Multi-Robot Task Scheduling

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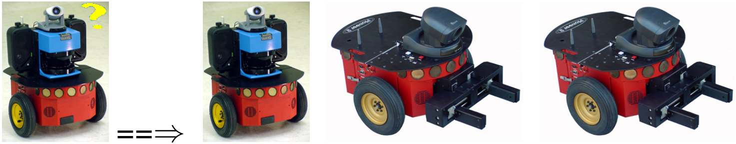
Multi-robot task scheduling

Multi-robot tasks:

Individual robots may not have all the required capabilities



Scheduling:

- A set of robots, $R = \{r_1, ..., r_i, ...\}$
- A set of tasks, $T = \{t_1, ..., t_l, ...\}$

Build a schedule to optimize a *func*, $\{R_l, s_l, p_l\}_l$

Multi-robot task scheduling

To represent a general scheduling problem: $P|T|func$

Multi-robot task scheduling:

- $P \rightarrow$ Multi-purpose processor
- $T \rightarrow$ Multi-processor task
- Restrictions:
  - Execution is non-preemptive
  - Robots are non-divisible

or the *MPM MPT* problem [Gerkey and Mataric, 2004]

Complexity of *MPM MPT*

With *func* $= \sum_l e_l$:

- *MPM*: polynomial-time solvable
- *MPT*: $\mathcal{NP}$-hard
- *MPT*2: $\mathcal{NP}$-hard

Two types of multi-robot tasks:

- Loosely coupled: reducible to single robot tasks (*MPM MPT* becomes *MPM*)
- Tightly coupled: ?

Efficient algorithms, preferably with solution bounds, are needed.

Scheduling for tightly coupled multi-robot tasks

Steps:

1. Reduce *MPM MPT* to *MPM*
2. Solve the *MPM* problem

When considering a coalition as a robot, *MPM MPT* becomes *MPM*.

However, coalitions can interfere with each other:

Coalition 1: $\{r_1, r_4, r_5\}$

Coalition 2: $\{r_4, r_6\}$

Contributions

- Considers the scheduling problem for multi-robot tasks at the coalition level
- Proposes four efficient heuristics to address the problem with provable solution bounds
- Provides formal analyses and simulation results to demonstrate and compare their performances

Notations

Table: NOTATIONS USED

R	Set of robots	r_i
C	Set of coalitions	c_j
T	Set of tasks	t_l
p_{jl}	Processing time of t_l by c_j	
e_l	End time of task t_l	

We consider $func = \sum_l e_l$

MinProcTime

Definition (*MinProcTime*)

At each step:

1. Find the assignment that has the smallest p_{jl}
2. Schedule the task at the earliest possible time

Theorem

The MinProcTime heuristic yields a solution quality bounded by $\frac{|T|+1}{2}$.

Tight solution bound

MinStepSum

Definition (*MinStepSum*)

At each step:

1. Find the assignment that increases $\sum_l e_l$ the least

Theorem

The MinStepSum heuristic yields a solution quality bounded by $\frac{|T|+1}{2}$.

Tight solution bound

InterfereAssign

To consider the interference between coalitions:

Definition (*Coalition Interference*)

For any two coalitions c_j and $c_{j'}$ ($j \neq j'$), c_j interferes (or conflicts) with $c_{j'}$ if and only if $c_j \cap c_{j'} \neq \emptyset$.

Consider the impact of an assignment $c_j \rightarrow t_l$ on $\sum_l e_l$:

1. The assignment's processing time p_{jl}
2. Tasks that are scheduled on c_j after t_l
3. Tasks scheduled on coalitions that interfere with c_j

InterfereAssign

For $c_j \rightarrow t_l$:

1. The assignment's processing time p_{jl}
2. Tasks that are scheduled on c_j after t_l

Together, contribute $I_{jl} \cdot p_{jl}$

I_{jl}: scheduling position for t_l on c_j

For example:

$c_2 : t_2 \Rightarrow t_1 \Rightarrow t_3$

For t_2, $I_{22} = 3$ (including influence on t_1 and t_3)

InterfereAssign

For $c_j \rightarrow t_l$:

3. Tasks scheduled on coalitions that influence with c_j

Upper bound is $|\cup_{c \in F_j} N_c| \cdot p_{jl}$

F_j: coalitions that interfere with c_j

N_c: set of tasks that c can accomplish

InterfereAssign

Convert *MPM MPT* to *MPM* by constructing an assignment problem:

- Create a task node for each task t_l
- Create a coalition-position node for each coalition c_j and position pair, with positions ranging from 1 to N_{c_j} for coalition c_j
- If a coalition c_j can accomplish a task t_l, connect t_l with all coalition-position nodes for c_j, and set the weights to be $(|\cup_{c \in F_j} N_c| + I_{jl}) \cdot p_{jl}$, respectively, based on I_{jl}

Now, solve this problem optimally.

InterfereAssign

Lemma

There exists a schedule that is no worse than the solution of the assignment problem.

Theorem

The schedule that is constructed from the solution of the assignment problem yields a solution quality bounded by $\max_j |\cup_{c \in F_j} N_c| + 1$.

- Quality dependent on complex structure of the problem instance
- Less coalition interference, better quality
- Optimal solution for single robot tasks

MinInterfere

In *InterfereAssign*:

- $|\cup_{c \in F_j} N_c|$ is an overestimation

Definition (*MinInterfere*)

At each step:

1. Compute β_{jl} and choose the assignment that minimizes it:

$$\beta_{jl} = e_{jl} + |\cup_{c \in F_j} N_c \setminus M_{jl}| \cdot p_{jl}$$

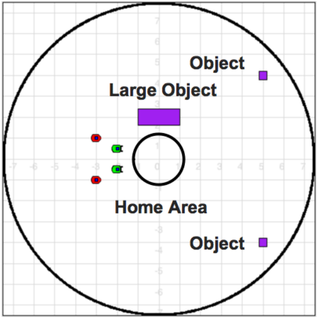
M_{jl}: $t_l \cup$ the set of tasks that are scheduled before $c_j \rightarrow t_l$

Summary

Table: SUMMARY OF DISCUSSED HEURISTICS

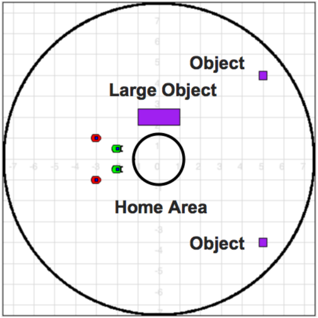
Name	**Solution Bound**	**Complexity**
Optimal	1	$O((\|C\|\|T\|)^{\|T\|}\|T\|!)$
MinProcTime	$\frac{\|T\|+1}{2}$	$O(\|C\|\|T\|^3)$
MinStepTime	$\frac{\|T\|+1}{2}$	$O(\|C\|\|T\|^3)$
InterfereAssign	$\max_j \|\cup_{c \in F_j} N_c\| + 1$	$O(\|C\|^3\|T\|^3)$
MinInterfere	Not Determined	$O(\|C\|\|T\|^3)$

A simple scenario



Task	Robots Required	Process Time
1) Object 1	One gripper, one localizer	6
2) Object 2	One gripper, one localizer	6
3) Large Object	Two grippers	5

A simple scenario



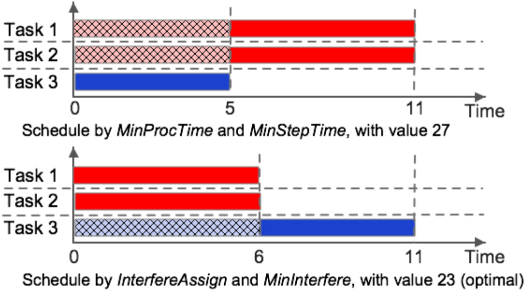


Figure: Schedules created by our heuristics

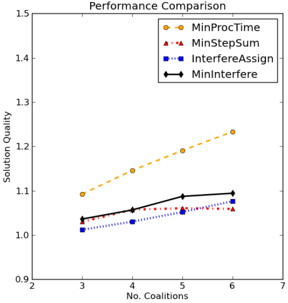
Heuristics that consider the interference produce the optimal solution

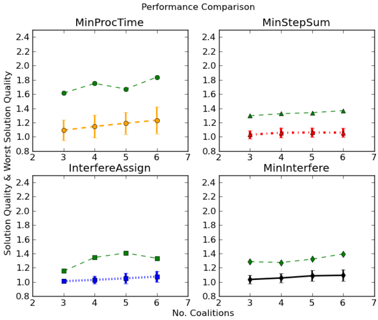
Parameters

Table: PARAMETERS USED IN THE SIMULATIONS

Parameter	Description
n_c	No. of coalitions
n_t	No. of tasks
n_f	Average no. of conflicting coalitions per coalition
n_e	Average no. of executable tasks per coalition
n_{min}, n_{max}	Minimum and maximum processing time

Varying n_c

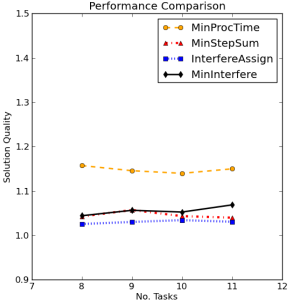


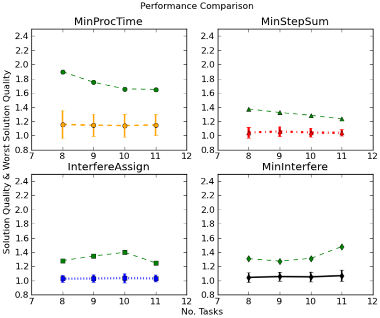


The average solution quality is better than the proven bounds

MinStepSum, *InterfereAssign* and *MinInterfere* perform similarly; *InterfereAssign* is better for smaller n_c (i.e., $3-4$)

Varying n_t

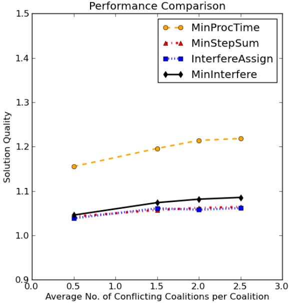


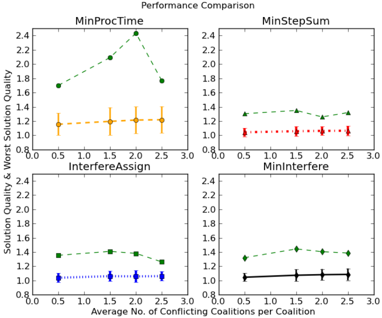


Similar observations

Since n_f stays as a constant, the curve formed is smoother

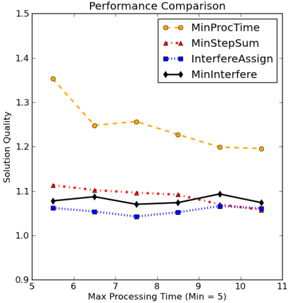
Varying n_f

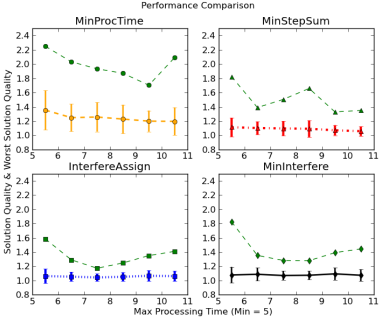




Performance decreases as the interference becomes more complex

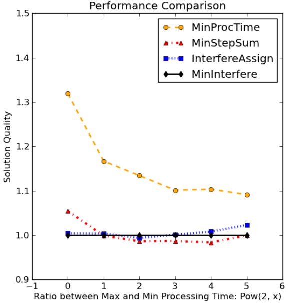
Varying n_{max}

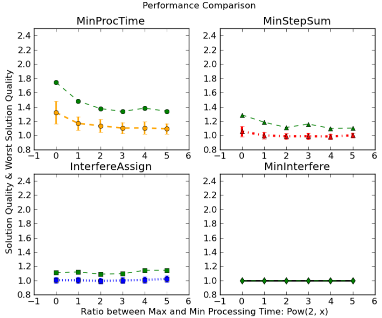




Increase of n_{max} does not always decrease the performance

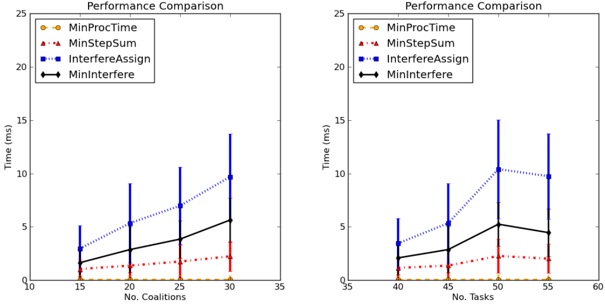
Varying n_{max}, with large n_c and n_t





MinStepSum performs slightly better with large n_c and n_t

Time analysis. Left: Varying n_c. Right: Varying n_t



Has potentials to be applied to large-size problems

Conclusions

- When there is less interference between coalitions, use InterfereAssign
- Otherwise, choose the best

Contributions

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- Proposes four efficient heuristics to address the problem with provable solution bounds
- Provides formal analyses and simulation results to demonstrate and compare their performances

References

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