Provable estimation in distribution grids: physics-informed statistical learning perspective

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Los Alamos National Lab

- Oldest DOE-NNSA Lab
- ~ 10,000 staff
- 7200 feet above sea level
- Ski-hill is 10 mins away
- Near Santa Fe, New Mexico







Los Alamos National Laboratory's Advanced Network Science Initiative.

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Overview

The Los Alamos National Laboratory's Advanced Network Science Initiative (ANSI) is an interdisciplinary initiative that enables fundamental and applied research to address long-term challenges in critical infrastructure design, operation, and security. The primary philosophy of ANSI is that combining insights from Theoretical Physics, Applied Mathematics, Computer Science, and Applied Engineering can result in novel computational methods that address a variety of emerging challenges in infrastructure networks.

Application Areas

To help motivate and inspire novel computational methods, ANSI studies a variety of challenging problems in critical infrastructure networks, such as Analysis of Extreme Events, Notwork baca Machina Larring and Data Analytica Control and





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Transmission Grid



Distribution Grid

• Final Tier in electricity transfer



Distribution Grid

• Final Tier in electricity transfer



Distribution Grid

• Final Tier in electricity transfer



Challenges

- Greater Variability/intermittent
- Lesser inertia/stability
- Needs real-time observability, control



Challenges

- Greater Variability/intermittent
- Lesser inertia/stability
- Needs real-time observability, control

Use

- Estimation
- Optimization
- Resilience





Electricity markets:

Forest fires

Challenges

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Solution

- Smart meters: PMUs, micro-PMUs, IoT
- Big Data: High fidelity measurements
- Over 2500 networked PMUs
- Sparse: Not everywhere in low voltage grids



Physics (Power-Systems) Informed Tuning

(Power System interpretable but repetitive & off-line, hand-controlled)





Physics-Free

Machine Learning

(automatic, training & execution efficient, but lacking Power System interpretability)

Speed

> Advantage: Provable results, Missing data extensions

Learning Problems in Distribution Grids

- Structure Learning
- Learning Line Impedances
- Incomplete observations

Theoretical guarantees:

what length of observations? how much noise? how much observability?





- Restrictions due to domain knowledge
 - 1. Structure of the grid: radial or large loops if meshed



- Restrictions due to domain knowledge
 - 1. Structure of the grid: radial or large loops if meshed
 - 2. Flow Physics (Static regime)

$$P_a + \hat{i}Q_a = \sum_{(a,b)} V_a e^{\hat{i}\theta_a} \frac{(V_a e^{-\hat{i}\theta_a} - V_b e^{-\hat{i}\theta_b})}{(R_{ab} - \hat{i}X_{ab})}$$

• First order expansion: LinDist Flow

$$\theta = H_{1/X}^{-1} P - H_{1/R}^{-1} Q,$$
$$V = H_{1/R}^{-1} P + H_{1/X}^{-1} Q$$

•
$$H_{1/R} = M^T R^{-1} M$$

wt. Laplacian matrix



• Static Regime:

• LinDist Flow:
$$\theta = H_{1/X}^{-1}P - H_{1/R}^{-1}Q,$$

 $V = H_{1/R}^{-1}P + H_{1/X}^{-1}Q$

• Dynamic Regime: Swing Equations

$$M_{a}\ddot{\theta}_{a} + D_{a}\dot{\theta}_{a} = \sum_{\substack{(a,b) \text{ is edge}}} B_{ab}(\theta_{b} - \theta_{a}) + P_{a}$$
Dynamics of state
variables
Net disturbance
imbalance

$$P_a, Q_a$$

 R_{ab}, X_{ab}
 θ_b, V_b
 B
 $Slack Bus$

- Frequency: $\omega_a= heta_a$
- Inertia (*M*) and Damping (*D*)

- 1. Structure of the grid:
 - radial or large loops
- 2. Flow Physics:
 - Static Regime (>1 m)
 - Dynamic Regime (<1 sec)



- Properties of large/finite data
- 1. Sufficient statistics:
 - Means, covariances
- 2. Concentration bounds:

If X_i are i.i.d.

 $< 2 \exp$

• How far are empirical estimates from true values?

 $\mathbb{P}[|\sum X_i - \sum E(X_i)| > t]$

Chernoff, Hoeffding, Markov bounds



Dvorkin et al., Uncertainty Sets For Wind Power Generation, *PES Letters 2016*

- 1. Structure of the grid
- 2. Flow Physics

Statistical Learning:

- 1. Sufficient statistics:
- 2. Concentration bounds:





Provable Learning solutions

- 1. Estimation algorithm *consistent* at infinite samples.
- 2. Correct *with high probability* at finite samples/noise etc.

Learning with nodal voltages

Data: Time-series Nodal voltages at all nodes (static regime)

$$\mu_V = \mathbb{E}[V]$$

$$\Omega_V = \mathbb{E}[V - \mu_V][V - \mu_V]^T$$

- Unobserved: all lines
- Estimate: Operational Topology



• Deka et al., Structure Learning in Distribution Networks, *IEEE Trans. Control of Networks*, 2017

Voltages in Radial Network

- Variance of voltage diff.: $\mathbb{E}[(V_a \mu_{V_a}) (V_k \mu_{V_k})]^2$
- Minimum along any direction reached at nearest neighbor



Topology Learning (No missing nodes)

Greedy Topology Learning:

1. Spanning Tree with edge weights given by

$$\phi_{ab} = \mathbb{E}[(V_a - \mu_{V_a}) - (V_b - \mu_{V_b})]^2$$

- NO additional information needed
- Works for *monotonic* flows (gas, water, heating)
- Computational complexity: O(|V|^2 log |V|)

Sample Complexity :

For a grid with constant depth and sub-Gaussian complex power injections, $O(|V|^2 \log(|V|/\eta))$ samples recovers the true topology with probability $1 - \eta$.



Topology Learning (No missing nodes)





33-bus test system, Matpower Reference: 12 KV substation voltage

Topology Learning with Missing Data

Missing nodes that are greater than 1 hop away (not adjacent)



Missing nodes that are greater than 1 hop away (not adjacent)

• Deka et al., Joint Estimation of Topology and Injection Statistics with Missing Nodes, *IEEE Trans. Control of Networks, 2020*

Topology Learning with Missing Data

- Missing nodes that are greater than 1 hop away (not adjacent)
- Learning Algo:
 - 1. Construct spanning tree
 - 2. Cluster matrix $\left[\phi_{ac_i} + \phi_{ac_j} \phi_{c_ic_j}\right]$
 - 3. Find missing parents and iterate.



• Deka et al., Joint Estimation of Topology and Injection Statistics with Missing Nodes, *IEEE Trans. Control of Networks, 2020*

Learning with *end-users*

- Data: Time-series Nodal voltages and injection samples at leaves
- Unobserved: all intermediate nodes & lines
- Estimate: Operational Topology + Line Impedance



End-user data

• Time-stamped voltage magnitudes (V)

$$\mu_V = \mathbb{E}[V]$$

$$\Omega_V = \mathbb{E}[V - \mu_V][V - \mu_V]^T$$



- Time-stamped nodal active & reactive injections (P &Q) $\mu_P, \ \Omega_P, \ \mu_Q, \ \Omega_Q, \ \Omega_{PQ}$
- Cross-covariances: Ω_{VP}, Ω_{VQ}



Learning with *end-users*

- Data: Time-series Nodal voltages and injection samples at leaves
- Algorithm:
- Compute *effective impedances* between leaf pairs

$$R_{eff}(a,b) = H_{1/R}^{-1}(a,a) + H_{1/R}^{-1}(b,b) - 2H_{1/R}^{-1}(a,b)$$

Recursive Grouping Algo (Anandkumar'11) to learn topology & distances from known effective impedances



Recursive Grouping Algo

1. *a*, *b* are leaf nodes with common parent iff d(a,c) - d(b,c) = d(a,c') - d(b,c') for all $c,c' \neq a, b$

2. a is a leaf node and b is its parent iff

d(a,c) - d(b,c) = d(a,b) for all $c \neq a, b$







3. Update distance



Recursive Grouping Algo





2. Introduce parents

3. Update distance



Recursive Grouping Algo









Estimating effective impedances

- Algorithm:
- Compute *effective impedances* between leaves

$$R_{eff}(a,b) = H_{1/R}^{-1}(a,a) + H_{1/R}^{-1}(b,b) - 2H_{1/R}^{-1}(a,b)$$

- Uncorrelated Injections

 $\begin{bmatrix} \mathbb{E}[v_a p_b] & \mathbb{E}[v_a q_b] \end{bmatrix} = \begin{bmatrix} H_{1/r}^{-1}(a, b) & H_{1/x}^{-1}(a, b) \end{bmatrix} \begin{bmatrix} \mathbb{E}[p_b^2] & \mathbb{E}[p_b q_b] \\ \mathbb{E}[q_b p_b] & \mathbb{E}[q_b^2] \end{bmatrix}$

Two equations with 2 unknowns

Effect of Correlated Injection

• Algorithm:

Compute *effective impedances* between leaves

$$R_{eff}(a,b) = H_{1/R}^{-1}(a,a) + H_{1/R}^{-1}(b,b) - 2H_{1/R}^{-1}(a,b)$$



• Algorithm:

Compute *effective impedances* between leaves

$$R_{eff}(a,b) = H_{1/R}^{-1}(a,a) + H_{1/R}^{-1}(b,b) - 2H_{1/R}^{-1}(a,b)$$

 $\succ \quad \text{Correlated Injections} \\ \begin{bmatrix} \mathbb{E}[v_{\mathcal{L}}p_{\mathcal{L}}^{T}] & \mathbb{E}[v_{\mathcal{L}}q_{\mathcal{L}}^{T}] \end{bmatrix} = \begin{bmatrix} H_{1/r}^{-1}(\mathcal{L},\mathcal{L}) & H_{1/x}^{-1}(\mathcal{L},\mathcal{L}) \end{bmatrix} \begin{bmatrix} \mathbb{E}[p_{\mathcal{L}}p_{\mathcal{L}}^{T}] & \mathbb{E}[p_{\mathcal{L}}q_{\mathcal{L}}^{T}] \\ \mathbb{E}[q_{\mathcal{L}}p_{\mathcal{L}}^{T}] & \mathbb{E}[q_{\mathcal{L}}q_{\mathcal{L}}^{T}] \end{bmatrix} \end{bmatrix}$

ML estimate (SPICE) for inverse:

$$\min_{\Omega \succ 0} - \det(\Omega) + \operatorname{tr} \left(\mathbb{E} \left[[p_{\mathcal{L}}^T \ q_{\mathcal{L}}^T]^T [p_{\mathcal{L}}^T \ q_{\mathcal{L}}^T] \right] \Omega \right) + \lambda |\Omega^-|_1$$

Uncorrelated :

For a grid with constant depth and sub-Gaussian complex power injections, $O(|V| \log(|V|/\eta))$ samples recovers the true topology with probability $1 - \eta$. Correlated :

 $O(|V|^2 \log(|V|/\eta))$ samples recovers the true topology with probability $1 - \eta$.

 Park et al., Learning with End-Users in Distribution Grids: Topology and Parameter Estimation, IEEE Trans. Control of Networks, 2020

Simulations: IEEE 33 bus graphs (Matpower samples)



What about

- 1. Loopy grids
- 2. Time-correlated voltages and injections



Probabilistic Distribution \rightarrow Graphical Model



Correlation

Conditional Dependence

Probabilistic Distribution \rightarrow Graphical Model

- Graphical Model: Graphical Factorization of Distribution
- Think Inverse Correlation instead of Correlation



Correlation of stock prices

Graphical Model of stock prices

Probabilistic Distribution of Nodal Voltages

• Distribution of injections:

$$\mathcal{P}(P) = \prod_{a \in \mathcal{V}} \mathcal{P}_a(P_a)$$

• Distribution of voltages:

$$\begin{aligned} \mathcal{P}(V,\theta) &= \frac{1}{|J_P(V,\theta)|} \prod_{a \in \mathcal{V}} \mathcal{P}_a(P_a) \\ &= \frac{1}{|J_P(V,\theta)|} \prod_{a \in \mathcal{V}} \mathcal{P}_a\left(\sum_{b:\{a,b\} \in \mathcal{E}} V_a(V_a^* - V_b^*)/z_{ab}^*\right) \\ &\text{Jacobian } J_P(V,\theta) = \left(\frac{\partial V,\theta}{\partial P,Q}\right) \end{aligned}$$

 $\theta = H_{1/X}^{-1} P - H_{1/R}^{-1} Q,$

 $v = H_{1/R}^{-1}P + H_{1/X}^{-1}Q$

voltages

 $\mu_P \ \Omega_P \ \mu_Q \ \Omega_Q \ \Omega_{PQ}$

Injections

Graphical Model of Voltages

- Distribution $\mathcal{P}(V,\theta)$ with $\begin{aligned} \theta &= H_{1/X}^{-1}P H_{1/R}^{-1}Q, \\ v &= H_{1/R}^{-1}P + H_{1/X}^{-1}Q \end{aligned}$
- Graphical Model: between voltage, phase



Graphical Model of Voltages

• Distribution $\mathcal{P}(V, \theta)$

• **Graphical Model:** Topology Edges + 2-hop neighbors



Graphical Model Estimation

- Variables: (V, θ)
- Gaussian voltage fluctuations

- 'Hybrid' Graph $(v_i) \theta_i$ $(v_k - \theta_k) - \theta_j$ (v_j)
- Inverse covariance $\Sigma_{(V,\theta)}^{-1}$ gives graphical model

- Graphical Lasso: (Yuan & Lin, 2007)
- Neighborhood Lasso: (Meinshausen, 2006)

$$\arg\min_{\boldsymbol{S}} -\log\det{\boldsymbol{S}} + \langle \boldsymbol{S}, \sum_{k} x^{k} x^{k^{T}} / n \rangle + \lambda \|\boldsymbol{S}\|_{1}$$

$$\left(\arg\min_{\substack{\beta_{ij}\\j\neq i}} \mathbb{E} \left[x_i - \sum_{j\neq i} \beta_{ij} x_j \right]^2 + \lambda \begin{bmatrix} \beta_{i1} \\ \beta_{i2} \\ \vdots \\ \vdots \end{bmatrix}_1 \right)$$

- How to distinguish true edges??
- Two schemes:
 - Neighborhood Counting
 - Thresholding
- Exact for radial networks
- Restrictions for loopy/meshed grids



• Deka et al., Graphical Models in Meshed Distribution Grids: Topology estimation, change detection & limitations, *IEEE Trans. Power Systems, 2020*

• Neighborhood Counting:



• Neighborhood Counting: topological separability

- Loopy Grid:
 - Recovers exact topology if cycle length greater than 6

• Deka et al., Graphical Models in Meshed Distribution Grids: Topology estimation, change detection & limitations, *IEEE Trans. Smart Grid*, 2020

• Thresholding:



• True edges have $\Sigma_{\theta\theta}^{-1}(i,j) + \Sigma_{VV}^{-1}(i,j) < \tau_2 < 0$

- Loopy Grid:
 - Recovers topology if cycle length greater than 3 (no triangles)
- Deka et al., Graphical Models in Meshed Distribution Grids: Topology estimation, change detection & limitations, *IEEE Trans. Smart Grid*, 2020

- Alg1: counting
- Alg2: thresholding
- 56 bus system



• Deka et al., Graphical Models in Meshed Distribution Grids: Topology estimation, change detection & limitations, *IEEE Trans. Smart Grid*, 2020

- Extends to 3-phase unbalanced system
- Deka et al., Topology estimation using graphical models in multi-phase power distribution grids, *IEEE Trans. Power Systems, 2020*





What about

- General Grids with triangles??
- Temporal Correlations??



What about

- General Grids with triangles??
- Temporal Correlations??



Dynamic Regime: Swing Equations

Fluctuations due to ambient noise in injections:

$$M_{a}\ddot{\theta}_{a} + D_{a}\dot{\theta}_{a} = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_{b} - \theta_{a}) + P_{a}$$
Dynamics of phase angles Net power imbalance

 P_b, θ_b

1.2

b

0.6

0.8

Time in seconds

• Frequency
$$\omega_a=\dot{ heta}_a$$

- Inertia (M) and Damping (D) from synchronous machines.
- Stochastic noise P_a

Power Flow

• General Form:

$$0 = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a$$

$$M_a \ddot{\theta}_a + D_a \dot{\theta}_a = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a$$

Graphical Model:
 Inverse Correlation Matrix



Inverse Power Spectral Density

$$\Sigma_{\theta}(r) = \mathbb{E}[\theta(t)\theta(t-r)^T]$$

Fourier Transform of delayed correlation



Power Flow

• General Form: $0 = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a$

$$M_a \ddot{\theta}_a + D_a \dot{\theta}_a = \sum_{(a,b) \text{ is edge}} B_{ab}(\theta_b - \theta_a) + P_a$$

• Graphical Model: Inverse Correlation Matrix

Inverse Power Spectral Density

• Finite samples:

Neighborhood Lasso

(Meinshausen, 2006)



$$\left(\arg\min_{\substack{\beta_{ij}\\j\neq i}} \mathbb{E} \left[\theta_i - \sum_{j\neq i} \beta_{ij} \theta_j \right]^2 + \lambda \begin{bmatrix} \beta_{i1} \\ \beta_{i2} \\ \vdots \\ \vdots \end{bmatrix}_1^2 \right)$$

Wiener Filter (non-causal) (Wiener, Kolmogorov 1950)

$$\left(\arg\min_{\substack{\beta_{ij}^r, j\neq i}} \mathbb{E}\left[\theta_i(t) - \sum_{\substack{j\neq a \\ -\infty < r < \infty}} \beta_{ij}^r \theta_j(t-r)\right]^2 + \lambda \begin{bmatrix} \vec{\beta_{i1}} \\ \vec{\beta_{i2}} \\ \vdots \\ \vdots \end{bmatrix}_{(2,1)}\right)^2 \right)$$

- Graphical Model of voltages: Topology Edges + 2-hop neighbors
- Dynamic regime (inverse power spectral density):
 - Neighborhood counting (cycle length > 6)
- Inverse PSD: $(\Omega_{\theta}^{-1}(j\omega))$ is function of frequency) □ Phase remains **constant** for spurious edges at all frequency



- Graphical Model of voltages: Topology Edges + 2-hop neighbors
- Dynamic regime (inverse power spectral density):
 - Neighborhood counting (cycle length > 6)
 - Phase based edge detection (all graphs)
 - Holds for colored (WSS or cyclo-stationary) inputs
 - S. Talukdar et al., Physics-informed learning in linear dynamical systems, *Automatica*, 2020.



- Graphical Model of voltages: Topology Edges + 2-hop neighbors
- Dynamic regime (inverse power spectral density):
 - Phase based edge detection (all graphs)
 - Any linear dynamical system: Eg. Buildings
 - S. Talukdar et al., Physics-informed learning in linear dynamical systems, *Automatica*, 2020.



- Graphical Model of voltages: Topology Edges + 2-hop neighbors
- Dynamic regime (inverse power spectral density):
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Learning in *under-excited* grids:

- Graphical Model of voltages: static or dynamic
 - Uses inverse voltage covariance or power spectral density
 - Needs fluctuations at all nodes ($\Sigma_{(P,Q)}^{-1} \Rightarrow \Sigma_{(V,\theta)}^{-1}$ to be defined)
 - What if zero-injection buses exist?
- Learning when 0-injection buses not adjacent



• Deka et al, Tractable learning in under-excited power grids, arxiv pre-print, 2020.

Learning in *under-excited* grids:



• Deka et al, Tractable learning in under-excited power grids, arxiv pre-print, 2020.

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Practical Applications:

- Use tractable/provable algorithms as a starting point
- Additional constraints from real data:
 - Prior structures /impedance values (monitor change instead)
 - Use threshold selection based on historical data
 - Learn noise levels
- Data-driven guided by real-data:
 - Matt Reno, Yang Wang, Ram Rajagopal, Reza Arghandeh, Sascha von Meier, Vijay Arya
- Direct Samples not statistics: (regression or active probing based)
 - Steven Low, Vassilis Kekatos, Guido Cavraro
- Statistical change detection:
 - Anuradha Annaswamy, Alejandro Garcia

When such methods do not work well?

- Non-linearity makes linear approximations inadequate
 - Kernel based methods (George Giannakis)
 - Koopman operators
 - Neural networks- physics-informed (Yue Zhang)
- Use case where NN works well:
 - Fault detection/ localization

 Wenting Li et al., Real-time Faulted Line Localization and PMU Placement in Power Systems through Convolutional Neural Networks, *IEEE Trans. Power Systems, 2019.*



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Thank You. Questions!

