Lecture 15

Predicate Logic
Announcements

• Stats:
  = Homework (each 4%) + Quiz (each 10%) + Test(40%) + Project (20%)
  - >95 – 3
  - >90 – 8
  - >80 – 22
  - >70 – 33
  - >60 – 39
  - < 60 – 4
  ➢ Average 77.6

• Project proposal due: Mon, 03/20, 2PM
  - About 4 people in a team
  - Sample projects will be discussed later
  - At most two teams can work on the same topic – will create a Google Doc
Project Ideas

< Sudoku like Puzzle >
• Hidato
• Nurikabe
• Dominos
• Nonogram

< Grid Puzzle >
• Minesfield
• Battleship
• Undercover underground
• Networks Puzzle

< Towards Practical Application >
• Exam Seating Chart Generator
• Schedule Creator
Project Ideas

< Non-grid Puzzle >
• Redefining Rubik’s
• Three in a row

< Game >
• King in Check
• Game of Tents

< Logic Clues >
• Football Problem

< Route / Path >
• Lattice Paths - Sweigart & Walz
• Mouse Maze - Weafer & Ditsworth
• Toxic Clean-up - Vlieger & Gonclaves
What we have covered so far

• Propositional logic
  – Syntax, recursive definition
  – Proof theory: Natural deduction
  – Model theory: semantics, satisfaction, tautology, entailment, equivalence
• Proof methods
  – Induction
The Need For a Richer Language

Propositional Logic:

- Study of declarative sentences, statements about the world which can be given a truth value
- Dealt very well with sentence components like: *not, and, or, if … then …*
- **Limitations:** cannot deal with modifiers like *there exists, all, among, only.*

**Example:** “*Every student is younger than some instructor.*”
The Need For a Richer Language

Propositional Logic:

- Study of declarative sentences, statements about the world which can be given a truth value
- Dealt very well with sentence components like: not, and, or, if ... then ...
- Limitations: cannot deal with modifiers like there exists, all, among, only.

Example: “Every student is younger than some instructor.”

- We could identify the entire phrase with the propositional symbol \( p \).
- However, the phrase has a finer logical structure. It is a statement about the following properties:
  - being a student
  - being an instructor
  - being younger than somebody else
Predicates, Variables, and Quantifiers

Properties are expressed by predicates. \( S, I, Y \) are *predicates*.

\[
S(\text{andy}): \text{ Andy is a student.} \\
I(\text{paul}): \text{ Paul is an instructor.} \\
Y(\text{andy}, \text{paul}): \text{ Andy is younger than Paul.}
\]

“Every student is younger than some instructor.”
Predicates, Variables, and Quantifiers

Properties are expressed by predicates. $S, I, Y$ are **predicates**.

$S(\text{andy})$: Andy is a **student**.
$I(\text{paul})$: Paul is an **instructor**.
$Y(\text{andy}, \text{paul})$: Andy is **younger than** Paul.

Variables are placeholders for concrete values.

$S(x)$: $x$ is a student.
$I(x)$: $x$ is an instructor.
$Y(x, y)$: $x$ is younger than $y$.

“**Every student is younger than some instructor.**”
Dealing with Quantifiers

Variables are placeholders for *any*, or *some*, unspecified concrete value.

\( \exists x \Phi \) We try to find some instance of \( x \) (some concrete value) such that \( \Phi \) holds for that particular instance of \( y \). If this succeeds, then \( \exists x \Phi \) evaluates to \( T \); otherwise (i.e. there is no concrete value of \( x \) that realizes \( \Phi \)) the formula evaluates to \( F \).

\( \forall x \Phi \) We try to show that for all possible instances of \( x \), \( \Phi \) evaluates to \( T \). If this is successful, \( \forall x \Phi \) evaluates to \( T \); otherwise (i.e. if there exists some instance of \( x \) that does not realize \( \Phi \)), the formula evaluates to \( F \).
Predicates, Variables, and Quantifiers

Properties are expressed by predicates. \( S, I, Y \) are predicates.

- \( S(\text{andy}) \): Andy is a student.
- \( I(\text{paul}) \): Paul is an instructor.
- \( Y(\text{andy}, \text{paul}) \): Andy is younger than Paul.

Variables are placeholders for concrete values.

- \( S(x) \): \( x \) is a student.
- \( I(x) \): \( x \) is an instructor.
- \( Y(x,y) \): \( x \) is younger than \( y \).

Quantifiers make possible encoding the phrase:

“Every student is younger than some instructor.”

Two quantifiers: \( \forall \) — forall, and \( \exists \) — exists.
Predicates, Variables, and Quantifiers

Not all birds can fly.

\[ \forall x (B(x) \rightarrow F(x)) \]
\[ \neg \forall x (B(x) \rightarrow F(x)) \]
\[ \exists x (B(x) \land \neg F(x)) \]

\[ B(x) : \ x \text{ is a bird} \]
\[ F(x) : \ x \text{ can fly.} \]