CSE 259: Logic in Computer Science (Spring 2017)

Final Study Guide

Final: In class, Monday, May 1, 2:30PM to 4:20PM (1 hour & 50min)

Comprehensive but with a bit more emphasis on the materials after mid-term

Syntax, Natural Deduction, and Semantics of Propositional Logic + Mathematical Induction

1. Strategy to succeed in the Final
   1) Go over lecture slides again and make sure you understand
   2) Read the text book
   3) Ask questions on Piazza; discuss with your peers, TA and instructor
   4) Work on the problems (in sides, home works, exercises) WITHOUT looking at the solution; refer to the solution only when you are stuck
   5) For natural deduction: Writing proofs from both sides helps – write down the premises or assumptions first, followed by the (intermediate) conclusion, and then fill in the steps in between.
   6) Practice, practice and practice!

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 Section 1: Syntax and Natural Deduction -------------------

2. Syntax of propositional logic
   1) Definition of declarative sentences
   2) Turning declarative sentences into formulas: atomic sentences, connectives, composite formulas
   3) Recursive definition of propositional formula
   4) Well-formed formulas and how to identify whether a formula is well-formed
   5) Uniqueness of parsing; conditions for unique parsing; connective priority.
   6) Parse tree; how to construct parse trees

3. Propositional Logic: Proof Theory
   1) Understand the structure of logic proof; understand how to construct logic proof from English sentences
   2) Understand proof rules and how they are used in everyday life
   3) Understand how to apply proof rules
   4) Know how to write logic proofs
   5) Understand derived rules and how they can be derived from basic rules
   6) Understand the scope (when using assumption) in proof
   7) Understand provably equivalence and bidirectional implication
   8) Understand how to apply logic rule to natural reasoning (similar to the proof showing that the square root of 2 is irrational)
4. Summary of logic rules discussed in class:

The basic rules of natural deduction:

<table>
<thead>
<tr>
<th></th>
<th>introduction</th>
<th>elimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\land$</td>
<td>$\frac{\phi \ \psi}{\phi \land \psi}$</td>
<td>$\frac{\phi \land \psi}{\phi}$ $\land e_1$</td>
</tr>
<tr>
<td>$\lor$</td>
<td>$\frac{\phi}{\phi \lor \psi}$ $\lor i_1$</td>
<td>$\frac{\psi}{\phi \lor \psi}$ $\lor i_2$</td>
</tr>
<tr>
<td>$\rightarrow$</td>
<td>$\frac{\phi \rightarrow \psi}{\phi}$ $\rightarrow i$</td>
<td>$\frac{\phi}{\phi \rightarrow \psi}$ $\rightarrow e$</td>
</tr>
<tr>
<td>$\neg$</td>
<td>$\frac{\neg \phi}{\bot}$ $\neg i$</td>
<td>$\frac{\phi}{\neg \phi}$ $\neg e$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Some useful derived rules:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{\phi \rightarrow \psi}{\psi}$ $\rightarrow l$</td>
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<tr>
<td></td>
<td></td>
<td>$\frac{\phi}{\neg \phi}$ $\neg l$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{\bot}{\phi}$ $\bot r$</td>
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</tbody>
</table>

5. Proof strategy:

<table>
<thead>
<tr>
<th>If your goal is to prove</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F \rightarrow G$</td>
<td>Assume $F$ and derive $G$; use $(\rightarrow l)$</td>
</tr>
<tr>
<td>$FA \land G$</td>
<td>Derive each of $F$ and $G$; use $(\land l)$</td>
</tr>
<tr>
<td>$F \lor G$</td>
<td>Try to derive $F$ or $G$ as a subgoal; use $(\lor l)$ (limited)</td>
</tr>
<tr>
<td>$\neg F$</td>
<td>Assume $F$ and derive $\bot$; use $(\neg l)$</td>
</tr>
</tbody>
</table>
6. Additional exercise problems for logic proof:

\[ p \land q \vdash q \land p \]
\[ (p \land q) \land r \vdash p \land (q \land r) \]
\[ p \rightarrow (p \rightarrow q), p \vdash q \]
\[ q \rightarrow (p \rightarrow r), \neg r, q \vdash \neg p \]
\[ \vdash (p \land q) \rightarrow p \]
\[ p \vdash q \rightarrow (p \land q) \]
\[ p \vdash (p \rightarrow q) \rightarrow q \]
\[ (p \rightarrow r) \land (q \rightarrow r) \vdash p \land q \rightarrow r \]
\[ q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r) \]
\[ p \rightarrow (q \rightarrow r), p \rightarrow q \vdash p \rightarrow r \]
\[ p \rightarrow q, r \rightarrow s \vdash p \lor r \rightarrow q \lor s \]
\[ p \lor q \vdash (p \lor q) \land r \]
\[ (p \lor (q \rightarrow p)) \land q \vdash p \]
\[ p \rightarrow q, r \rightarrow s \vdash p \land r \rightarrow q \land s \]
\[ p \rightarrow q \vdash ((p \land q) \rightarrow p) \land (p \rightarrow (p \land q)) \]
\[ \vdash q \rightarrow (p \rightarrow (p \rightarrow (q \rightarrow p))) \]
\[ p \rightarrow q \land r \vdash (p \rightarrow q) \land (p \rightarrow r) \]
\[ (p \rightarrow q) \land (p \rightarrow r) \vdash p \rightarrow q \land r \]
\[ \vdash (p \rightarrow q) \rightarrow ((r \rightarrow s) \rightarrow (p \land r \rightarrow q \land s)) \]

-------------------- Section 2: Semantics of Propositional Logic --------------------

7. Understand the precise definitions related to the semantics of propositional logic; should be able to apply them to examples and to proofs:
1) Understand interpretations
2) Understand truth values of a formula assigned by an interpretation (valuation)
3) Understand how to create truth tables to evaluate formulas
4) Satisfaction: does this interpretation satisfy the formula?
5) Tautologies: is this formula a tautology? How do you check? How to check that a formula is not a tautology
6) Equivalent formulas: are these formulas equivalent to each other? How do you check?
7) How do you simplify formulas using the equivalence relationship?
8) Satisfiability: is this set of formulas satisfiable?
9) Understand the soundness and completeness of natural deduction. Understand the relationship between natural deduction and its semantic meaning.
10) Understand how and when to apply the soundness and completeness of natural deduction to prove or disprove conclusions.

----------------------------- Section 3: Mathematical Induction -----------------------------

8. Understand the idea of mathematical induction and know how to apply them to prove conclusions
   1) What is the base case; why it is important; what if you cannot prove the base case
   2) What is the inductive step; what is inductive hypothesis;
   3) Understand what we are aiming to prove in the inductive step
   4) Understand intuitively why mathematical induction produces a valid proof
   5) Understand the differences between mathematical induction (weak induction) and course of values induction (strong induction)
   6) Understand how to apply mathematical induction to construct proofs
   7) Be able to find out and explain when and where a mathematical induction is applied incorrectly

----------------------------- Section 4: GNU Prolog -----------------------------

9. Understand Prolog and how to use it to program
   1. Understand facts and rules; syntax to create facts (with or without arguments) and rules.
   2. Understand variables in prolog and how they are used
   3. Understand list and list operations
   4. Understand unification
   5. Understand how to create rules to implement basic functionalities, such as ‘negation’ and etc.
   6. Understand the cut operator
   7. Understand basic prolog operators, such as ->, =, is, \+ and etc.
   8. Understand depth-first backtracking search
   9. Understand the recursive nature of prolog
  10. Understand how to turn sentences into prolog statements and use it to solve problems
  11. Understand how to use prolog to solve puzzles, such as 8-queen, Hanoi tower puzzle and etc.
  12. Know how to create/complete programs to solve a problem
  13. Additional exercises:
      https://www.doc.gold.ac.uk/~mas02gw/prolog_tutorial/prologpages/
Section 5: Syntax of Predicate Logic

10 Understand the syntax of predicate logic

1. Understand what are terms and formulates in predicate logic
2. Understand the differences between propositional and predicate logic
3. Know how to create parse trees in predicate logic
4. Understand the scope of variables
5. What are bound and free variables
6. Understand substitutions
7. Understand how to turn natural language statements into predicate formulas
8. Additional exercises:

2. Use the predicate specifications

\[ B(x, y) : \ x \text{ beats } y \]
\[ F(x) : \ x \text{ is an (American) football team} \]
\[ Q(x, y) : \ x \text{ is quarterback of } y \]
\[ L(x, y) : \ x \text{ loses to } y \]

and the constant symbols

\[ c : \text{ Wildcats} \]
\[ j : \text{ Jayhawks} \]

to translate the following into predicate logic.

(a) Every football team has a quarterback.
(b) If the Jayhawks beat the Wildcats, then the Jayhawks do not lose to every football team.
(c) The Wildcats beat some team, which beat the Jayhawks.
4. Let $F(x, y)$ mean that $x$ is the father of $y$; $M(x, y)$ denotes $x$ is the mother of $y$. Similarly, $H(x, y)$, $S(x, y)$, and $B(x, y)$ say that $x$ is the husband/sister/brother of $y$, respectively. You may also use constants to denote individuals, like ‘Ed’ and ‘Patsy.’ However, you are not allowed to use any predicate symbols other than the above to translate the following sentences into predicate logic:
   (a) Everybody has a mother.
   (b) Everybody has a father and a mother.
   (c) Whoever has a mother has a father.
   (d) Ed is a grandfather.
   (e) All fathers are parents.
   (f) All husbands are spouses.
   (g) No uncle is an aunt.
   (h) All brothers are siblings.
   (i) Nobody’s grandmother is anybody’s father.
   (j) Ed and Patsy are husband and wife.
   (k) Carl is Monique’s brother-in-law.

4. Let $\phi$ be $\exists x (P(y, z) \land (\forall y (\neg Q(y, x) \lor P(y, z))))$, where $P$ and $Q$ are predicate symbols with two arguments.
   * (a) Draw the parse tree of $\phi$.
   * (b) Identify all bound and free variable leaves in $\phi$.
   (c) Is there a variable in $\phi$ which has free and bound occurrences?
   * (d) Consider the terms $w$ ($w$ is a variable), $f(x)$ and $g(y, z)$, where $f$ and $g$ are function symbols with arity 1 and 2, respectively.
     i. Compute $\phi[w/x], \phi[w/y], \phi[f(x)/y]$ and $\phi[g(y, z)/z]$.
     ii. Which of $w$, $f(x)$ and $g(y, z)$ are free for $x$ in $\phi$?
     iii. Which of $w$, $f(x)$ and $g(y, z)$ are free for $y$ in $\phi$?
   (e) What is the scope of $\exists x$ in $\phi$?
   * (f) Suppose that we change $\phi$ to $\exists x (P(y, z) \land (\forall x (\neg Q(x, x) \lor P(x, z))))$. What is the scope of $\exists x$ now?

----------------------- Section 6: Proof Theory of Predicate Logic -----------------------

11 Understand the syntax of predicate logic

1. Understand the differences between natural deduction in propositional and predicate logic
2. Understand the additional rules in predicate logic
3. Understand how to derive predicate logic proofs
4. Additional predicate logic proof exercises:
1. (a) $\neg \forall x \phi \vdash \exists x \neg \phi$
   (b) $\neg \exists x \phi \vdash \forall x \neg \phi$.

2. Assuming that $x$ is not free in $\psi$:
   
   (a) $\forall x \phi \land \psi \vdash \forall x (\phi \land \psi)$
   (b) $\forall x \phi \lor \psi \vdash \forall x (\phi \lor \psi)$
   (c) $\exists x \phi \land \psi \vdash \exists x (\phi \land \psi)$
   (d) $\exists x \phi \lor \psi \vdash \exists x (\phi \lor \psi)$
   (e) $\forall x (\psi \rightarrow \phi) \vdash \psi \rightarrow \forall x \phi$
   (f) $\exists x (\phi \rightarrow \psi) \vdash \forall x \phi \rightarrow \psi$
   (g) $\forall x (\phi \rightarrow \psi) \vdash \exists x \phi \rightarrow \psi$
   (h) $\exists x (\psi \rightarrow \phi) \vdash \psi \rightarrow \exists x \phi$.

3. (a) $\forall x \phi \land \forall x \psi \vdash \forall x (\phi \land \psi)$
   (b) $\exists x \phi \lor \exists x \psi \vdash \exists x (\phi \lor \psi)$.

4. (a) $\forall x \forall y \phi \vdash \forall y \forall x \phi$
   (b) $\exists x \exists y \phi \vdash \exists y \exists x \phi$. 