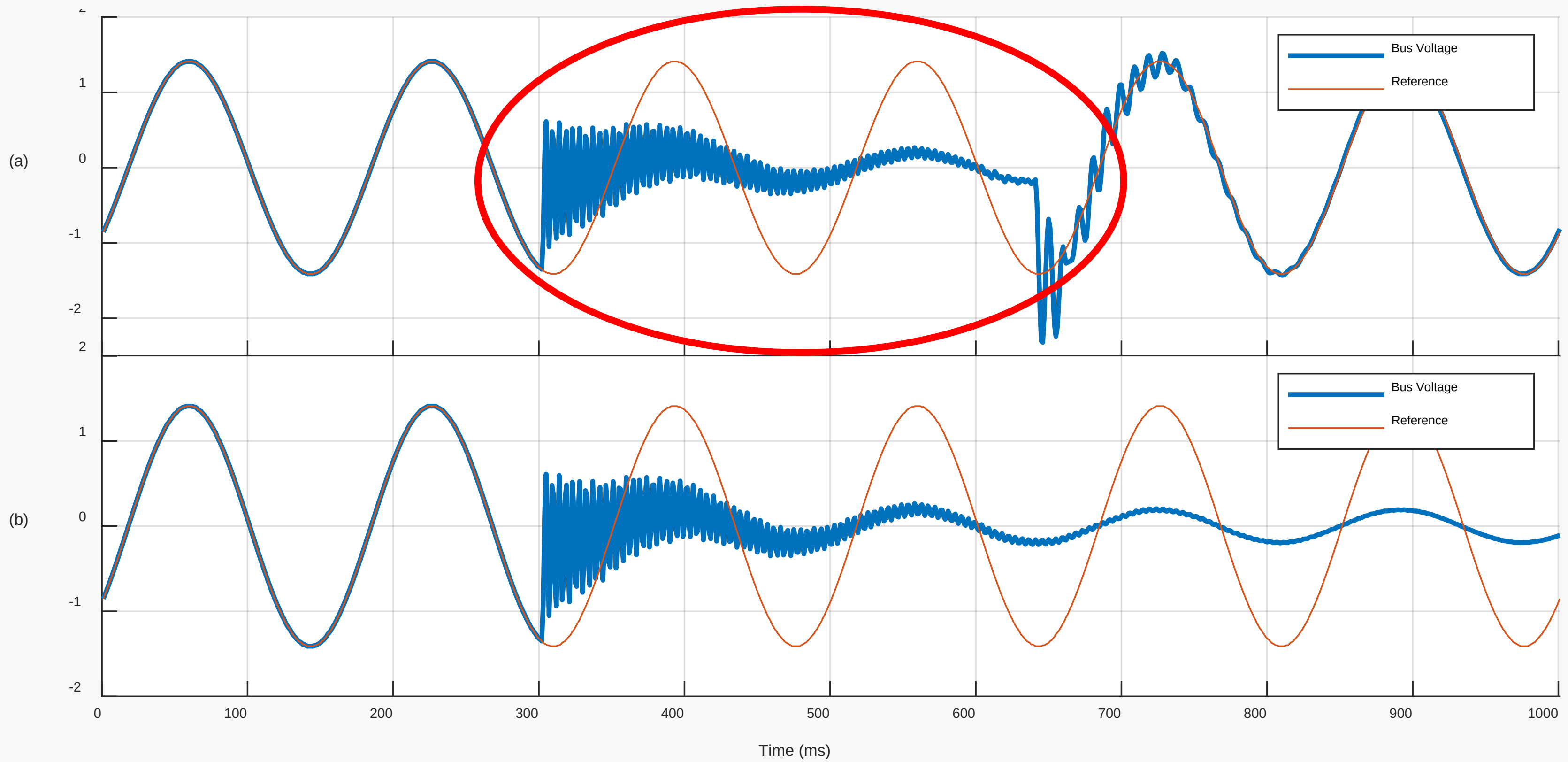
- Not all entries of $[\Delta V]$ may be measured !
  There are a limited number of PMUs, not every bus is equipped with a PMU.

Addressing Challenges

- Use Prony Analysis to estimate the post-fault steady state voltage based on the limited transient samples captured by the PMU.

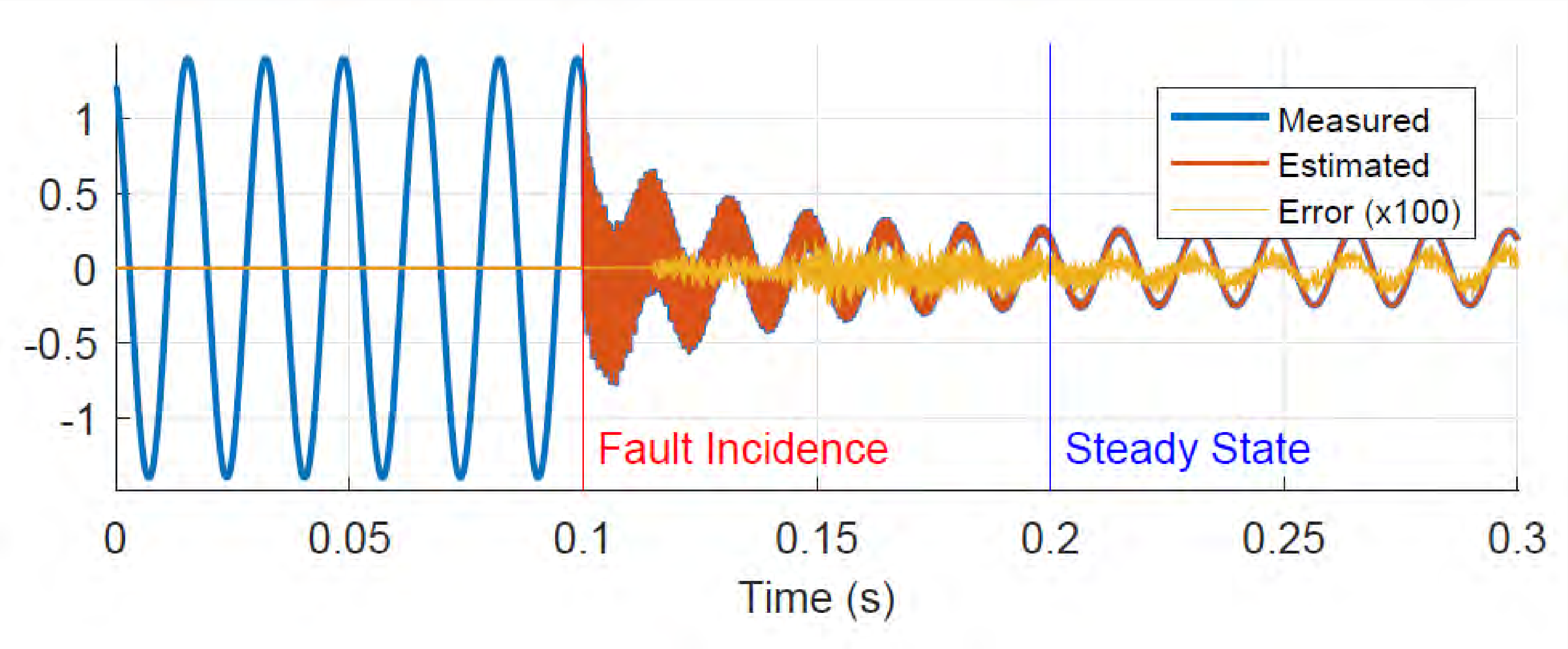
- Use sparse estimation to determine the virtual current injections at the faulted branch terminal buses and determine fault location based on these currents.

Prony Analysis

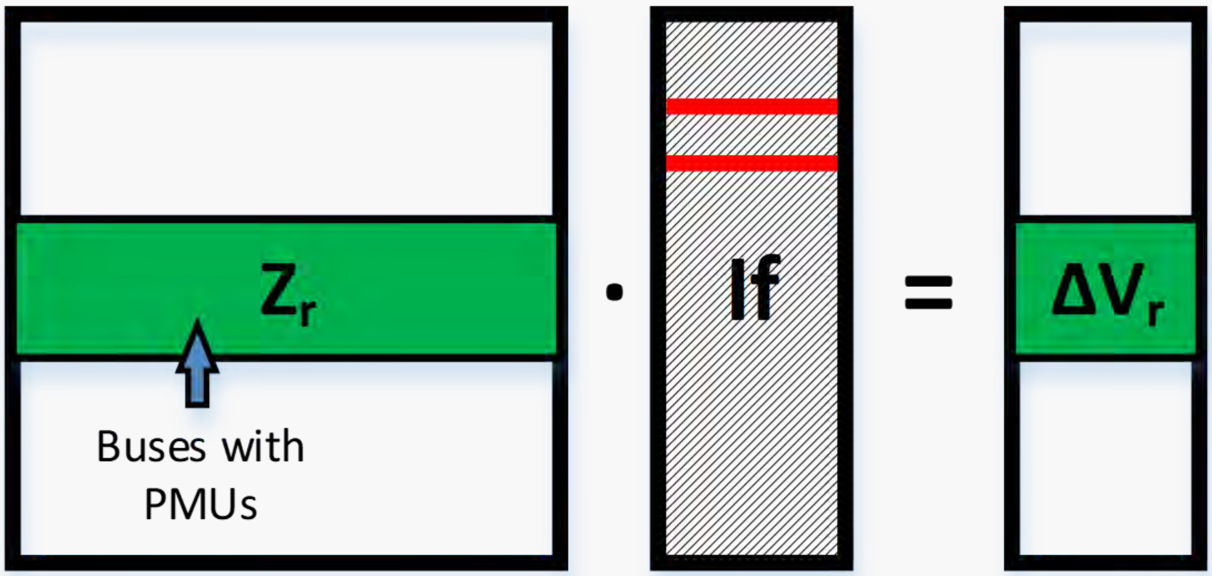


Mouco, A. and Abur, A., "Improvement of Fault Location Method Based on Sparse PMU Measurements," *2017 North American Power Symposium (NAPS)*, Morgantown, WV, 2017, pp. 1-5.

Prony Analysis

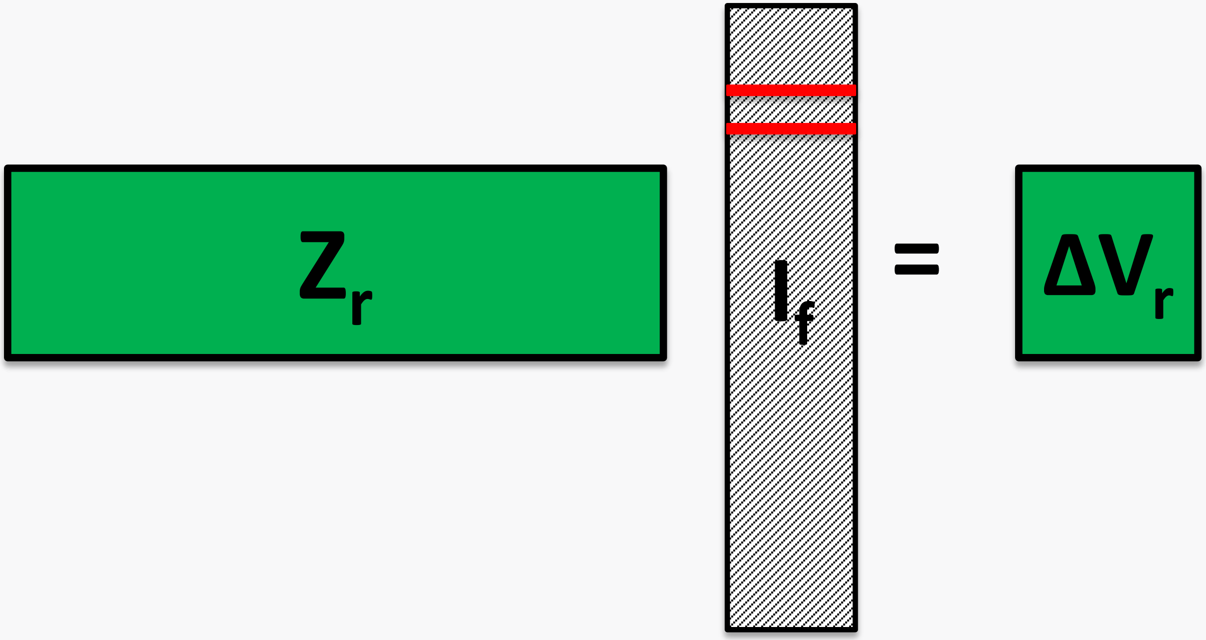


Sparse Estimation



$$[Z_r].[I_f] = [\Delta V_r]$$

Sparse Estimation



$$[Z_r].[I_f] = [\Delta V_r]$$

Sparse Estimation

LASSO - least absolute shrinkage and selection operator

$$\hat{x}(\lambda) = \arg\min_{x} \frac{1}{2}||y - Ax||_2^2 + \lambda||x||_1 \text{ s.t. } \lambda > 0$$

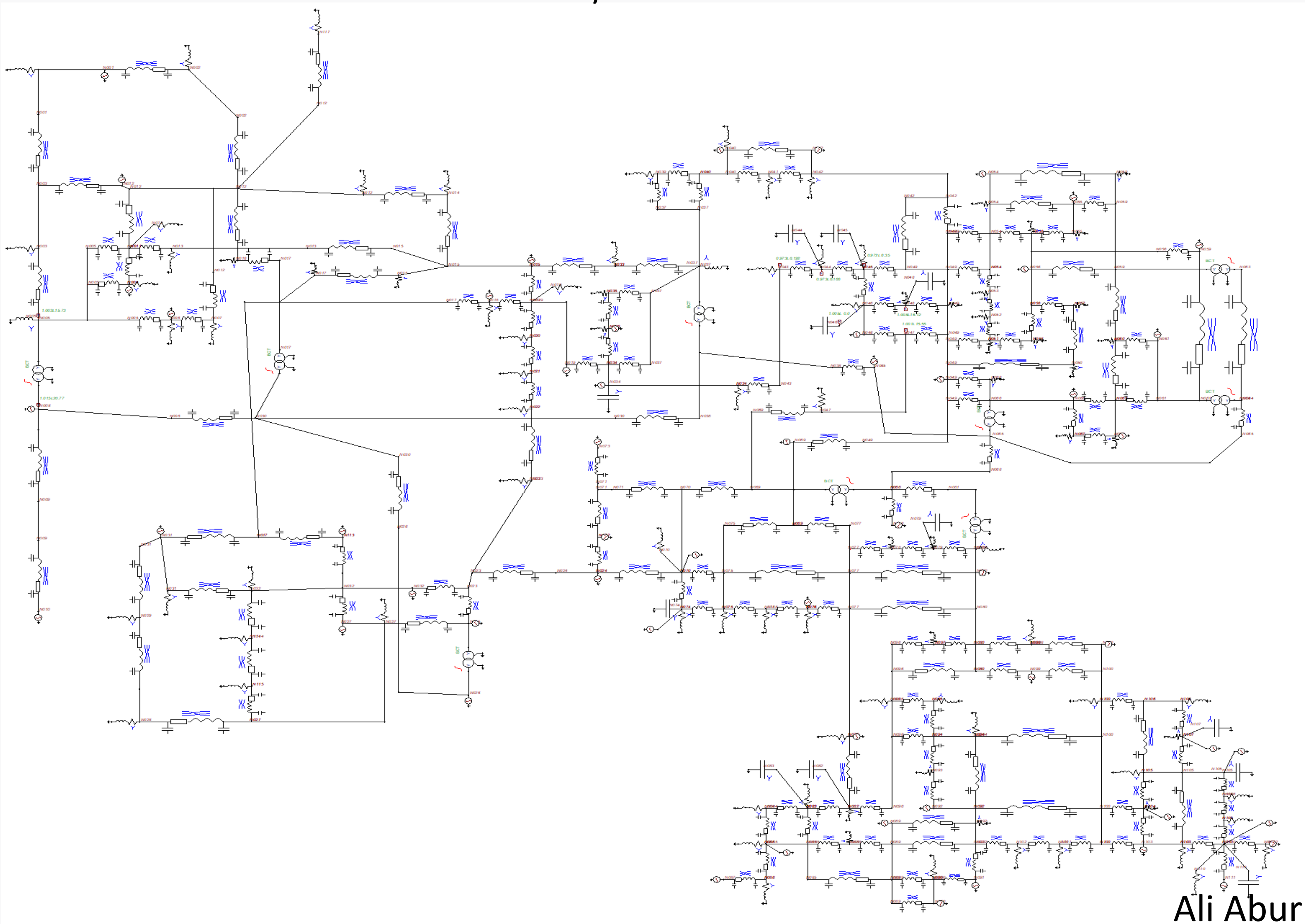
$$[Z_r].[I_f] = [\Delta V_r]$$

$$\begin{bmatrix} \Re(Z_{node}) & -\Im(Z_{node}) \\ \Im(Z_{node}) & \Re(Z_{node}) \end{bmatrix} * \begin{bmatrix} \Delta\Re(I_{node}) \\ \Delta\Im(I_{node}) \end{bmatrix} = \begin{bmatrix} \Delta\Re(V_{node}) \\ \Delta\Im(V_{node}) \end{bmatrix}$$

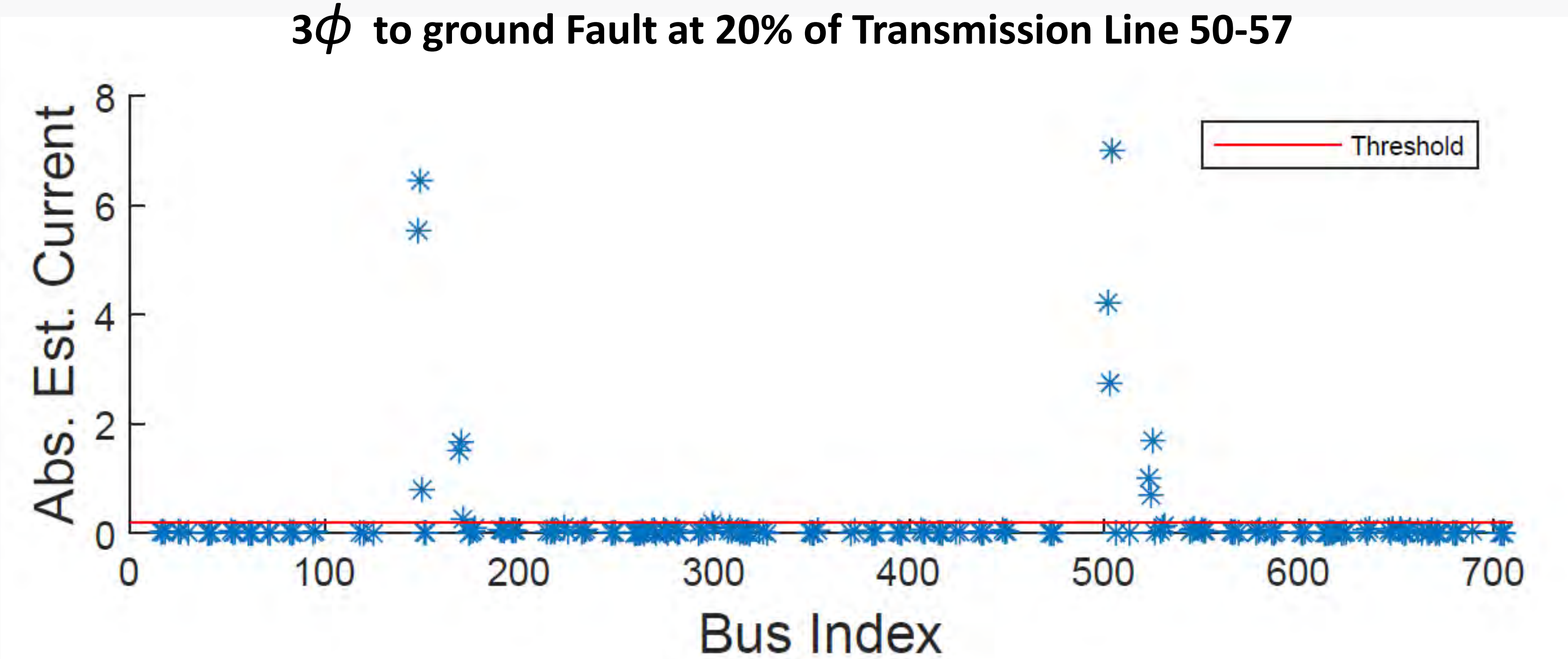
Test System

- Only 31 measured buses
- 3 cycles of fault transient data
- Several fault types tested

3-Phase Model for the IEEE 118 Bus test System



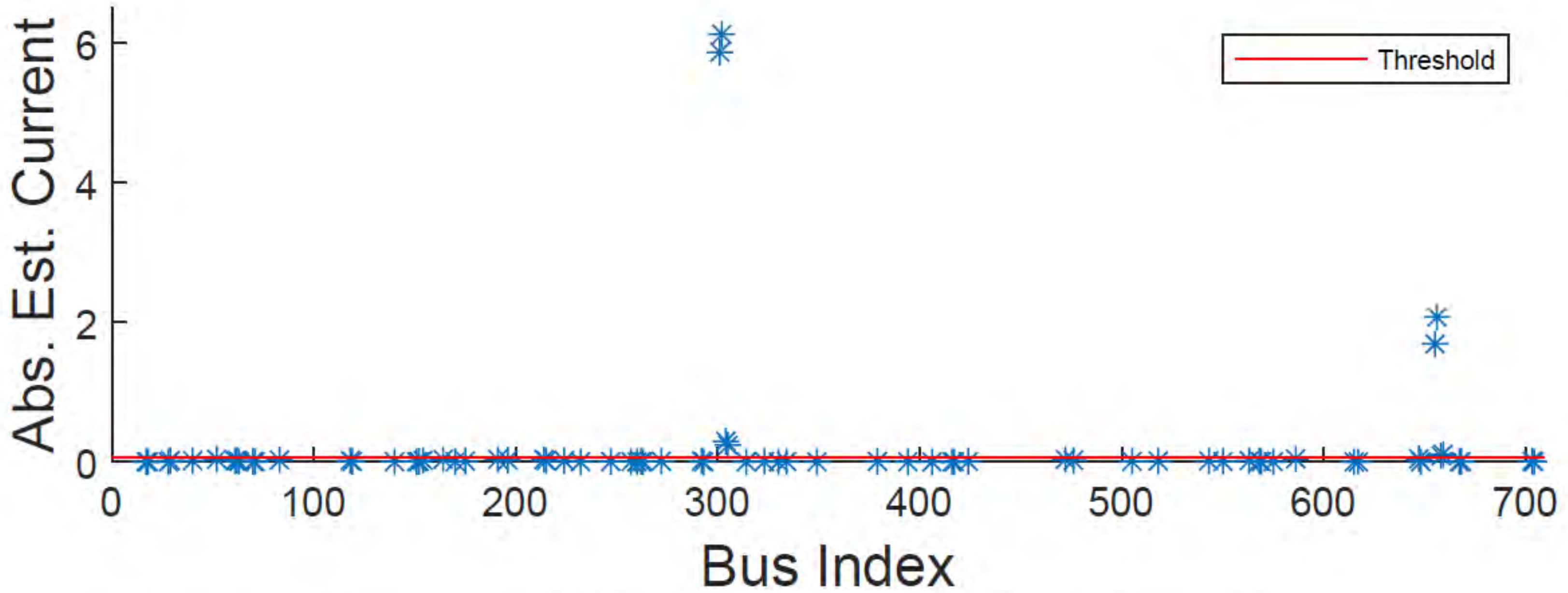
Simulation Results



Bus	Phase	Est. Current	Est. Fault Location
50	A	$5.5416 - 4.2220i$	20.74%
50	B	$-6.4571 - 2.7499i$	20.52%
50	C	$0.8004 + 7.0102i$	19.58%
57	A	$1.5136 - 1.0149i$	79.29%
57	B	$-1.6724 - 0.6971i$	79.48%
57	C	$0.2637 + 1.6973i$	80.43%

Simulation Results

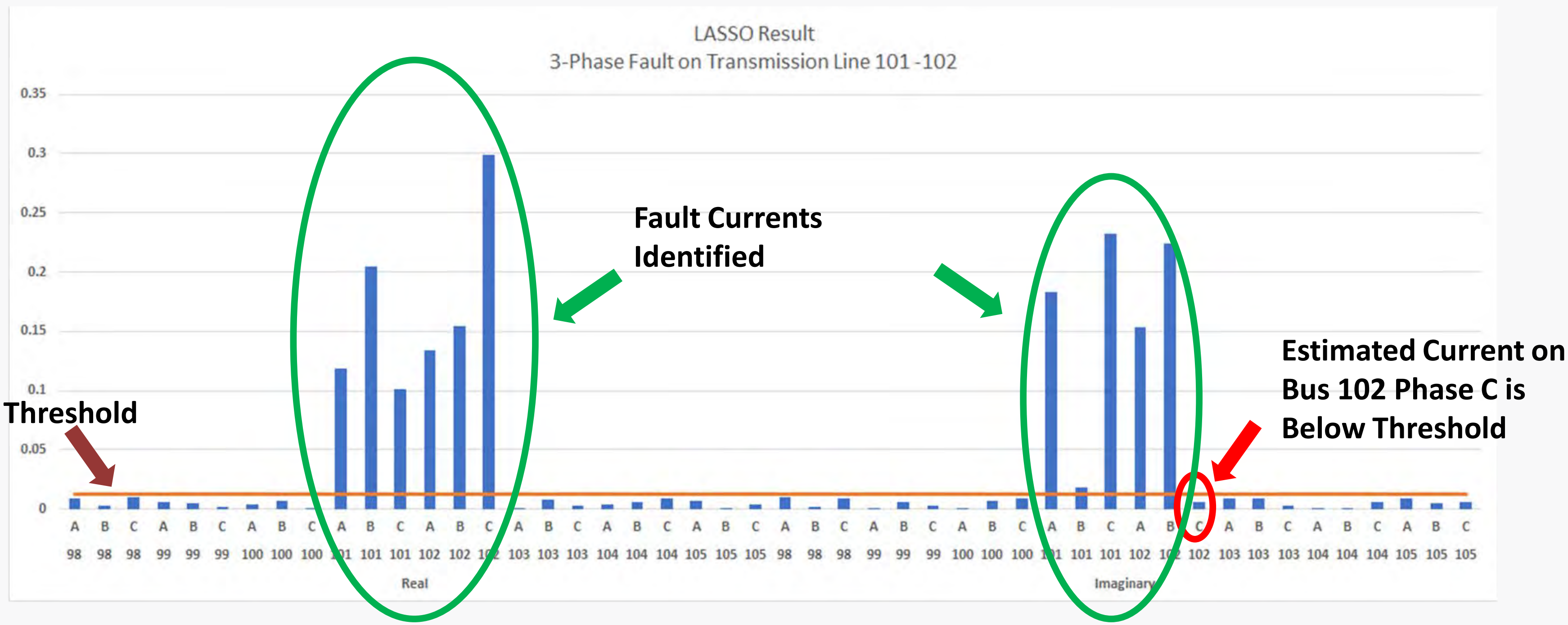
ϕ to ϕ Fault at 5% of Transmission Line 101-102



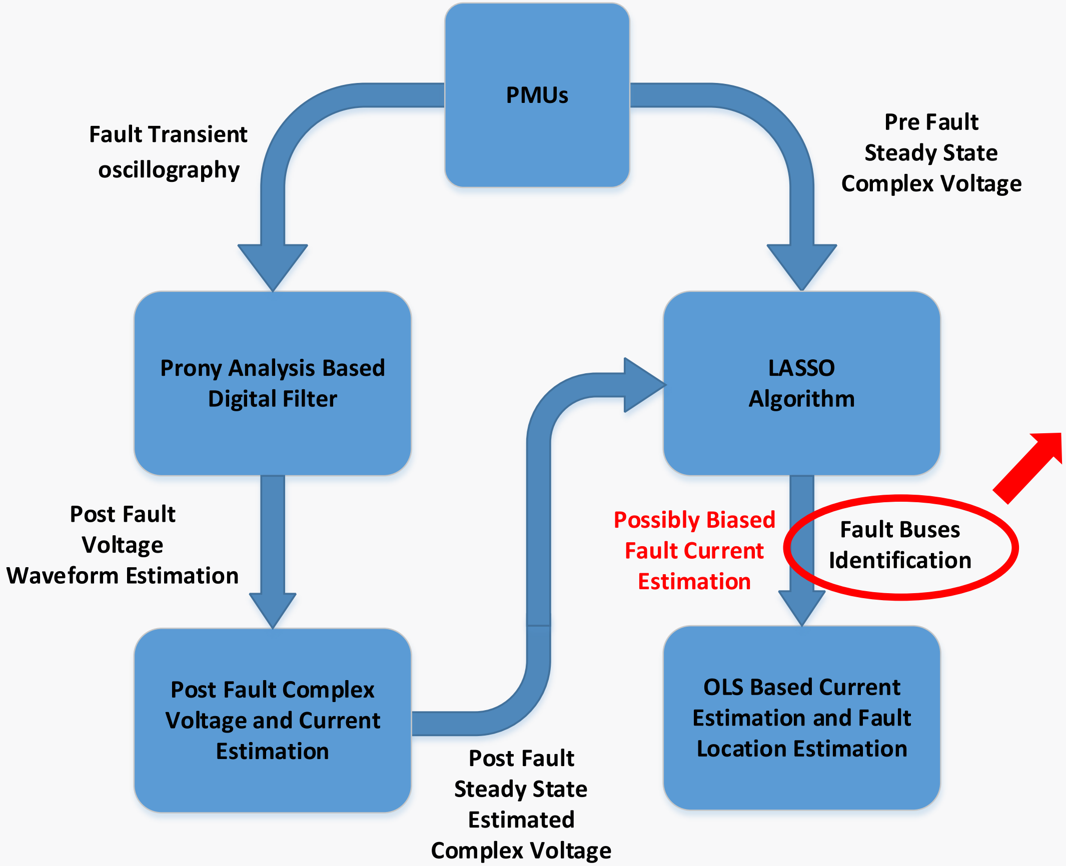
Bus	Phase	Est. Current	Est. Fault Location
101	A	$5.8731 - 1.6913i$	4.80%
101	B	$-6.1302 - 2.0764i$	3.93%
102	A	$0.2956 - 0.0875i$	95.20%
102	B	$-0.2402 - 0.1105i$	96.10%

Limitations

LASSO Algorithm



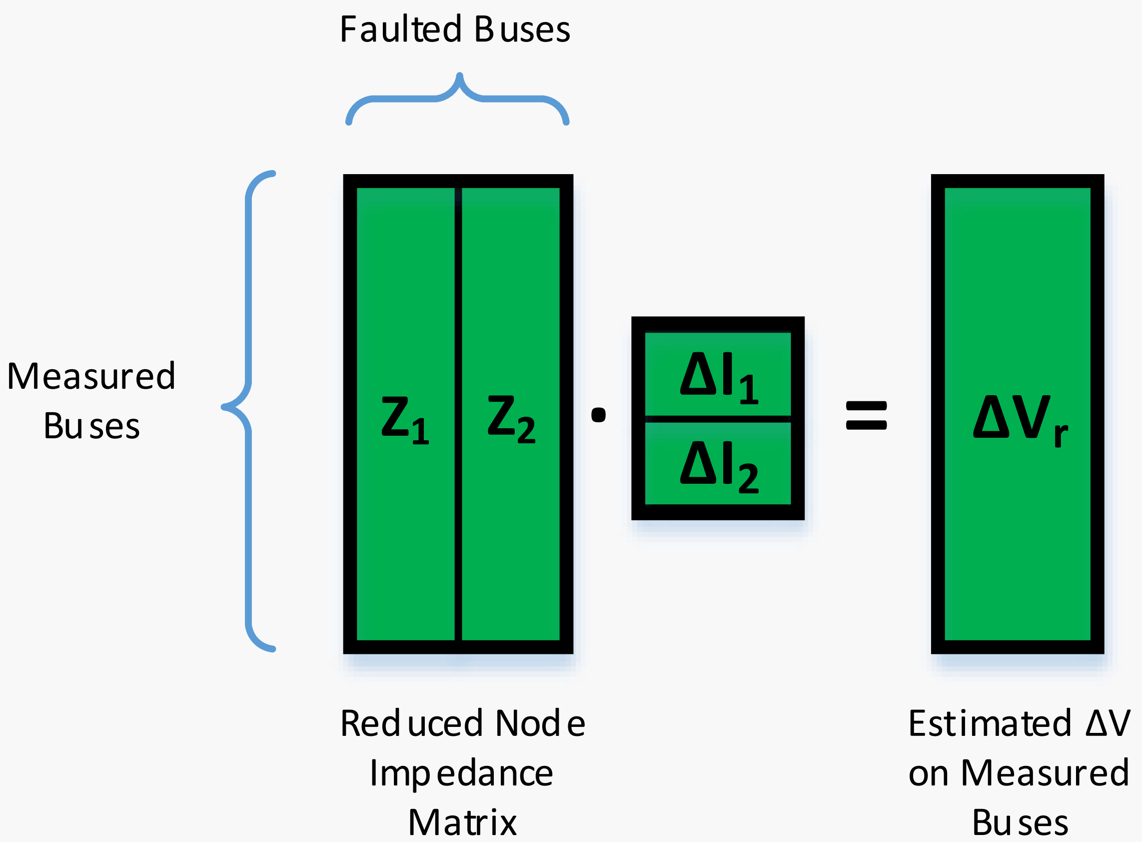
LARS + OLS



- **Current Magnitude from LARS Estimation**
- **System Topology**

Mouco A. and Abur, A., "Improving the wide-area PMU-based fault location method using ordinary least squares estimation," Electric Power Systems Research, vol. 189, pp. 1–7, Dec. 2020.

OLS Solution



LARS + OLS Estimator Results

LASSO Est. Result - Fault at 5% of Line 101-102 - 3ϕ *to ground*

Type	Bus	Phase	Est. Current
Real	6	A	0.0917
Real	78	B	−0.1265
Real	101	A	5.8978
Real	101	B	−5.7855
Real	101	C	−0.1386
Real	102	A	0.4063
Real	102	B	−0.3320
Imaginary	78	A	0.0989
Imaginary	101	A	−3.3033
Imaginary	101	B	−3.5980
Imaginary	101	C	6.8924
Imaginary	102	A	−0.1467
Imaginary	102	B	−0.1721
Imaginary	102	C	0.3244

OLS Est. Result - Fault at 5% of Line 101-102 - 3ϕ *to ground*

Bus	Phase	Est. Current	Est. Fault Location
101	A	$6.1822 - 3.2711i$	4.08%
101	B	$-5.9551 - 3.6761i$	4.20%
101	C	$-0.2138 + 6.9854i$	4.29%
102	A	$0.2385 - 0.1780i$	95.96%
102	B	$-0.2606 - 0.1621i$	95.80%
102	C	$0.0121 + 0.3129i$	95.72%

LARS + OLS Estimator Results

LASSO Est. Result - Fault at 10% of Line 50-57 - 3ϕ *to ground*

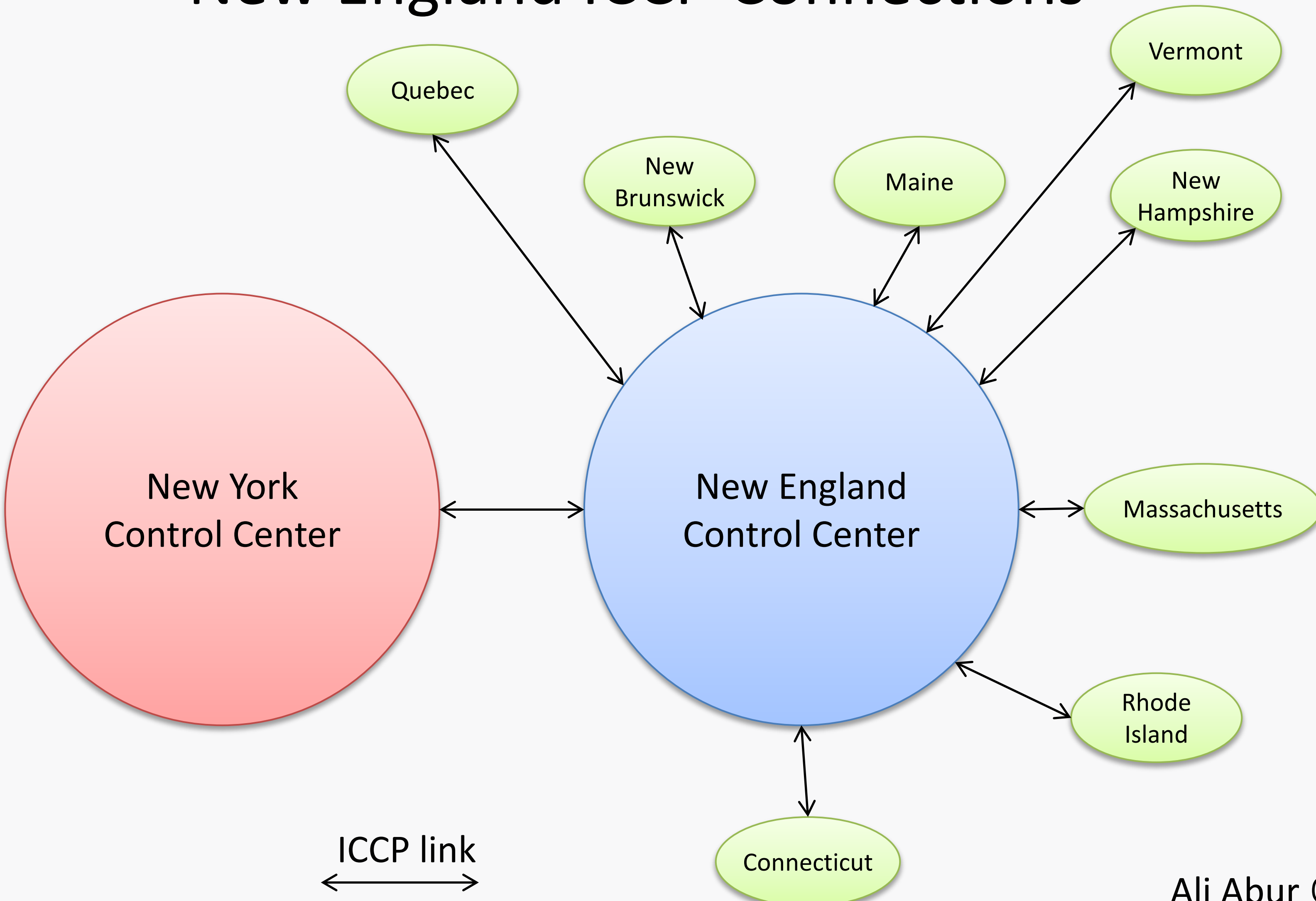
Type	Bus	Phase	Est. Current
Real	50	A	6.6690
Real	50	B	−7.6427
Real	50	C	0.9032
Real	57	A	0.8495
Real	57	B	−0.8322
Imaginary	50	A	−4.7212
Imaginary	50	B	−3.4832
Imaginary	50	C	7.6721
Imaginary	51	C	1.5850
Imaginary	57	A	−0.6394

OLS Est. Result - Fault at 10% of Line 50-57 - 3ϕ *to ground*

Bus	Phase	Est. Current	Est. Fault Location
50	A	7.0160 - 4.7574i	8.71%
50	B	-7.6005 - 3.6256i	9.21%
50	C	0.6838 + 8.4229i	9.24%
57	A	0.5537 - 0.5877i	91.48%
57	B	-0.8150 - 0.2547i	90.87%
57	C	0.1908 + 0.8378i	90.85%

Tracking External Network Model

New England ICCP Connections



External Area Line Outage Detection

Can we detect outages in the external system

- using PMU measurements at area boundary buses

and

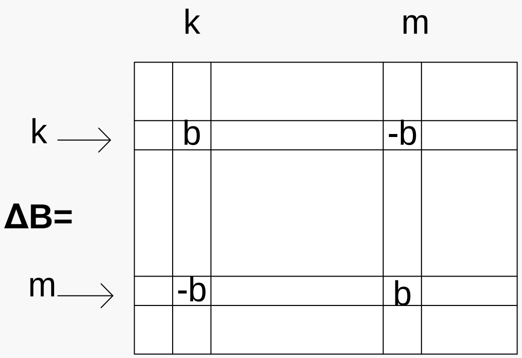
- without receiving any SCADA measurement updates from the external system ?

Dönmez, B. and Abur, A., "Sparse Estimation Based External System Line Outage Detection," Proceedings of the Power System Computations Conference (PSCC), Genoa, Italy, June 20-24, 2016.

Line Switching Modeled by Bus Injections

$$B\theta_0 = P_0$$

$$(B + \Delta B)\theta_1 = P_0$$



$$B\theta_1 = P_0 - \Delta P$$

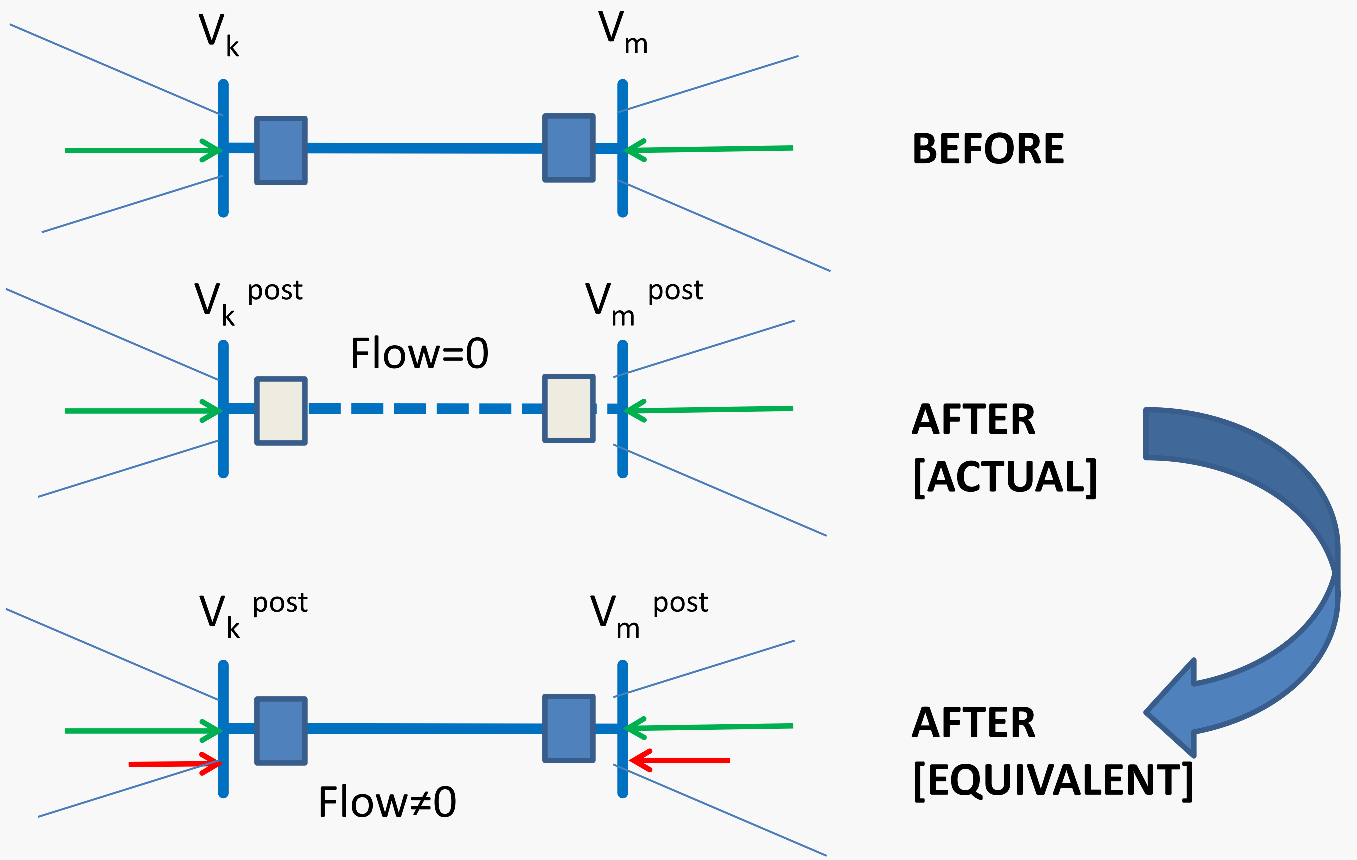
Sparse Bus Injection Vector

Pre-outage topology

Post-outage state of the system

$$\Delta P = \left[0 \ldots 0\, p\, 0 \ldots 0 - p\, 0 \ldots 0\right]^T$$

Problem Formulation



Problem Formulation for Line Outages

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_n \end{bmatrix} = \boldsymbol{B} \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \vdots \\ \Delta \theta_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \quad (1)$$

$$\boldsymbol{B_{st}} = \begin{cases} -\dfrac{1}{x_{st}}, & \text{for } (s,t) \in L \text{ and } s \neq t \\ \displaystyle\sum_{t \in N_s} \dfrac{1}{x_{st}}, & \text{if } s = t \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$\boldsymbol{B}$ is the DC jacobian, x_{st} is the reactance of branch s-t.

Problem Formulation for Line Outages

Partition the system w.r to internal and external buses:

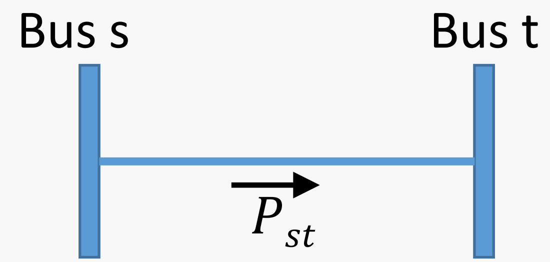
$$\begin{bmatrix} \Delta P_i \\ \Delta P_e \end{bmatrix} = \begin{bmatrix} B_{ii} & B_{ie} \\ B_{ei} & B_{ee} \end{bmatrix} \begin{bmatrix} \Delta \theta_i \\ \Delta \theta_e \end{bmatrix} + \begin{bmatrix} e_i \\ e_e \end{bmatrix} \quad (3)$$

An external line outage will lead to changes in $\boldsymbol{B}$ matrix and the bus angle vector $\boldsymbol{\theta}$.

Problem Formulation for Line Outages[*]

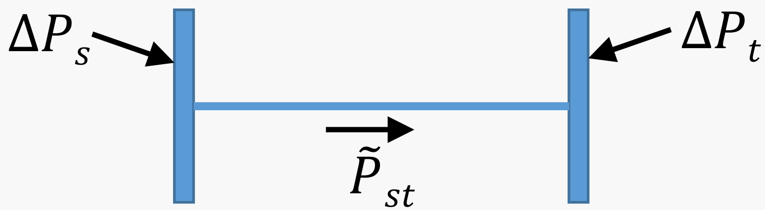
Representing the outage by equivalent injections

$$\Delta P_s = -\Delta P_t = \tilde{P}_{st} \qquad (4)$$



Pre-contingency flow on branch s-t

$$\tilde{P}_s = \left[\frac{1}{1 - \frac{1}{x_{st}}(B_{ss} + B_{tt} - 2B_{st})}\right] P_{st} \qquad (5)$$



Post-contingency flow on branch s-t

[*] Allen J. Wood, Bruce F. Wollenberg, Gerald B. Sheble, “Power Generation, Operation and Control,” 3rd Edition, Wiley (Book)

Problem Formulation for Line Outages

Post-contingency $\boldsymbol{\Delta P_e}$ will have the form:

$$\boldsymbol{\Delta P_e} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \Delta P_s \\ 0 \\ \vdots \\ 0 \\ \Delta P_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (6)$$

where the two non-zeros will correspond to the outaged line.

Problem Formulation for Line Outages

Eliminating $\boldsymbol{\Delta\theta_e}$ in (3):

$$J = M \Delta P_e + e \quad (7)$$

$$J = (B_{ie} B_{ee}^{-1} B_{ei} - B_{ii}) \Delta\theta_i + \Delta P_i \quad (8)$$

$$M = B_{ie} B_{ee}^{-1} \quad (9)$$

$$e = e_i - B_{ie} B_{ee}^{-1} e_e \quad (10)$$

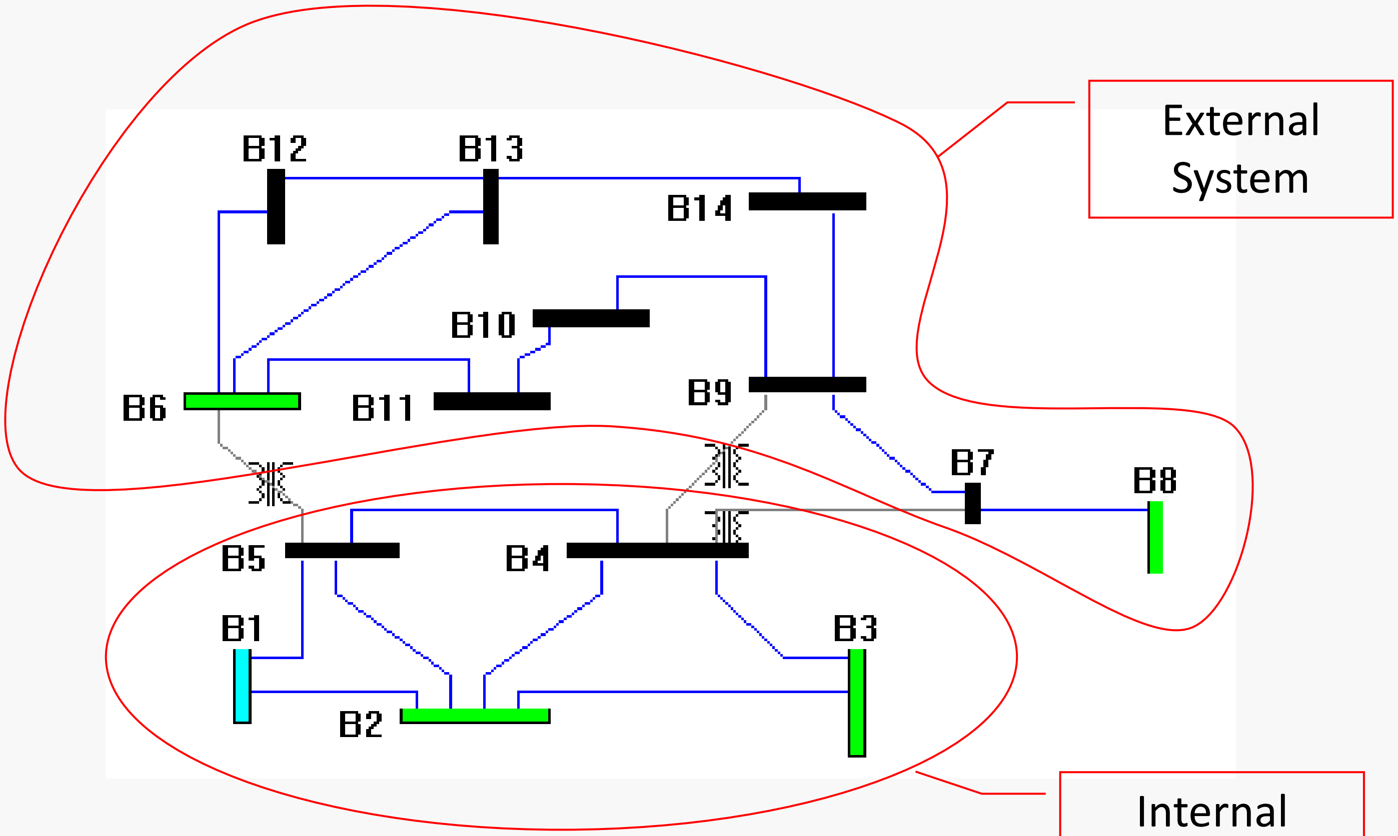
Sparse Estimation

$$\Delta P_e := \underset{\Delta P_e}{\operatorname{argmin}} \|J - M\Delta P_e\|_2^2 + \lambda \|\Delta P_e\|_1 \qquad (11)$$

Note about the choice of λ :

The algorithm starts with a large λ and reduces it exponentially in each subsequent iteration.

Example: IEEE 14-bus case



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Example: IEEE 14-bus case

Actual Outage **Branch** / $\tilde{P}_{st}$ (MW)	Detected Outage Iteration No. / **Branch** / $\tilde{P}_{st}$ (MW)		
7-9 / -103.2	1 / **7-9** / -109.9		
9-10 / 101.4	1 / **9-10** / 95.8		
10-11 / -24.1	1 / **9-10** / -22.4	2 / **10-11** / -23.1	
6-11 / 144.0	1 / **9-10** / 162.7	2 / **10-11** / -167.5	3 / **6-11** / 139.0
6-12 / -87.5	1 / **6-12** / -90.3		
6-13 / -211.3	1 / **6-12** / -630.0	2 / **6-13** / -212.2	
12-13 / -56.8	1 / **6-12** / -114.5	2 / **6-13** / -38.6	3 / **12-13** / -51.3

Example: IEEE 118-bus case

System Configuration

- Internal buses: {1-45,113,114,115,117}
- External buses: {46-112,116,118}
- Detection is tested for 98 branch outages in the external system

Simulated Cases

- Case 1: No PMUs in the external system
- Case 2: PMUs installed at buses 93 and 109
- Case 3: PMUs installed at buses 93, 109 and 50

Example: IEEE 118-bus case

Case 1: No PMUs in external system

- 79 out of 98 outages are detected
- 19 undetected outages are electrically far from the internal system boundary

Case 2: PMUs at 93 and 109

- 5 undetected outages

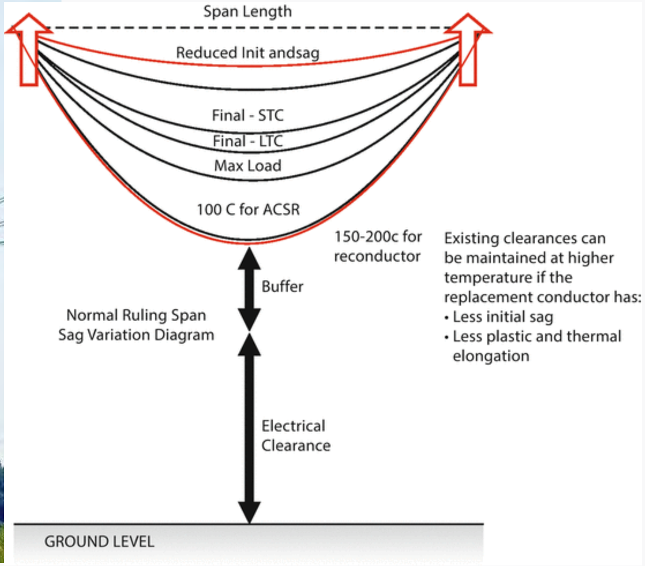
Branch No.	From Bus	To Bus
54	49	50
60	54	56
67	63	64
165	62	67
177	68	81

Case 3: PMUs at 93, 109, 50

- 3 undetected outages

Branch No.	From Bus	To Bus
67	63	64
130	101	102
177	68	81

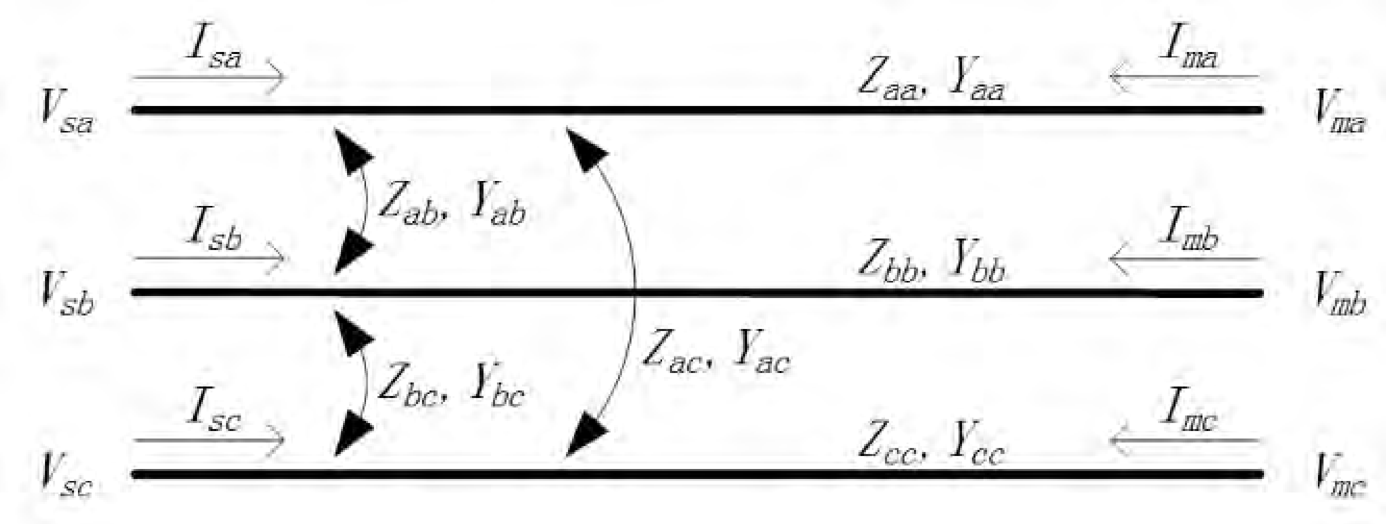
Online Tracking Transmission Line Parameters Using PMU Measurements



Ren, Pengxiang, Lev-Ari H., and Abur, A., "Tracking Three Phase Un-transposed Transmission Line Parameters Using Synchronized Measurements" IEEE Transactions on Power Systems, vo. 33, no. 4, pp.4155-4163, July 2018.

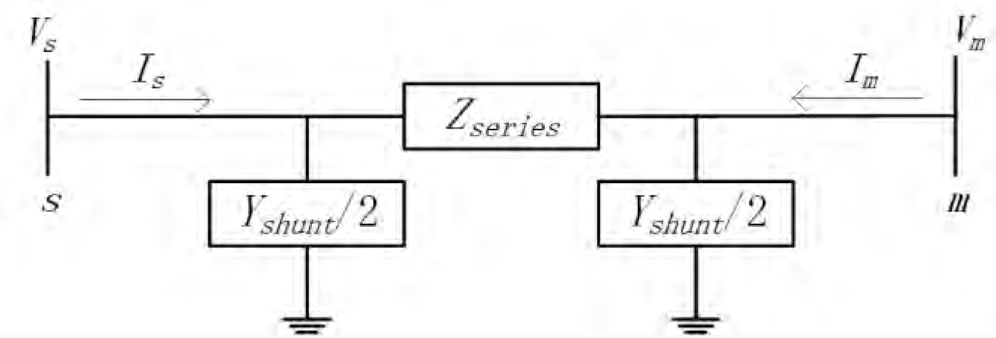
Line Model

PMU measurements at both ends of the line



Line Model

For the equivalent pi-model



$$V_s = \begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix}, \qquad I_s = \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \end{bmatrix},$$

$$V_m = \begin{bmatrix} V_{ma} \\ V_{mb} \\ V_{mc} \end{bmatrix}, \qquad I_m = \begin{bmatrix} I_{ma} \\ I_{mb} \\ I_{mc} \end{bmatrix},$$

$$\widetilde{V} = \begin{bmatrix} V_s \\ V_m \end{bmatrix}, \qquad \widetilde{I} = \begin{bmatrix} I_s \\ I_m \end{bmatrix}.$$

The nodal equations will be:

$$I = YV$$

$$Y = \begin{bmatrix} Y_{shunt}/2 + Z_{series}^{-1} & -Z_{series}^{-1} \\ -Z_{series}^{-1} & Y_{shunt}/2 + Z_{series}^{-1} \end{bmatrix}$$

Un-transposed Line with Terminal PMU Measurements

Number of independent parameters:

6 in Z_{series} , 6 (only imaginary) in Y_{shunt}

→ Total: 2 * 6 + 6 = **18 real unknowns**, hence the dimension of the parameter vector *p* will be **18**.

$$Z_{series} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \qquad Y_{shunt} = \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ab} & Y_{bb} & Y_{bc} \\ Y_{ac} & Y_{bc} & Y_{cc} \end{bmatrix}$$

Measurement Equations

- Using rectangular coordinates:

  $$I_{\text{(6-by-1 complex vector)}} = Y_{\text{(6-by-6 complex matrix)}} V_{\text{(6-by-1 complex vector)}}$$

  $$I_{\text{(12-by-1 real vector)}} = H_{p\ \text{(12-by-12 real matrix)}} V_{\text{(12-by 1 real vector)}}$$

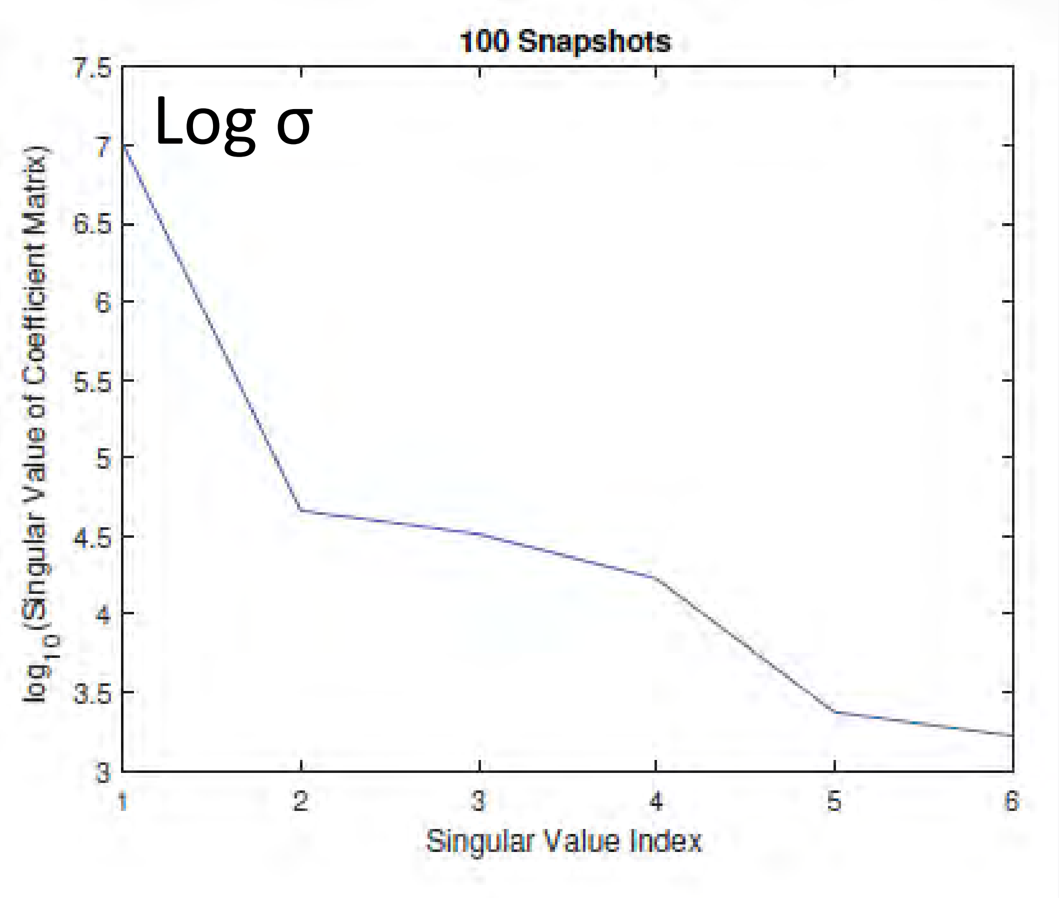
- $I = H_p V$ can also be rearranged as $I = H_V p$ where
  - I is a 12-by-1 vector that contains the current measurements in rectangular coordinates from both terminals of the line.
  - p is the 18-by-1 unknown parameter vector.
  - H_V is the 12-by-18 rearranged coefficient matrix consisting of measured voltages at the line terminals.
- More unknowns p (18) than measurements I (12)!

Parameter Tracking

Multiple measurement snapshots CANNOT be effectively used since H_V will still have low rank ! (voltages do not vary much between scans)

Cond no = σ_{max} / σ_{min}

~ 10^4 Large !



Parameter Tracking

Alternative solution:

Use a Kalman filter to track parameter variations

Discrete-time model for parameters

$$p_{k+1} = p_k + w_{p,k}$$

Discrete-time measurement equations:

$$z_{p,k} = I_k = H_{V,k}\, p_k + v_k$$

Voltage Estimation

Measurement (PMU) equations:

$$\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} Identity\ matrix \\ H_p \end{bmatrix} x + e$$

x : 12x1 voltage vector in rectangular form,

H_p : 12x12 coefficient matrix dependent on parameters

e : measurement noise.

- Solve the estimation problem by WLS method
- Parameters used in H_p come from the parameter tracking algorithm.

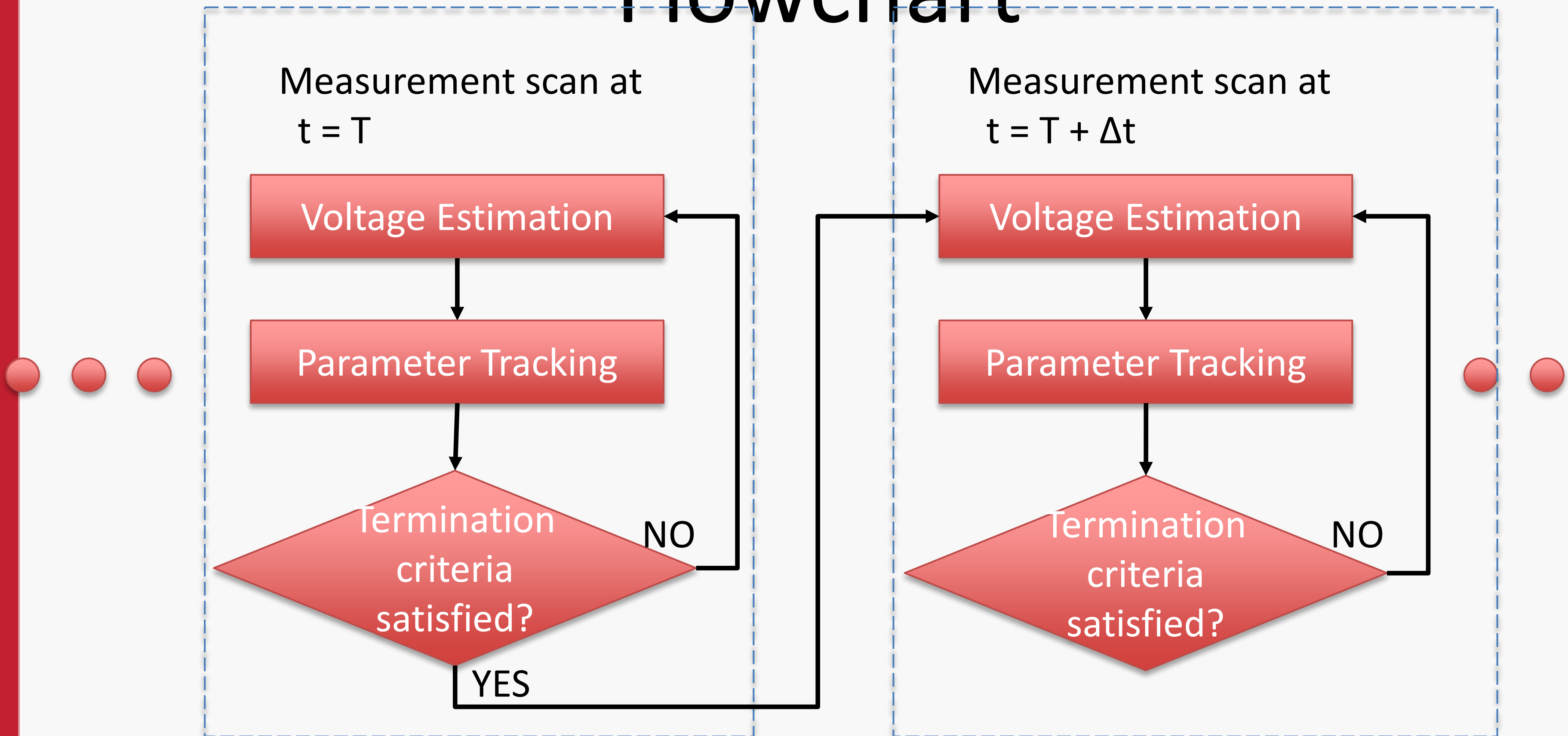
Voltage – Parameter Interdependency

- Voltages are estimated using parameters and current measurements
- Parameters are estimated using voltages and current measurements

Iterative Solution

- For each scan, iterate between state estimation and parameter tracking
  - **voltage estimation**, which relies on the most recent parameter estimates and suppresses voltage and current measurement noise, and
  - **parameter tracking**, which relies on most recent voltage estimates and further suppresses current measurement noise.
- Terminate iterations when two successive parameter estimates are close enough or reach the allowable limits on iteration.

Flowchart

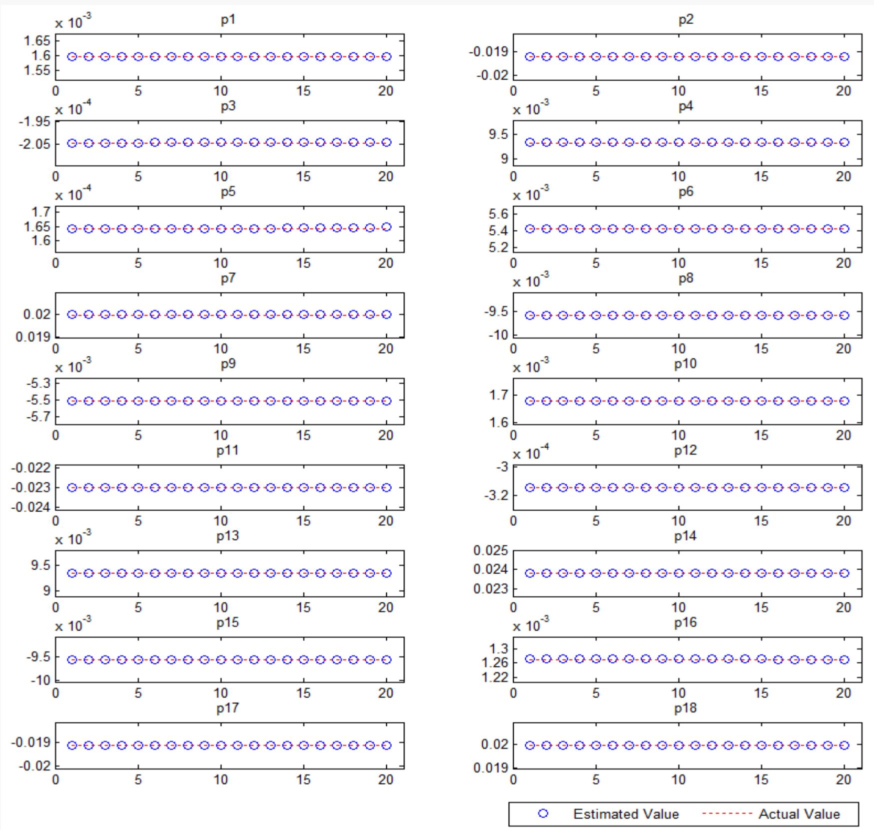


By iteratively processing voltage estimation and parameter tracking, and controlling the convergence criteria, the mismatch between actual and estimated parameters can be reduced.

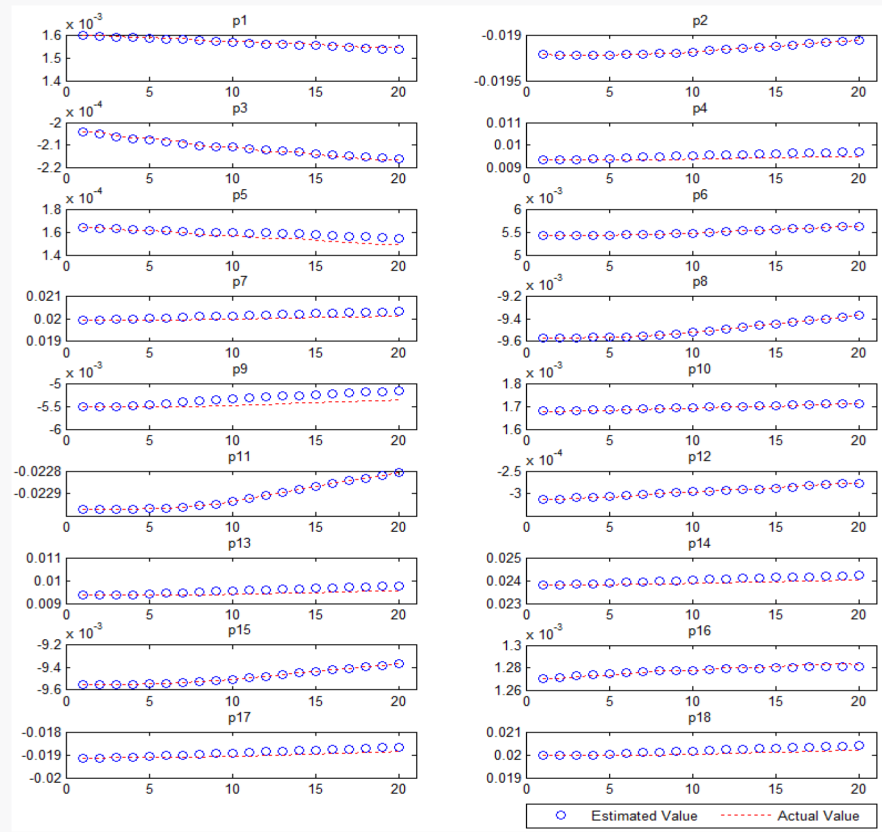
Simulation Results

Algorithm is verified on simulated data

Constant parameters

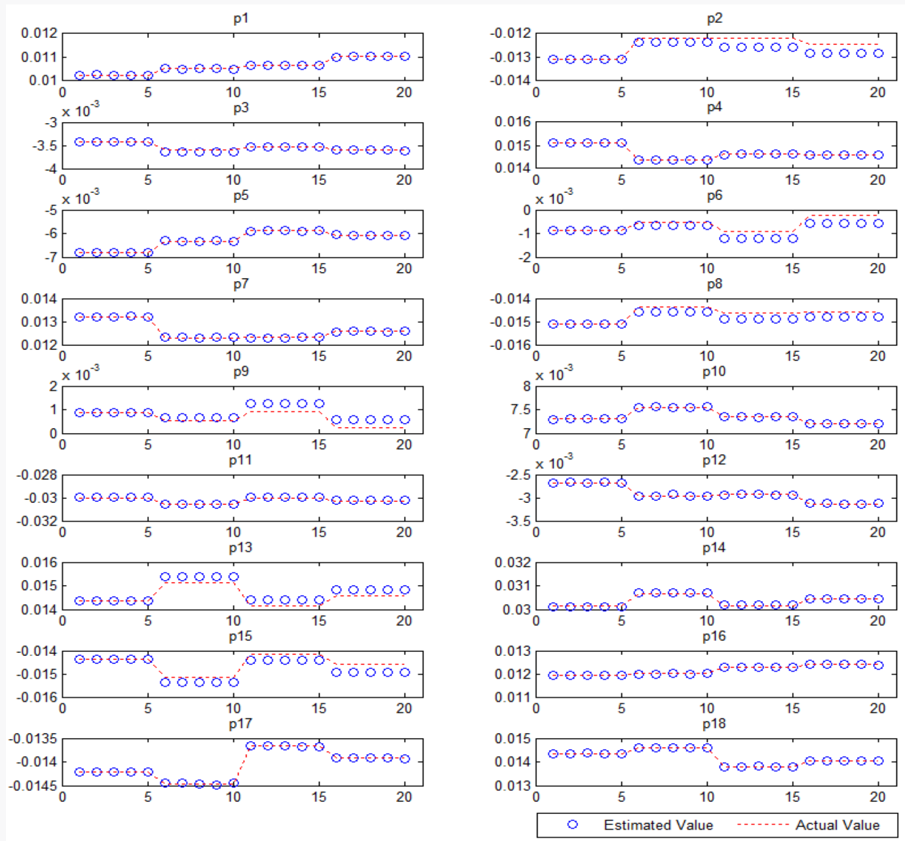


Varying parameters



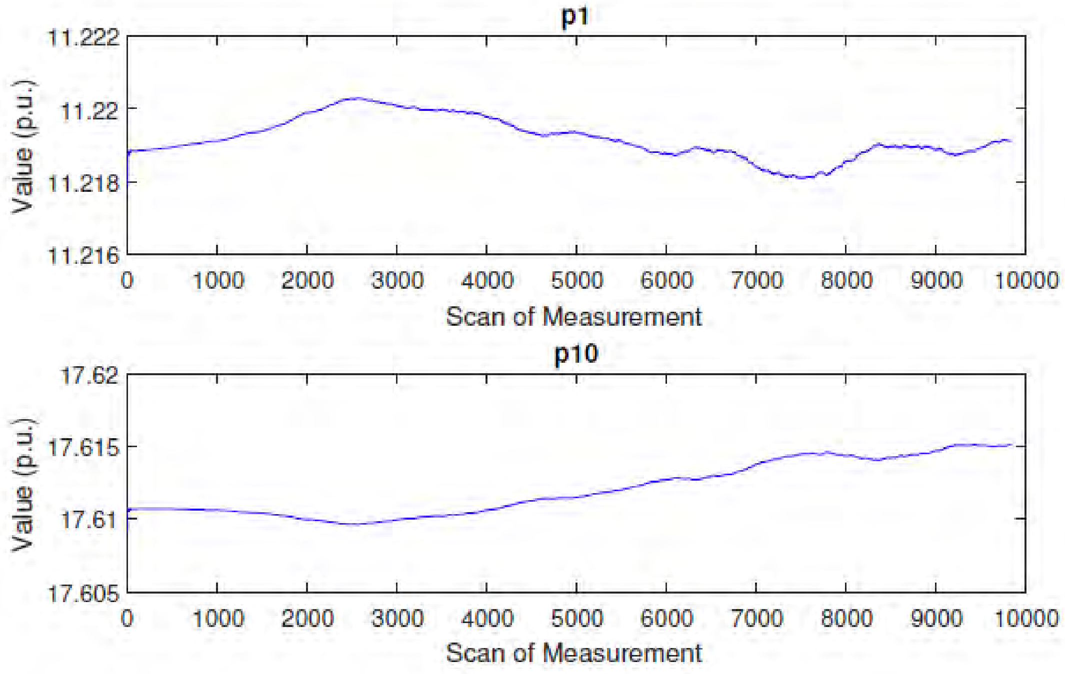
Simulation Results

Parameters with abrupt changes



Recorded Measurements from ISO

9000+ measurement snapshots



Parameters are Tracked successfully !

Parameter	R	X	B
Database value (p.u.)	0.000481	0.006223	0.11114
Estimated value mean	0.000498	0.006475	0.11440

Tracking Parameters with Limited Number of PMUs

How to track parameters of a line even when the line is not measured from both ends by PMUs?

How to track parameters of any line by using sparsely placed PMUs in the system?

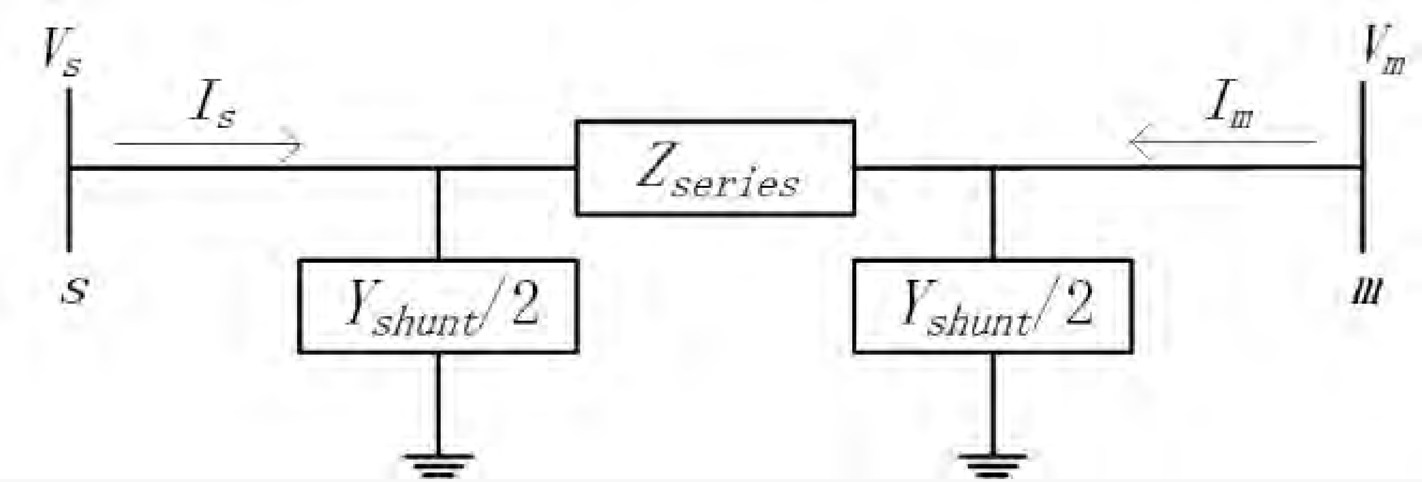
TTL Grid Parameter - Model of the Entire Grid

- Assume a power grid with PMUs installed strategically
  - System observability is satisfied
  - Each branch has at least one incident current measurement

- Each line parameters can be tracked
  - using one or two current measurements

- State estimation can be carried out for the whole system
  - incorporating zero injections to improve redundancy

Single Phase Transmission Line Model (Positive Sequence Only)



Single Phase Transmission Line Model

A linear estimation problem in three parameters for each line: series conductance g, series susceptance b and shunt susceptance B:

$$\begin{bmatrix} I_s \\ I_m \end{bmatrix} = Y_{s-m} \begin{bmatrix} V_s \\ V_m \end{bmatrix} = \begin{bmatrix} g + jb + jB & -g - jb \\ -g - jb & g + jb + jB \end{bmatrix} \begin{bmatrix} V_s \\ V_m \end{bmatrix}$$

$$= \begin{bmatrix} V_s - V_m & jV_s - jV_m & jV_s \\ V_m - V_s & jV_m - jV_s & jV_m \end{bmatrix} \begin{bmatrix} g \\ b \\ B \end{bmatrix}$$

Parameter Tracking Formulation

Consider the measured current phasors at k:

$$z_{I,k} = H_{V,k}\, p_k + v_{I,k}$$

$z_{I,k}$: measured line current; p_k: line parameters ;

$v_{I,k}$: measurement noise

- Using the assumed parameter dynamics:

$$p_{k+1} = p_k + w_{p,k}$$

- Develop a Kalman filter to track the parameters

PMU Placement

Measured voltages may not be directly used to form $H_{V,k}$:

A system-wide state estimator needs to be implemented in order to estimate bus voltages.

Given the limited number of PMUs, not all voltages are measured:

Best locations for PMUs in order to estimate the states and track the parameters of all lines must be determined.

Phasor only State Estimation

Consider a system observable by PMUs and its measurement model for state estimation:

$$z_k = \begin{bmatrix} z_{I,k} \\ z_{V,k} \\ z_{0,k} \end{bmatrix} = H_{p,k}\, x_k + v_k$$

$z_{I,k}$, $z_{V,k}$:current, voltage measurements

$z_{0,k}$: zero injections

Note that, vector "p" in H_p is unknown!

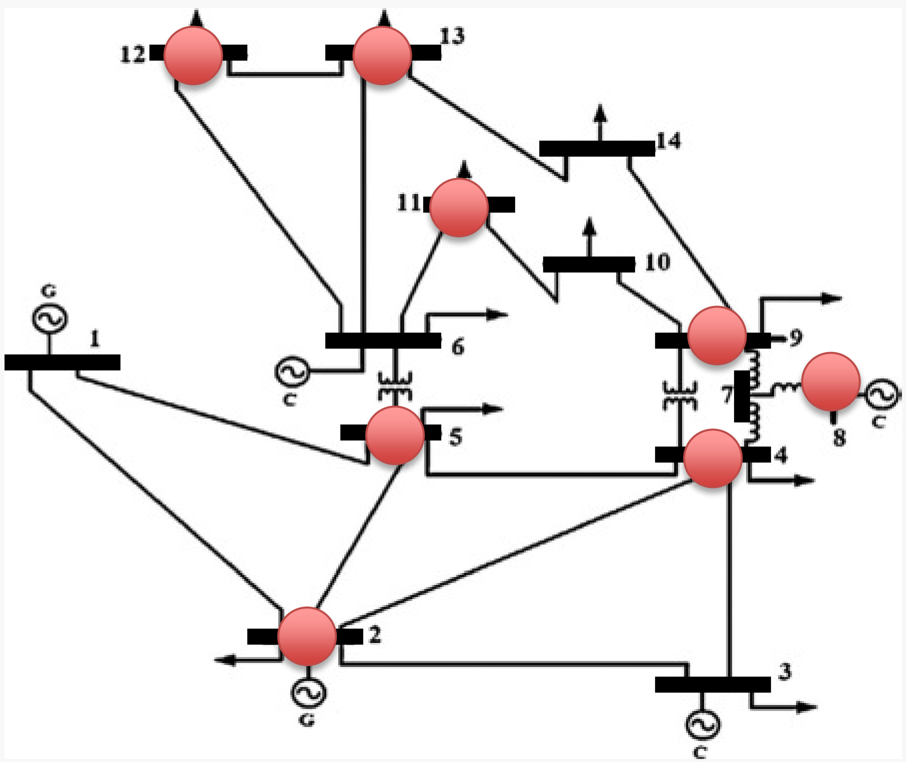
PMU Placement

- Identified the following requirements
  - at least one current measurement / line;
  - all current measurements must be redundant (in the state estimation sense);
  - install a minimum number of PMUs.
- An integer programming optimization problem can then be formulated and solved.

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{N_b} q_i \\
\text{subject to} \quad & |\tilde{A}_1| q \geq \mathbf{1} \\
& q_j = 1, \text{for all } j \in TB \\
& q_i \in \{0, 1\}
\end{aligned}
$$

PMU Placement Example

The given PMU locations ensure not only state observability but also parameter tracking for all lines



Optimal PMU Placement Results

OPPTP: optimal PMU placement for tracking parameters

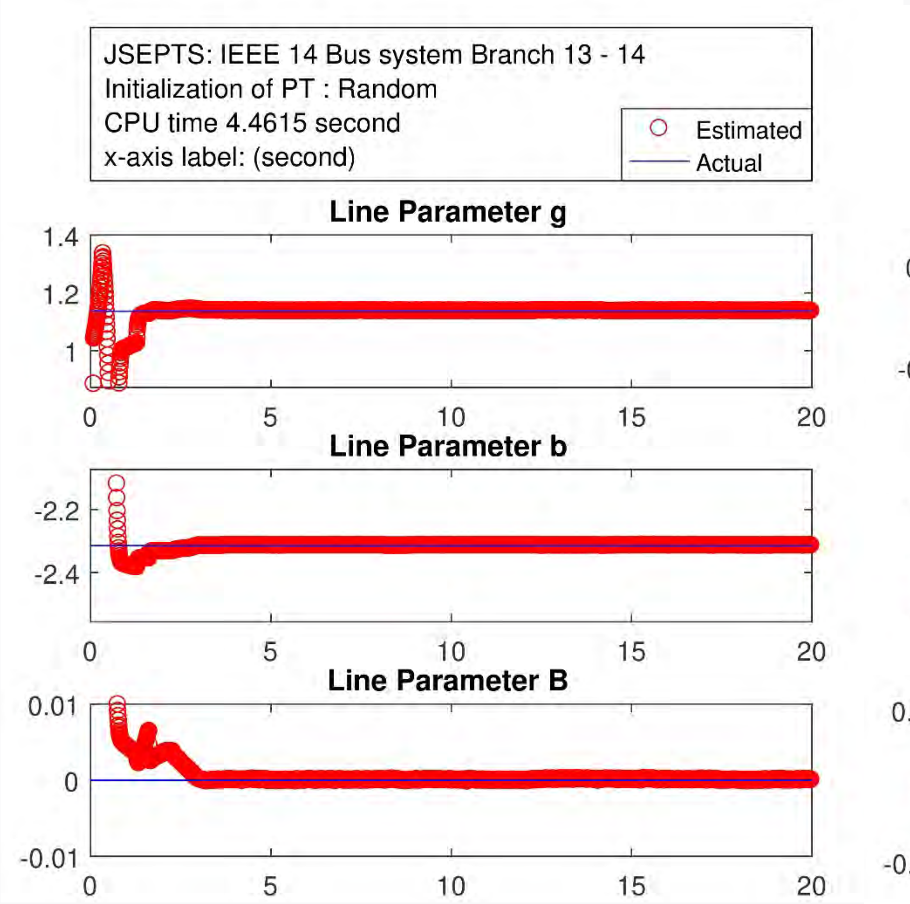
OPP : optimal PMU placement for observability.

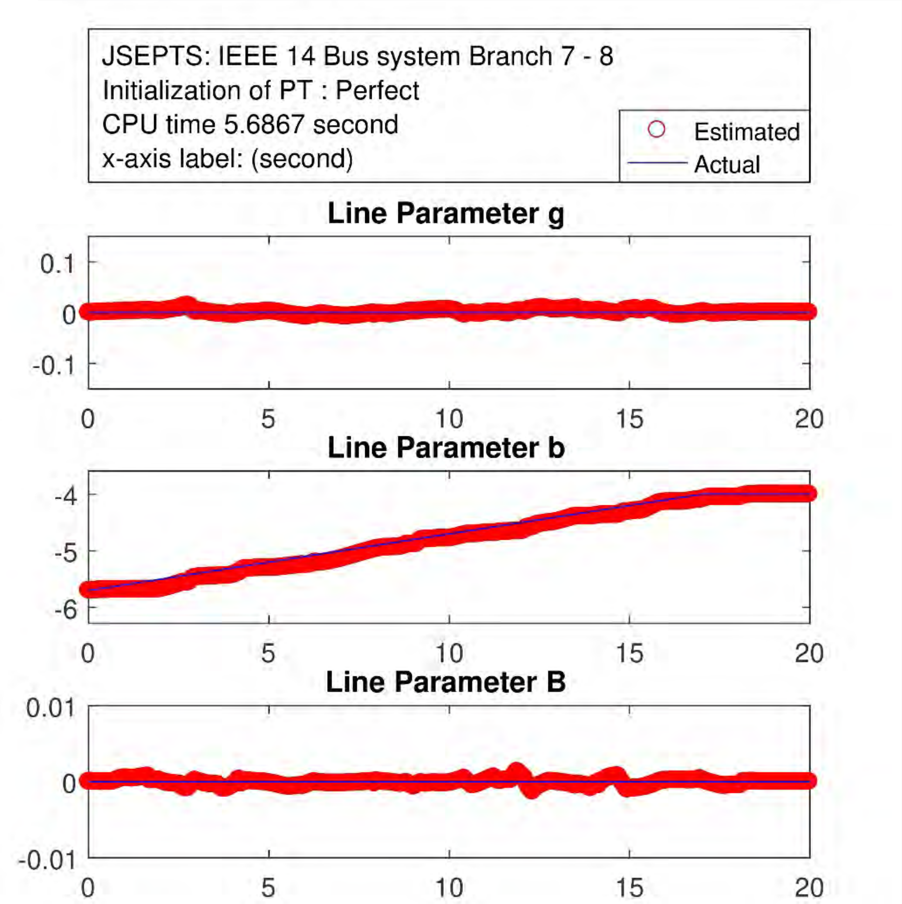
Nearly twice as many PMUs are required!

System	OPPTP	OPP
IEEE 14 bus system	8	4
IEEE 30 bus system	17	10
IEEE 118 bus system	64	32
IEEE 300 bus system	187	87
2071 bus system	1207	634

Simulation Case 1 & 2

- 1) Constant parameter   2) Varying parameter





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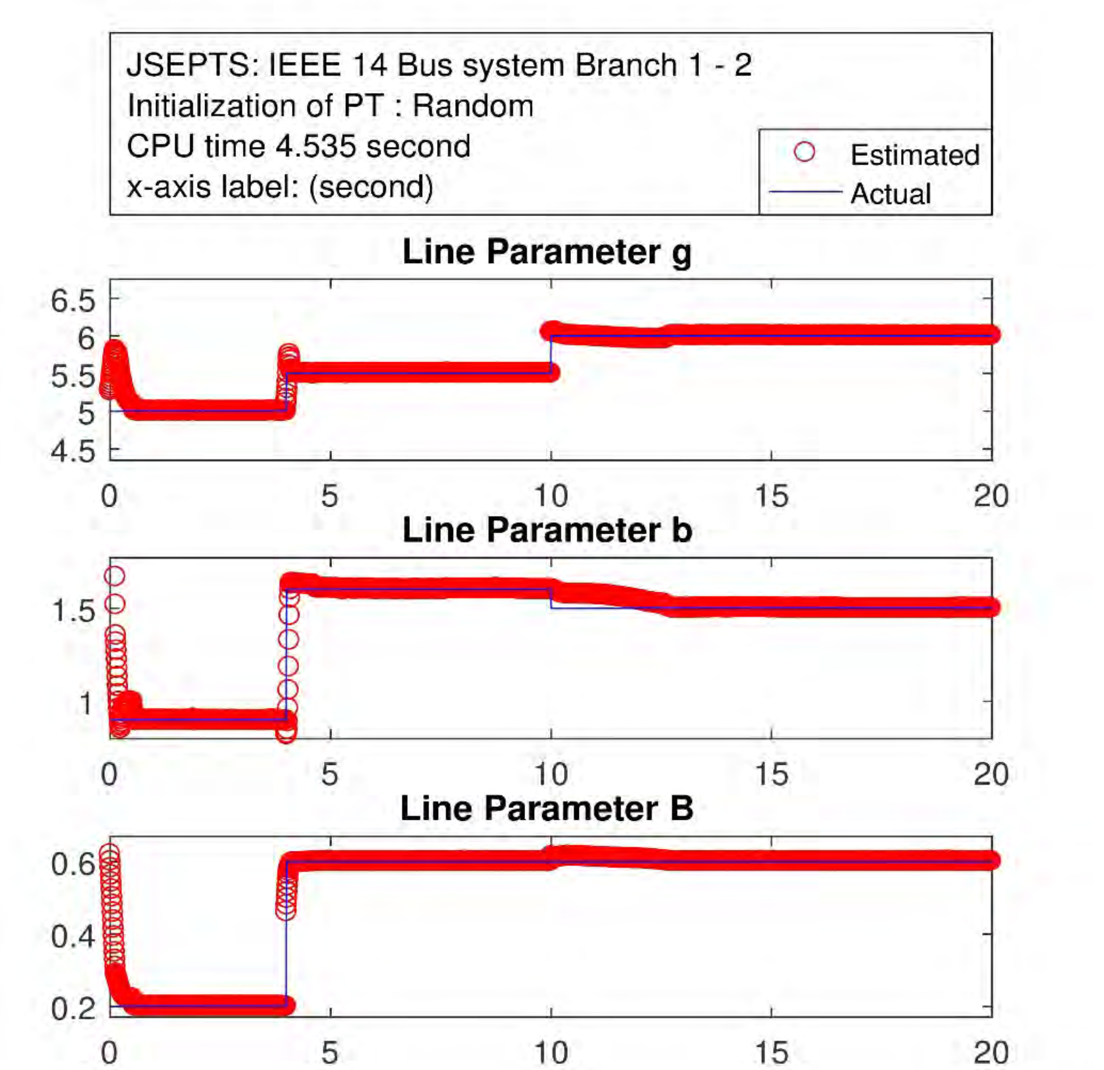
Simulation Case 3

3) Parameters with abrupt changes



Final Remarks

- Sparse estimation methods can be exploited to solve various power system problems. A couple of illustrative examples are shown.
- Complete set of model parameters for non-transposed transmission lines can be dynamically tracked using PMUs at each line terminal.
- Given enough PMUs, a three-phase state estimator combined with a dynamic parameter estimator can track all transmission line parameters in a system.
- Network model / parameter errors ought to receive equal attention as other sources of errors due to their significant impact on various network applications.

Questions?